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# GEOMETRY OF ENGINEERING DRAWING

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# GEOMETRY OF ENGINEERING DRAWING

## DESCRIPTIVE GEOMETRY BY THE DIRECT METHOD

BY

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# PREFACE

Descriptive Geometry is the outstanding subject for training the imagination to visualize structures and objects that are being designed. It also gives training in analyzing structures, and in representing them in drawings. The increasing complexity and refinement in present-day design require increasing ability to visualize thoroughly and to represent accurately.

In this textbook an attempt has been made to approach the subject in a direct way. The method here adopted is not an experiment; it has been used for the last six years in teaching this subject to more than eleven hundred students.

In agreement with engineering practice, the planes of projection, quadrants, the ground line, traces of planes, and projections are not used. Attention is directed to the object itself. By using this method, the student loses none of the mental training in imagining and thinking that is to be derived from studying the subject by the older method. The direct method fosters direct habits of thought, and develops a facility in drawing that is usually beyond the ability of the student of the projection method.

The change in methods has suggested that a new name, more nearly descriptive of the subject, might be adopted. The name Geometry of Engineering Drawing, seems fitting. The subject as here described might be called Descriptive Geometry *by the Direct Method.*

The general plan used in this textbook is, first, to have the student visualize and draw simple concrete objects, so that he may become familiar with the vocabulary of the subject and with the methods of drawing principal, auxiliary, and oblique views. This is followed by an analysis of the elements of structures, a consideration of the geometrical relations of these elements, and the development of the principles of representing these relations in drawings. This plan is in agreement with modern methods of teaching. The order may be varied as desired.

The continuity of the text is maintained by separating the problems from the text. The 1160 problems are assembled, in classified groups, in a problem section following the text. Each group of problems is referred to in the logical place in the text. Any desired group may be readily found in the problem section by means of the group number that is placed on the upper corners of the pages of the problem section.

The large number of problems offers a considerable variety of choice, and is sufficient for a number of years without duplication. Many practical problems are included. The problems are stated in a variety of ways in order to develop the ingenuity of the student and to give him training in following specifications. The student must read accurately, and visualize what he reads.

A simple coordinate system, together with coordinately ruled problem paper, is adopted to provide means for quickly and accurately locating the given data of problems. A coordinately ruled blackboard in the classroom is a great convenience.

The coordinate problem paper permits the accurate solution of problems, either with or without the use of the drawing board and tee-square. The necessary tools—triangles, pencil, and compass—can readily be carried by the student, so that he can solve problems anywhere. In case unruled paper is used for solving problems, it will be necessary for the student also to use a drawing board, a tee-square, and a scale. The printed coordinate paper is recommended, since it saves time both in laying out problems and in drawing problems.

Following the preface is a reproduction, to a reduced scale, of the letter-size problem sheet. The actual size of the problem space is 8 inches square. Experience indicates that this is satisfactory in size and shape. Samples of printed problem sheets may be had from the publishers, who can supply this paper in any quantity.

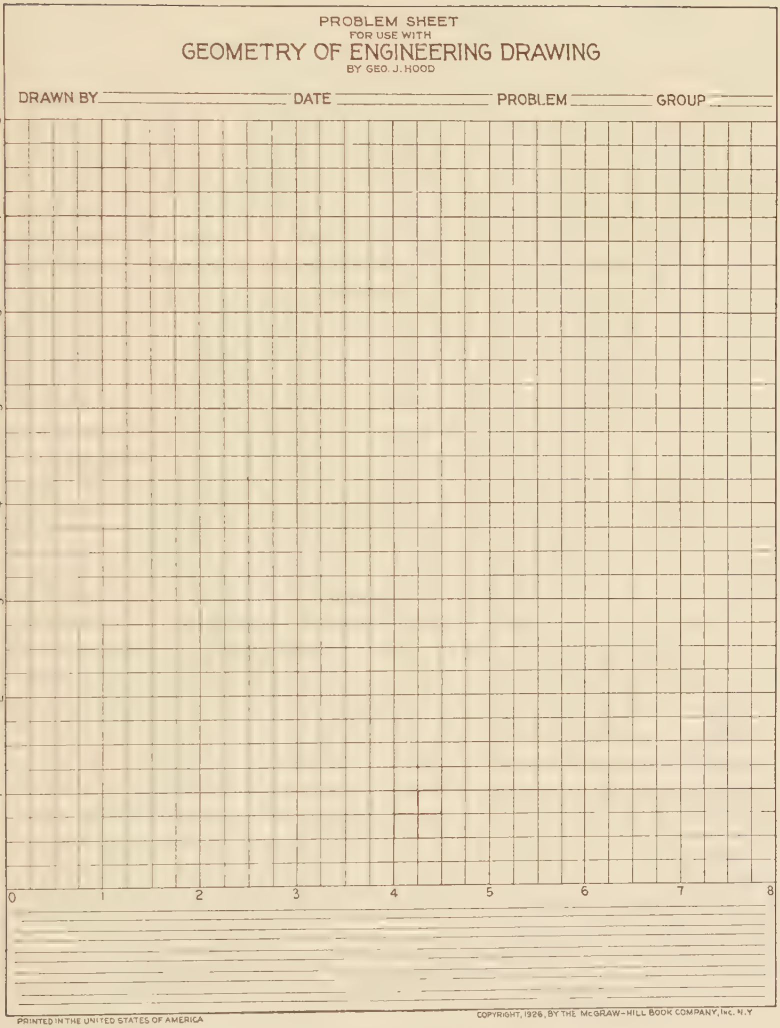
All problems are designed to be solved within the limits of the problem space. An attempt has been made to state each problem so that but one solution is possible. When drawn on coordinate paper, the accuracy of the solution may be gaged within very close limits. This makes much easier the work of the teacher who checks the solutions of the problems. A convenient key for correcting problems is printed inside the back cover of this book. It is recommended that problems be drawn in pencil only. This gives valuable training in the proper use of the lead pencil.

Grateful appreciation is expressed to Prof. Thomas E. French for helpful advice, to Prof. Richard L. Grider for criticizing the chapter on mining problems and for other suggestions, and to Prof. Frank E. Jones for solving all of the problems as a check on the original solutions.

Criticisms, corrections, and suggestions will be greatly appreciated by the author.

George J. Hood.

Lawrence, Kansas,
*May*, 1926.



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FOR USE WITH
GEOMETRY OF ENGINEERING DRAWING
BY GEO. J. HOOD

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Problem Sheet

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# GEOMETRY OF ENGINEERING DRAWING

## DESCRIPTIVE GEOMETRY BY THE DIRECT METHOD

## CHAPTER I

## INTRODUCTORY

**1.** The engineer adapts the materials and controls the forces of nature for the convenience and comfort of mankind. He does this by designing structures of many kinds: machines, buildings, bridges, factories, tools, tunnels, roads, railways, ships, instruments, apparatus, etc.

When designing a structure the engineer first visualizes the appearance of the proposed structure as clearly as he is able. He then describes it, by means of words and drawings, to the draftsman who makes the assembly drawings and the detail drawings of the structure. The assembly and detail drawings, usually in the form of blue prints, convey to the workman the information that he needs to make the various parts of the structure and to assemble it as a whole.

The engineer thus conveys his ideas to others by means of a spoken and written language and by means of drawings. He should have adequate training in both of these mediums of expression. When describing even a simple structure, the spoken and written language soon becomes inadequate and resort must be had to drawings. Drawing is a language much used by the engineer.

**2.** The Geometry of Engineering Drawing has for its purpose the exact description of the shape of structures by means of drawings.

Engineering Drawing has for its purpose the exact description of both the shape and the size of structures. Engineering Drawing thus makes use of the methods of the Geometry of Engineering Drawing and, in addition, is concerned with placing dimensions, explanatory notes, and symbols on drawings.

The student of engineering requires training both in visualizing the appearance of structures and in describing the exact shape of structures by means of drawings. Such training may be acquired by studying the geometry of engineering drawing, and by solving problems. The student of this subject obtains very valuable training in thinking in terms of three dimensions; an ability which is much rarer than the ability to think in two dimensions.

This textbook should be of aid to the practicing engineer in solving many practical problems connected with the geometry of structures and their representation in drawings.

The geometry of engineering drawing has many practical applications. It is used: in describing structures of three dimensions, in solving problems related to the exact representation of structures, in analyzing structures into their elements, in drawing structures of unusual shape, in showing structures in oblique positions, in determining actual lengths of the parts of structures, in determining the geometrical relations between the parts of structures, in developing patterns for sheet-metal structures, in determining the lines of intersection between surfaces, in drawing unusual surfaces, in making perspective drawings, in finding shades and shadows, in solving mining problems.

**3. To the Student.**—Many students find the geometry of engineering drawing very difficult during the first few weeks. It has, however, been a very common experience that those who have continued to do their best suddenly have a vision of the subject, which from that time on makes the study relatively easy for them. Such a vision does not come to those who do not work; neither does it come to those who give up too soon.

The solution of a problem in the geometry of engineering drawing often requires a considerable amount of thinking, of ingenuity, and of resourcefulness. The more complicated problems require continued mental effort, involving the application of general principles and a consideration of many small details.

Memorizing the appearance of the solution of a problem gives little aid in solving similar problems. To solve a problem readily it is necessary to visualize the object that is to be drawn, and to understand the elementary principles that underlie the solution. Each problem should be analyzed into its simple elements. The more complex problems are but combinations of simpler problems. When studying the subject, the illustrations of the text should receive as much attention as do the words of the text.

The student should not handicap himself by inattention in class, by getting behind in his work, by doing poor work, or by letting others think for him. He should develop his own mental ability by solving his own problems. He should go to others for aid only after he has exhausted his own mental resources. He should learn to think for the pleasure it will give him and for the financial returns it will bring him. The successful engineer must think out the solutions of his own problems.

## CHAPTER II

## PRINCIPAL VIEWS

**4.** The great majority of structures built by man are essentially rectangular, and are usually in such a position that the edges and faces are vertical and horizontal. In this general classification are included right cylindrical, conical, pyramidal, and similar structures in which the axis is perpendicular to the base. Structures not rectangular and in oblique positions attract special attention because they are unusual.

The custom of making structures rectangular is probably due to the ease and economy of building this type of structure. Also, the geometry, and hence the visualization and planning of rectangular structures, is relatively simple.

**5. Dimensions of Structures.**—All structures occupy space and have three dimensions. Rectangular structures have the three dimensions of height, width, and depth. Width is measured in a horizontal direction from right to left. Height is measured in a vertical direction, or up and down. Depth, as here used, is measured in a horizontal direction from front to back. The dimensions of width, height, and depth may be called the rectangular dimensions of a structure. Each of these dimensions is measured in a direction perpendicular to the other two.

It should be understood that the terms width, height, and depth are general terms, used to indicate dimensions taken in certain directions. It is not intended that they shall be used to indicate only the extreme dimensions of the object.

The generally rectangular nature of structures and the relative simplicity of considering the geometry of space as rectangular permit the use of a simple and exact method of representing all structures by means of views, as described below.

**6. Method of Describing Structures.**—The fundamental problem of the geometry of engineering drawing is to describe exactly the shape of structures of three dimensions by means of drawings that are made on a flat surface having but two dimensions. This purpose is accomplished by drawing front, top, and side views of the structure. These views, together with the seldom-used bottom and rear views, may be called the "principal views." Each one of the principal views can show but two of the three rectangular dimensions of a structure. At least two views will be needed to show all dimensions of the structure. Frequently other views, known as auxiliary views and oblique views, are necessary to describe completely a structure or to solve some problem relating to it.

While it is the purpose of the geometry of engineering drawing to describe exactly the shape of structures, it is impossible to do this without considering and measuring the dimensions of structures. The actual placing of dimensions on drawings, however, is a function of engineering drawing. In the illustrations of this text a few dimensions have been added, when necessary to explain the method of representing the shape of a structure.

**7. Example.**—Take as an example of a simple rectangular structure or object, the slide block shown in Fig. 1. The block is here illustrated by means of a single pictorial view that shows the three dimensions of the block. Pictorial views, however, do not show the exact geometrical relations between the parts of structures. Such views are relatively difficult to draw, and are used mainly for purposes of illustration, to show the general appearance of the structure.

In Fig. 1 the dimensions of height, width, and depth of the block are indicated. The slide block is assumed to be in its normal position, right side up, and with the edges and faces vertical and horizontal. The face A B C D is taken as the front face.

**8. Front View.**—To see the appearance of the block from the front, consider the block as stationary and imagine that the observer has taken a position directly in front of the block so that he can look squarely at it in the direction of the arrow marked "front," Fig. 1. This arrow is perpendicular to the front face. Mathematically speaking, the observer is supposed to be an infinite distance from the block, so that the rays of light, leading from the corners of the block to his eyes, are parallel, as illustrated by the light-weight lines parallel to the front arrow. Practical considerations make it easier to imagine the observer within a reasonable seeing distance of the block. It is always necessary, however, to imagine the rays as parallel. The point from which an object is viewed is called the "station point" or "point of view."

From his position in front of the block, the observer will be able to see the height and width of the block. In other words, he can see how far one corner of the block is above or below another corner, and also how far one corner is to the right or left of another corner; but from the front position he cannot tell how far one corner is in front of or behind another corner.

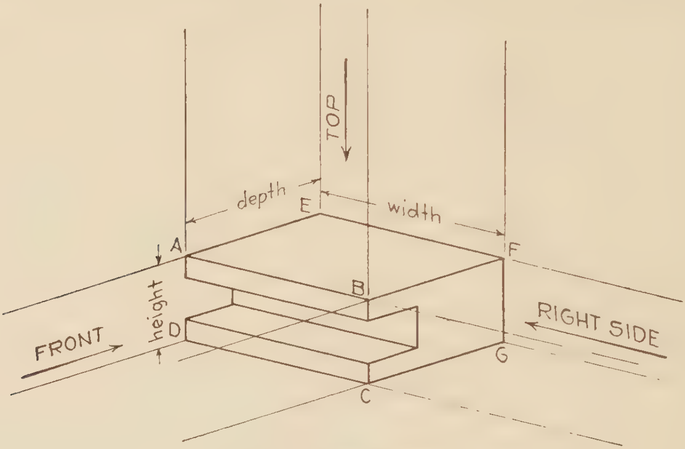


Fig. 1.—Slide block.

The front view of the slide block, sometimes called the "front elevation," is shown in Fig. 2. The reader of this view should imagine that he is directly in front of the object itself, and that he sees the entire object and not merely the front face of the object. The corners A, E, B, and F are all seen at the same upper level. Not one of these points is either above or below the other three. The corners D, H, C, and G are at a lower level. The distance between the upper and lower levels is equal to the height of the structure. Also, the front view shows that the points B, F, C, and G are to the right of the corners A, E, D, and

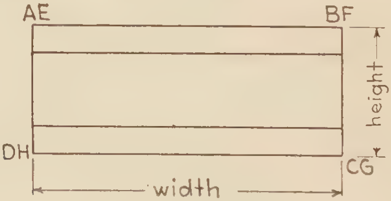


Fig. 2.—Front view.

H a distance equal to the width of the block. The groove is shown, but the corners of the groove have not been lettered. The reader of the front view sees the edge view of the horizontal plane faces of the block, and also of the vertical end faces of the block.

The front views of the points A and E coincide. As a matter of convenience in reading a drawing, points that coincide on a drawing should be lettered in the order of their distance from the observer. The point A is nearer to the observer than is E; therefore, in the front view, these should be lettered A E, but not E A.

In the front view the block appears to be rectangular, but it is impossible to tell from this view how far E is behind A and whether or not A is directly above D. Only two of the three rectangular dimensions of the block can be seen in this view. The depth of a structure, or distance from front to back, cannot be measured in the front view. In order to show the depth, it is necessary to draw a top or side view.

**9. Top View.**—To see the top view of the slide block the observer should imagine himself directly above the block and looking down at the block in a vertical direction indicated by the arrow marked "top," in Fig. 1. From this position he will see the top view that is shown in Fig. 3. The top view is always drawn with the front face toward the front. The front face should never be located in the rear. The top view is sometimes called the "plan."

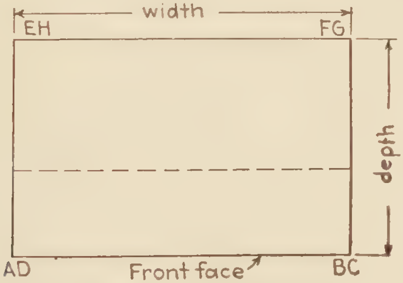


Fig. 3.—Top view.

The reader of a drawing, when looking at the top view, sees the width and depth of the structure. The points A, D, B, and C are seen to be in the front face of the block and the points E, H, F, and

G in the rear face. At the front left corner of the block are lettered the points A and D. The order of the letters indicates that the point A is nearer to the observer than is D; that is, A is directly above D, but the exact vertical distance cannot be seen in this view. Like conditions prevail at the other corners. In the top view the reader sees the edge view of the vertical plane faces of the block. The top view, in common with the front view, indicates the width of the block.

The depth of the groove is indicated by means of the broken dash line, because the object is opaque and this line cannot be seen in the top view. The shape of the groove, whether round or square bottomed, cannot be seen in either the front or the top view. A side view is necessary to show this.

**10. Side View.**—The side view may be observed either from the right side or the left side of the structure. The right-side view is more generally used. To see the right-side view of the slide block, imagine that the observer has taken his position to the right of the block and that he is looking in the direction of the arrow marked "right side," in Fig. 1. From this position he will be able to see the height and the depth of the block, and the front face of the block will be to his left.

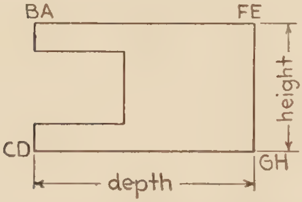


Fig. 4.—Right side view.

The right-side view, sometimes called the side elevation, is shown in Fig. 4. In this side view the points B, A, C, and D appear in the front face of the block, and the points F, E, G, and H in the rear face. This agrees with what is seen in the top view. The point B is nearer to the observer than is the point A. The points B, A, F, and E are in the top face, and the points C, D, G, and H are in the bottom face of the block. These last two statements agree with what was observed in the front view. The height as seen in the side view also agrees with the height as seen in the front view, and the depth agrees with the depth as seen in the top view. The side view shows the edge view of the horizontal faces and of the vertical front and rear faces. The shape of the groove is seen to be rectangular, and the exact depth and height of the groove are shown.

**11. Grouping of Views.**—Custom and convenience in making and reading drawings require that the different views of a structure be grouped in a single drawing, according to a definite, normal and natural arrangement that is described below. The grouping described is that most commonly used in this country. Some other countries use different groupings, as explained in Chapter XV.

In Fig. 5 the correct grouping of the front, top, and side views of the slide block is shown. The top view is placed directly above the front view, and the right-side view is placed to the right of the front view. The light-weight lines connecting the front and top views and also those connecting the front and side views are called "rays," and represent the rays of light or lines of sight along which the observer looks when he sees the different views of the structure. The rays lead from the corners or points of the structure to the eye of the observer, as was explained in connection with Fig. 1.

**12. Adjacent Views.**—Two views connected by rays are called "adjacent views." In Fig. 5 the front and top views are adjacent each to the other. Also the front and side views are adjacent each to the other. Each view should be imagined as representing the structure itself.

**13. Rule for Grouping Views.**—A rule for correctly placing each view in its normal and natural relation to its adjacent view may be stated as follows:

**Place each view of every point on the ray in which it is seen.**

For example, in Fig. 5 the front view may be taken as representing the object itself. To see the top view of the object, the observer takes his place above the object as represented by the front view. He sees the different corners or points of the object with the aid of the vertical parallel rays that lead upward from the points of the object to his eye. The top view of the point A is seen in the vertical ray leading upward from the point A as represented in the front view. The top view of A is then naturally placed on this ray. The top view of each point is similarly placed on the ray in which it is seen. These vertical rays do not extend below the object. Hence, in every case the top view of a point will be directly above its front view. Care must be taken not to reverse the top view. The front face is always in front.

In turn, the top view may be taken as representing the object. To see the front view, the observer takes his place directly in front of the object that is now represented by the top view. He sees the different corners of the object by means of the parallel rays that lead in a horizontal direction forward from the object to his eye. The front view of each point is then placed on the ray in which it is seen. These rays do not extend behind the object. Hence, in every case the front view of a point will be directly in front of its top view.

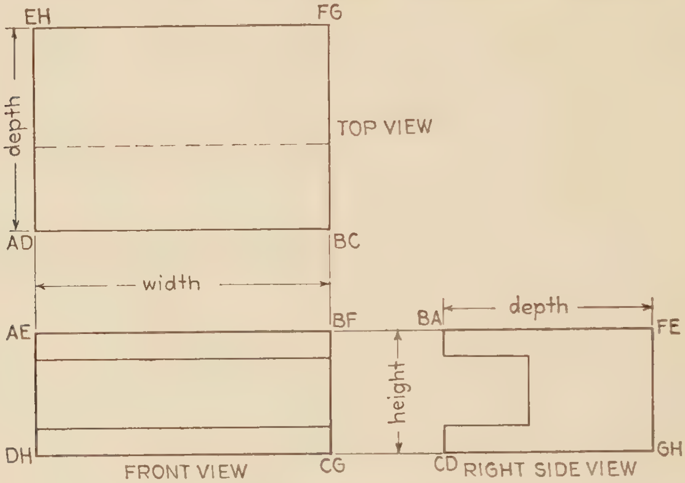


Fig. 5.—Correct grouping of views.

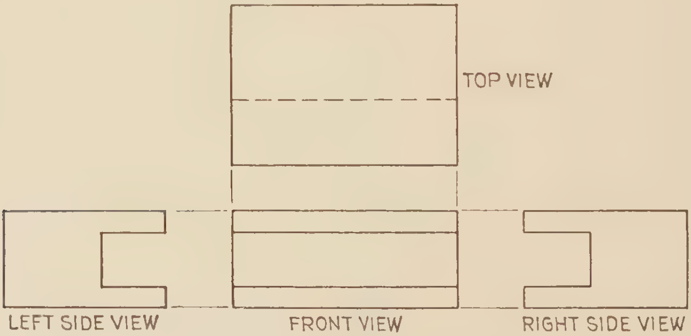


Fig. 6.—Grouping of side views.

The rays that enable the observer to see the right-side view of the block extend from the block as represented by the front view, to the right. Hence, the right-side view must be placed to the right of the front view. The right-side view of the block in Fig. 5 is placed to the right of, and at the same level with the front view, because in this location each point of the view is placed on the ray in which it is seen. Care must be taken that the right-side view of the block is not reversed. The observer sees this view with the front face of the block to his left, and the right-side view must be drawn in this position. If it were desired to draw the left-side view of this block, the left view would be placed to the left of the front view. The view from the left will be in a reversed position to the view from the right, as is shown in Fig. 6.

A right-side view of the slide block might be placed, as shown in Fig. 7, to the right of the top view, in agreement with the rule for locating the views of points. This arrangement is sometimes a convenience but is not generally used, because in this location the side view is not right side up.

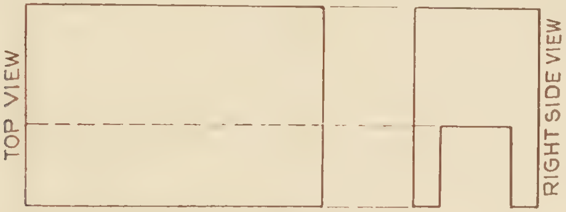


Fig. 7.—Top and side view.

The rule, requiring that each view of every point be placed on the ray in which it is seen, thus provides a simple means of determining a natural grouping of the views of a structure. Views grouped according to this rule are easily made and easily read.

**14. Complementary Relations of Views.**—Consider the rays that connect the adjacent front and top views of Fig. 6. When drawing or reading the top view, the rays are considered as vertical and leading upward from the object as represented by the front view; but when drawing or reading the front view, these rays are considered as being horizontal and extending forward from the object as represented in the top view.

In every view the correct distance relations between points and the correct measurements of width, height, and depth must be maintained. The rays connecting any two adjacent principal views automatically maintain the equality of one of the three rectangular dimensions of those views. If any two views disagree, at least one of the views does not tell a true story.

**15. Drawing Principal Views.**—Views are drawn by locating the different corners or points of the structure, and then the lines of the structure are drawn. The various points of any single view may be located in any convenient order.

Front views are drawn by locating each point of the structure at its proper distance above or below, and to the right of or to the left of each of the other points of the structure. Top views are drawn by locating each point of the structure at its proper distance to the right of or to the left of, and in front of or behind each of the other points of the structure. Side views are drawn by locating each point of the structure at its proper distance above or below, and in front of or behind each of the other points of the structure.

The distance between two views is immaterial, except that for the sake of convenience in making and reading drawings the views should not be too far apart, and for the sake of clearness they should not overlap.

**16. Problems.**—The problems for this chapter are designed to give the student preliminary training: in visualizing objects of three dimensions; in thinking in terms of three dimensions; in understanding the language of drawing; in correctly grouping different views; in measuring width, height, and depth in the proper view; in making neat drawings; in reading drawings; and in following specifications. The experience gained in solving these problems will aid the student in understanding the methods used in solving the more difficult problems.

All problems are assembled in groups in the back part of the book. Before attempting to solve problems of any group, any general statement preceding the group should be read. The general instructions for drawing problems, pages 185 and 186, should be observed for all problems. The coordinate system for locating points is explained on page 193. The problems for Chapter II are grouped as follows:

**17. Preliminary Problems.**

Group 1. Drawing Simple Objects from Description.
Group 2. Drawing Objects by Discarding Specified Parts.
Group 3. Drawing Objects in Specified Directions.

**18. Preliminary Coordinate Problems.**

Group 4. Locating Data by Coordinates.
Group 5. Locating Points by Measurements.

## CHAPTER III

## AUXILIARY VIEWS

**19.** In addition to drawing the principal views of a structure, it is frequently necessary to draw other views, called "auxiliary views," in order: to represent structures having oblique faces, to represent structures in oblique positions, and to solve many of the problems connected with the accurate representation of structures.

**20. Summary.**—The front, top, right-side, and left-side views, together with the seldom-used bottom and rear views, are called the principal views. Each of these views shows two of the three rectangular dimensions of the object. The observer sees the principal views from six different station points, and each station point requires the use of its own group of parallel rays. The six groups of parallel rays extend in three mutually perpendicular directions. That is, each of these directions is perpendicular to the other two.

**21. Auxiliary Views.**—An auxiliary view of an object is seen by an observer when he looks in a direction that is perpendicular to only one of the directions used in seeing the principal views.

Auxiliary views may be taken in a great many directions, limited only in that the direction must be perpendicular to one of the directions in which one of the principal views is taken. The principal views are limited in number to six, but auxiliary views are unlimited in number, and yet partly limited in direction. The particular direction in which an auxiliary view is taken is determined by the conditions of the problem that is under consideration. An auxiliary view can show but one of the three rectangular dimensions of an object.

Auxiliary views should not be confused with oblique views. The latter are taken in a direction that is not perpendicular to any of the directions in which the principal views are taken, and show none of the rectangular dimensions of the object.

Auxiliary views may be considered as being of three types, depending upon whether the direction in which the auxiliary view is taken is perpendicular to the rays used in seeing the front, top, or side view.

**22. Auxiliary Elevations.**—One type of auxiliary view may be explained with the aid of Fig. 8. This figure shows the front and top views of the slide block. The four long horizontal arrows, pointing at the block as seen in the top view, represent the directions in which four of the principal views are seen. The shorter horizontal arrows represent directions in which an observer may look in order to see one type of auxiliary view. Since these shorter arrows are horizontal, they are perpendicular to the vertical rays used in seeing the top view of the block. They are not, however, perpendicular to the rays used in seeing the front or side views. Other similar arrows may be drawn between those shown.

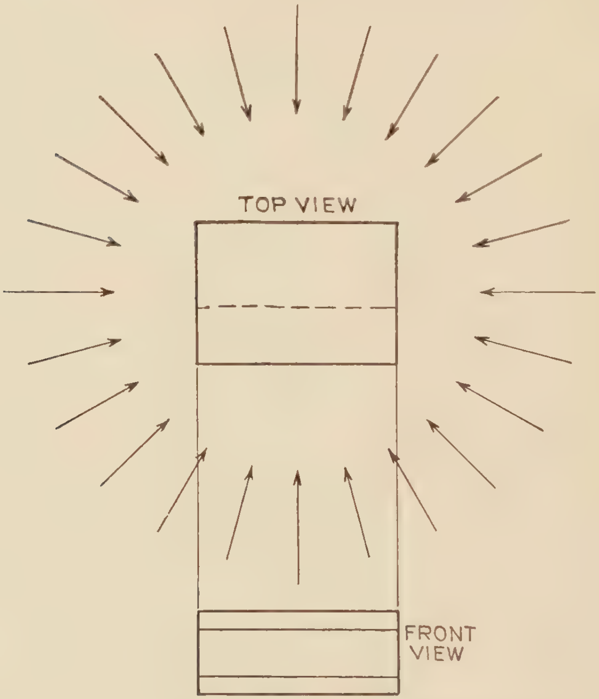


Fig. 8.—Directions in which auxiliary elevations are seen.

The observer may imagine that he walks around the block and in turn looks at it in each of the directions indicated by the shorter arrows. He looks at the block only in a horizontal direction, and from each station point he sees the height of the block. Since auxiliary views of this type are taken in a horizontal direction and show the height of the block, they may be called "auxiliary elevations." The front, rear, and side views, which are seen by looking in the directions of the longer arrows, are also elevations, but they are principal views and not auxiliary views. The particular direction in which any one auxiliary elevation is taken depends upon the requirements of the problem that is under consideration.

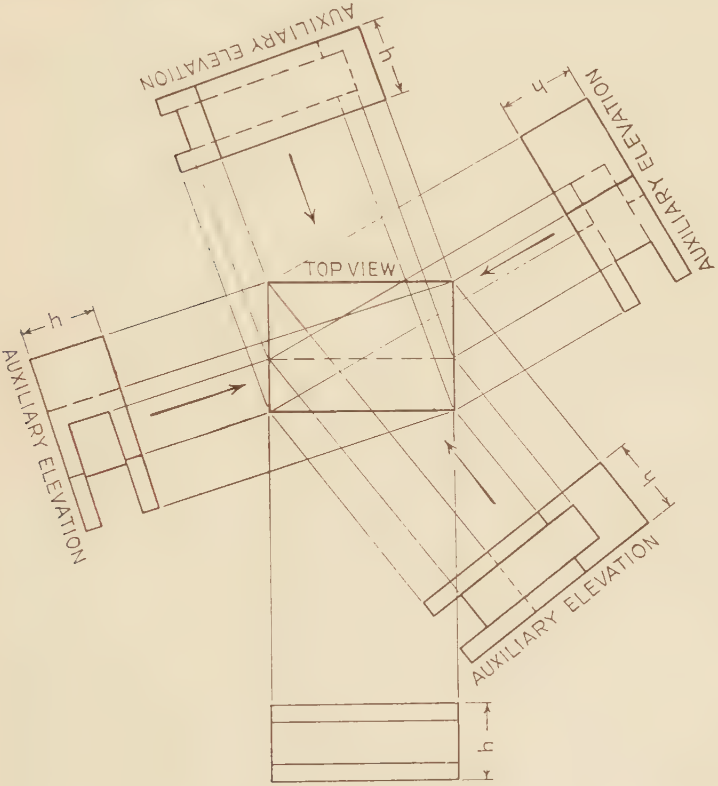


Fig. 9.—Auxiliary elevations.

In Fig. 9 are shown four auxiliary elevations of the slide block. Each auxiliary elevation is taken in a chosen direction and each shows the height of the block. An auxiliary elevation is always adjacent to the top view, because the direction in which it is taken can be seen only when looking downward at the object. Each point of the auxiliary elevation is placed on the ray in which it is seen. In every auxiliary elevation, the block is drawn right side up with its top face nearest the top view.

When studying any one of the auxiliary elevations of Fig. 9, it is well to turn the book around so that the particular auxiliary elevation that is being studied will be directly in front of the top view. In this position the auxiliary elevation is right side up, and may temporarily be considered as if it were a front view. In an auxiliary elevation no attempt should be made to measure the width or depth of the block.

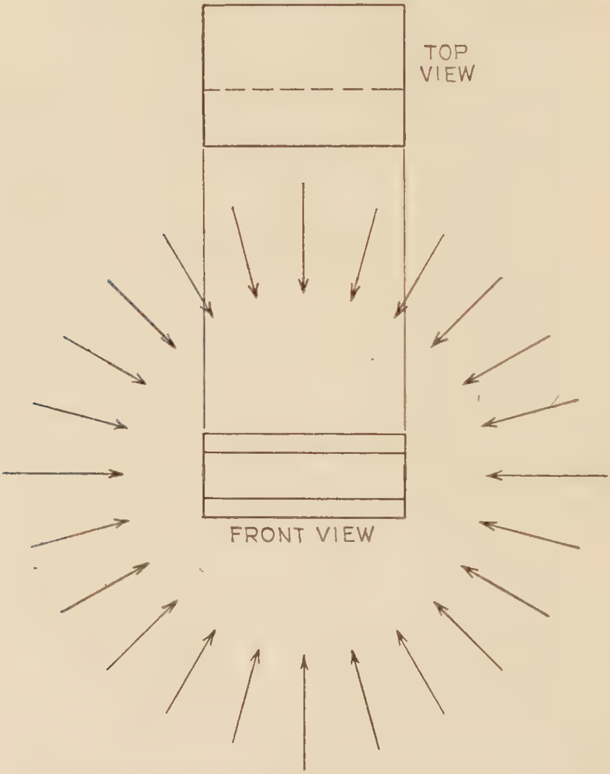


Fig. 10.—Directions in which right auxiliary and left auxiliary views are seen.

**23. Right- and Left-auxiliary Views.**—A second type of auxiliary view is explained with the aid of Fig. 10. The long arrows here indicate the directions in which the observer looks at the block when he sees the top, bottom, and side views. The shorter arrows are perpendicular only to the rays used in seeing the front view of the block. These arrows indicate directions in which the observer will see right- and left-auxiliary views of the block. Each of these auxiliary views will show the depth of the block. Since these auxiliary views are taken in an inclined direction from the right or from the left, they may be called "right-auxiliary" or "left-auxiliary views."

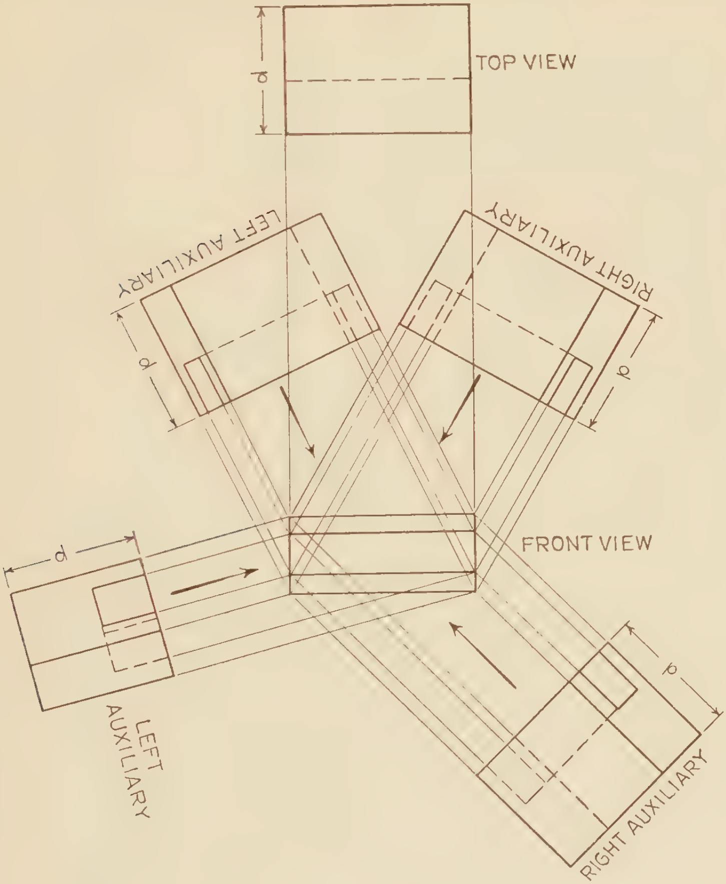


FIG. 11.—Right auxiliary and left auxiliary views.

Figure 11 shows two right-auxiliary and two left-auxiliary views. The depth of the block is shown in each of these views. Each right- or left-auxiliary view is adjacent to the front view,

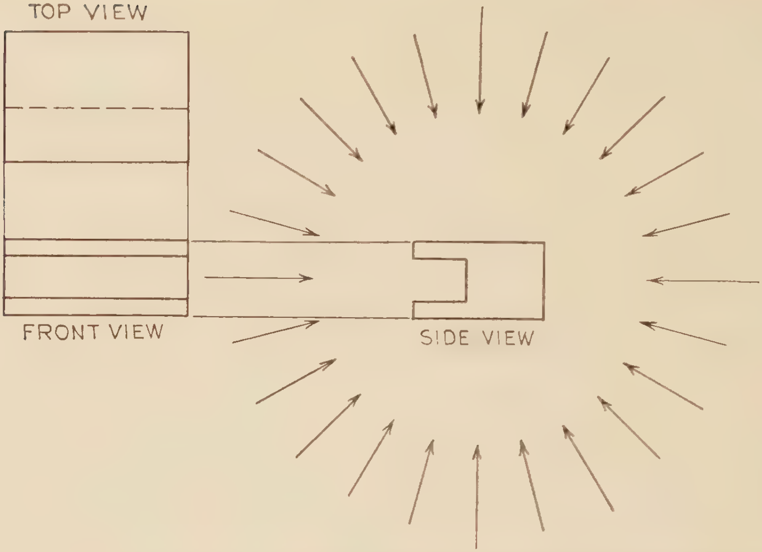


Fig. 12.—Directions in which front auxiliary and rear auxiliary views are seen.

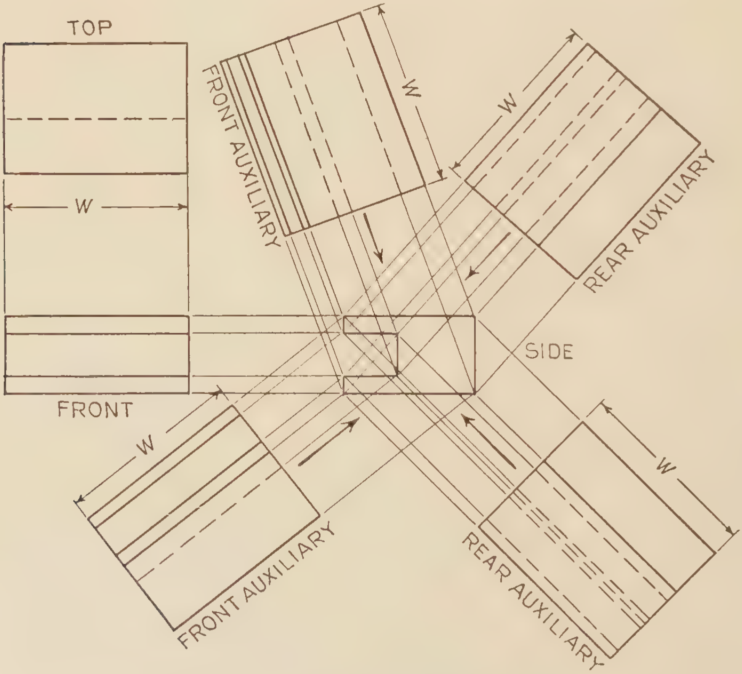


Fig. 13.—Front auxiliary and rear auxiliary views.

with its front face nearest the front view. As usual, each point of every view is placed on the ray in which it is seen.

**24. Front- and Rear-auxiliary Views.**—The third type of auxiliary view is explained with the aid of Fig. 12. The shorter arrows, perpendicular only to the rays used in seeing the side views, indicate various directions in which front- or rear-auxiliary views may be seen. Each view of this type will show the width of the block. Since these views are taken in an inclined direction from the front or rear, they may be called front-auxiliary or rear-auxiliary views.

In Fig. 13 are shown two front-auxiliary and two rear-auxiliary views of the slide block. Each of these views is adjacent to the side view, and each point is on its proper ray.

**25. Example of an Auxiliary View.**—As an example of one of the uses of auxiliary views, Fig. 14 shows the front and top views and an auxiliary elevation of an inclined brace block. The auxiliary view is necessary here in order to show the location and spacing of the three holes in the inclined face. This view is taken in a horizontal direction, perpendicular to the inclined face of the block. When drawing the auxiliary elevation, it is necessary to know the height of the brace block. This may be obtained from the front view.

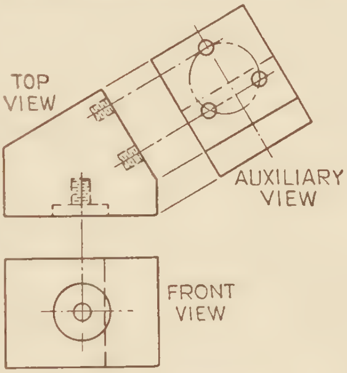


Fig. 14.—Brace block.

**26. Reading Drawings.**—To one who is trained in the art of reading drawings, the different views of a drawing should give a complete mental picture of the object represented. The reader of a drawing should first glance quickly from one view to another in order to obtain a general idea of the appearance of the object. He should then direct his principal attention to the view that best shows the characteristic shape of the object, but he should frequently refer to the other views for information not given in the most representative view.

The reader of a drawing should imagine that he is observing the object itself. Each view should appear not as a flat drawing, but as an object having three dimensions. The reader takes the place of the observer, and as the observer he must learn to imagine that he moves quickly from one station point to another as often as is necessary while he is reading the different views. When doing this he must not lose his sense of direction. From each different station point the reader should visualize the structure as clearly as possible.

In Fig. 15 many views of a right hexagonal pyramid are shown as an illustration of the correct method of grouping all possible types of views. In this figure, each view is correctly placed with reference to its adjacent view, in agreement with the rule that each view of a point must be placed on the ray in which it is seen. Corresponding corners bear similar letters in every view. As a matter of convenience, the dimensions of height, width, and depth of the pyramid are indicated in the views in which they appear.

The student should make a thorough study of Fig. 15. Such a study will give a clear knowledge of certain fundamentals that will be constantly needed in solving problems. A method of studying this figure is outlined in the following paragraphs.

The reader should first glance from one view to another, until he obtains some idea of the general shape of the object represented. Suppose, then, that in looking at Fig. 15, the reader first observes the top view. This might appear to be a view of a hexagon and its diagonals, but a glance at the front view will immediately show that the object has a considerable height, that it is broad at the base, that it comes to a point at the top, and that it is symmetrical. The reader of the drawing now has sufficient information to enable him to visualize the object as a pyramid having a horizontal hexagonal base with the vertex directly over the center of the base. In other words, the two views represent a right hexagonal pyramid standing on its base. The top view should now appear to the reader, not as a hexagon and its diagonals, but as a pyramid with the vertex pointing toward the reader, who is above the pyramid and is looking down at it, and the triangular faces of the pyramid should seem to slope from the vertex downward and outward.

When the observer looks at an object in the direction indicated by the rays which lead to either one of two adjacent views, he will see the object as shown in the other adjacent view. For example, the top view and the auxiliary elevation are adjacent views. When looking in the direction of the vertical rays which lead to the auxiliary elevation, the observer will see the pyramid as shown in the top view. Conversely, when the observer looks

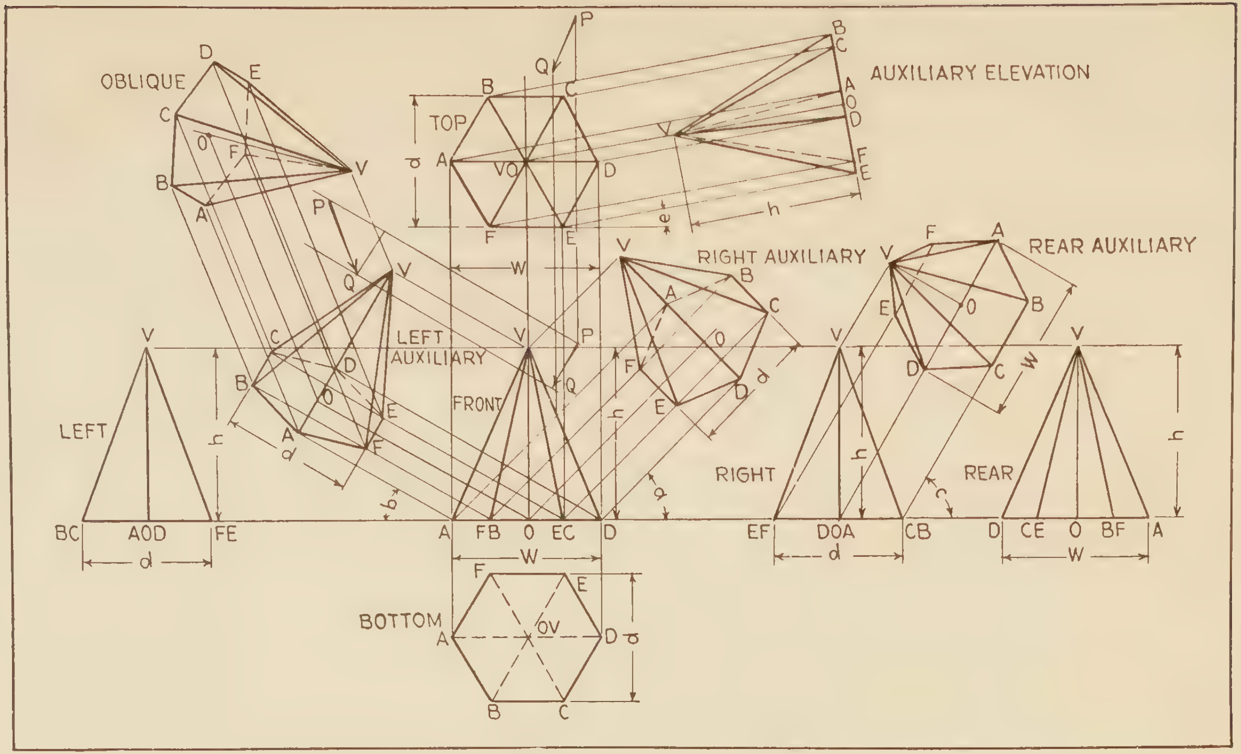


FIG. 15.—Pyramid. All types of views.

at the pyramid in the direction of the horizontal rays which lead to the pyramid as shown in the top view, he will see the pyramid as shown in the auxiliary elevation. Adjacent views are thus complementary to each other. An understanding of the above statement should greatly aid the reader of drawings.

The front and top views are sufficient to represent completely such a simple object as this pyramid, but various other views are given to illustrate the different types of views and to show the correct relation of one view to another.

**27. Reading Auxiliary Views.**—As the reader of the drawing studies each view of Fig. 15, he should assume that the pyramid is stationary and he should imagine that he himself has moved to a position from which he can observe the pyramid as it appears in the view he is studying. For example, when looking at the right-auxiliary view the reader of the drawing should imagine that he is directly to the right of the pyramid, above the level of the pyramid, and looking down at the pyramid in an inclined direction that makes the angle $a$ with the horizontal. As an aid in studying this view it may be well to turn the book so that the line V D of the right-auxiliary view is vertical. Turning the head will accomplish the same purpose. The name "right-auxiliary view" indicates a view taken from the right in a direction inclined to the horizontal.

To see the left-auxiliary and the right-auxiliary views, the observer must look at the object in slanting directions, inclined to the horizontal. The rays used in obtaining these views remain perpendicular to the rays used in seeing the front view. To see the rear-auxiliary view, the observer looks in a direction inclined to the horizontal, but perpendicular to the rays used in seeing the side views. To see the auxiliary elevation, the observer looks at the object in a horizontal direction. The angles $a$, $b$, $c$, and $e$ all depend upon the requirements of the particular problem under consideration.

**28. Drawing Auxiliary Views.**—To illustrate the method of drawing auxiliary views, take, for example, the right-auxiliary view of Fig. 15. Draw from each point of the pyramid as seen in the adjacent front view, the parallel rays which make a chosen or specified angle $a$ with the horizontal. These rays are perpendicular to the rays that are used in seeing the front view. The observer, looking along these rays at the pyramid as represented by the front view, sees the depth of the pyramid. In other words, he sees how far one point is in front of or behind another point. In locating points in the right-auxiliary view, the depth of the pyramid is measured parallel to the inclined rays just drawn. Obtain the required depth *d* from the top or any other view in which the depth may be measured. The top view shows that the points V, O, A, and D are halfway between the front points F and E and the rear points B and C. Since none of the points V, O, A, and D is in front of or behind the others, locate each of these points on its own ray, and on a line that is perpendicular to the rays. Next, locate F and E each on its own ray and at a distance of one-half the depth of the pyramid in front of V, A, and D. Locate the points B and C, each on its own ray and a distance one-half the depth *d* behind the points V, A, and D. Letter each point as soon as it is located. Care must be taken not to reverse this view. The observer of this view sees the front corners of the pyramid at his left and not in the reversed position.

In this auxiliary view connect by lines the points which are connected by lines in the front and top views. Hidden lines are represented by broken or dash lines. Check to see that the view is complete and correct. When a view is completed, always "size up" the view and notice whether or not it really is a view of the object that is being drawn. The object should not appear distorted in any view, nor should it ever be drawn in a reversed position.

No reversal of a view should occur if the object is clearly visualized in an adjacent view. A rule for preventing the reversal of views may be stated as follows: *The corners of an object that are nearest the observer of any one view should always be drawn nearest that view in every adjacent view.* For example, in the top view, the top corners of the object are nearest the observer; hence, in all views that are adjacent to the top view, the top corners of the object should be drawn nearest the top view. Also, in all views that are adjacent to the front view, the front corners are nearest the front view. These statements may be checked in Fig. 15. The above rule is universal, and may be applied to any view. It is sometimes a convenient rule to use when rapidly checking a drawing. Reversal of views should be prevented by clear visualization, rather than by making use of the above rule.

**29. Dimensions of Views.**—Each view in a drawing serves to communicate to the reader of the drawing certain information as to the dimensions of the object represented. Each principal view shows two of the three rectangular dimensions of the object. The third dimension may be found in an adjacent view. Two adjacent principal views have one dimension in common.

All views adjacent to any one view show the same rectangular dimension. For example, all views adjacent to the front view of the pyramid show the dimension $d$, and all views adjacent to the right-side view show the dimension $w$. This information aids in checking drawings.

The student should not memorize a list of the dimensions that appear in each view. He should visualize so clearly the direction in which the observer must look to see a certain view that he will have no difficulty in determining what dimension may be seen in that view. It is a waste of time to try to measure in a view a dimension that cannot be seen in that view.

**30. Number of Views.**—When a structure is to be described by means of a drawing, it is necessary to decide what views and how many views shall be drawn. This is a matter of judgment. The views that best represent the structure should be drawn. It is a commonly accepted rule that enough views, no more and no less, should be drawn. If some necessary view is missing, the structure is not fully described. If unnecessary views are drawn, the time of the draftsman and of the reader of the drawing is wasted. Any view that aids the draftsman in making the drawing will generally aid the workman in reading the drawing.

In many of the problems of this text unnecessary views are called for in order to give the student experience in drawing all types of views. On working drawings, however, only a sufficient number of views should be shown.

**31. Oblique Views.**—In Fig. 15, the oblique view of the pyramid shows the pyramid as seen by the observer when he is looking at the pyramid in an oblique direction. The arrow P Q, shown in the front and top views, indicates the direction in which this particular oblique view is taken. The method of making oblique views is explained in the next chapter.

**32. Problems.**

Group 6. Auxiliary Views.

# CHAPTER IV

## OBLIQUE VIEWS

**33.** Some problems may be solved by drawing principal views, many can be solved only with the aid of auxiliary views, and other problems can be solved only by means of oblique views. Oblique views are used: to obtain a view of an object in an oblique direction, to show objects in oblique positions, and to solve problems related to the exact description of structures.

Oblique views may be taken in any direction that is not perpendicular to any of the groups of rays used in seeing the principal views. An oblique view can show none of the rectangular dimensions of the object. It should be remembered that rectangular dimensions are to be measured only in directions parallel to the rays used in seeing the principal views.

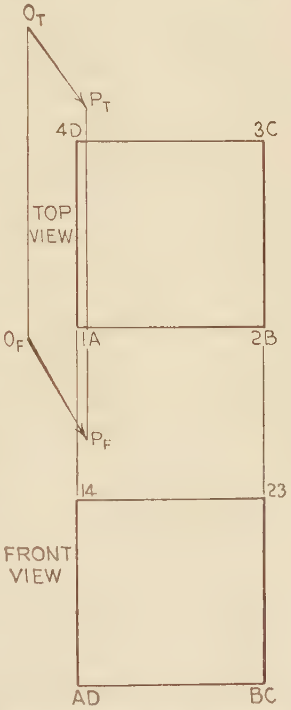


Fig. 16.—Front and top views of cube.

**34. Oblique View in a Specified Direction.**—As an example, take the cube shown in Fig. 16. The arrow O P indicates the direction in which the oblique view of the cube is to be taken. It is not possible to draw the oblique view without first drawing an auxiliary view of the cube and arrow, taken in a direction perpendicular to the arrow. Let this view be an auxiliary elevation taken in a direction perpendicular to the arrow as shown in the top view of Fig. 17. In this auxiliary elevation the ends of the arrow are equidistant from the observer, and the slope of the arrow here indicates the exact direction in which the oblique view is to be taken. The distance $h$ that $O_A$ is above $P_A$ in the auxiliary elevation is equal to the distance $h$ that $O_F$ is above $P_F$ in the front view. Heights, or differences in level, are shown in both views.

The oblique view of the cube may now be seen by the observer if he will look at the cube in the direction indicated by the arrow in the auxiliary elevation. The oblique view of the cube is shown in Fig. 18. The rays are drawn parallel to the arrow $O_A$ $P_A$, preparatory to locating each point of the cube in the oblique view.

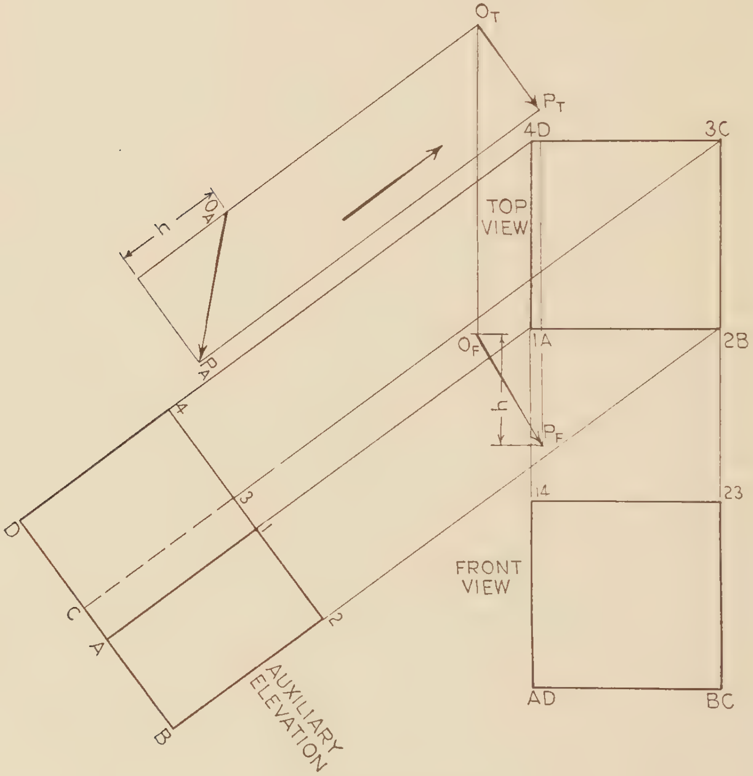


Fig. 17.—Auxiliary elevation of cube.

The observer should now go back temporarily to the cube as seen in the top view of Fig. 18, and he should look at the cube in the direction of the arrow $O_T$ $P_T$. He should realize that he is to the rear of, to the left of, and above the cube; that he is looking down at the cube in a slanting direction; and that he sees the oblique view of the cube. He will observe that the corners A and 1 are at his right, that D and 4 are a certain distance to the left of A and 1, that B and 2 are farther to the left, and that C and 3 are at the extreme left. These distances should be marked off on a straight strip of paper, as shown at S.

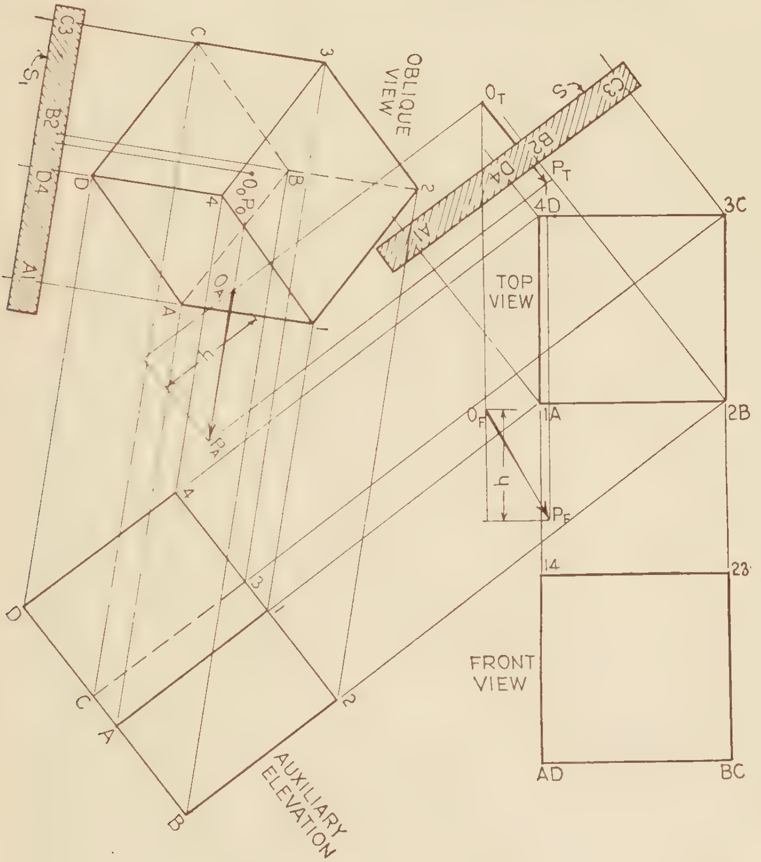


Fig. 18.—Oblique view of cube.

The edge of this paper scale should be perpendicular to the arrow; and the name of each point should be marked on the scale, and near the scale mark that indicates its proper position. The scale shows the distribution of points from right to left as seen by the observer of the oblique view.

The scale S is now moved adjacent to the auxiliary elevation and is placed at $S_1$ parallel to the arrow $O_A$ $P_A$ in Fig. 18. Care must be taken in placing the scale $S_1$ so that there will be a short distance between the oblique view and the auxiliary elevation. The exact distance is immaterial. The views should not overlap. The scale distances are transferred to the drawing paper and each point is carried to its proper ray by means of lines drawn perpendicular to the rays. Letter each corner of the oblique view of the cube as soon as it is found, and, after all points are located, draw the lines of the cube. The proper lines should be dotted. Care should be taken not to reverse the oblique view. Points that are seen to the right must be kept to the right. In the oblique view the observer is looking in the direction of the arrow. He sees the end view of the arrow, as shown at $O_O$ $P_O$.

**35. Orientation.**—The student should not simply memorize the method of measuring the distances in the top view and transferring them to the oblique view. He should study the figure and visualize the object until he understands why the observer sees the same dimensions in the top and oblique views. Figure 19 will aid in understanding the method of drawing the oblique view. In this figure the preceding drawing has been oriented so that the auxiliary elevation is directly below the top view. In this position the auxiliary elevation may momentarily be considered as if it were the front view. The oblique view, then, becomes a left-auxiliary view. To the student who understands auxiliary views the oblique view should offer no difficulties. An oblique view has the same relation to an adjacent auxiliary view that an auxiliary view has to the adjacent principal view.

When making or reading views, it is always necessary for the observer to orient himself with regard to the structure he is viewing, but he must not lose his sense of direction when moving from one station point to another. When turning a drawing, as was done in Fig. 19, care must also be taken not to lose the sense of direction.

When it is specified that a view is to be taken in a certain direction, O P for example, the observer is expected to look in the direction of the line O P, and from O toward P. The arrowhead is placed on the point last named.

Since the draftsman and the reader of the drawing usually work at a horizontal desk, and since auxiliary elevations are viewed in a horizontal direction, it is probably easier to visualize and draw auxiliary elevations than it is to draw other auxiliary views. The top view of the object is directly below the desk worker and the auxiliary elevation may readily be drawn from the top view. The oblique view may then be drawn adjacent to the auxiliary elevation. Oblique views may, however, be drawn adjacent to any auxiliary view that is taken in a direction perpendicular to the arrow. It is possible to draw oblique views adjacent to principal views, but this is neither customary nor desirable. Oblique views will be applied to the solution of problems as needed throughout the text.

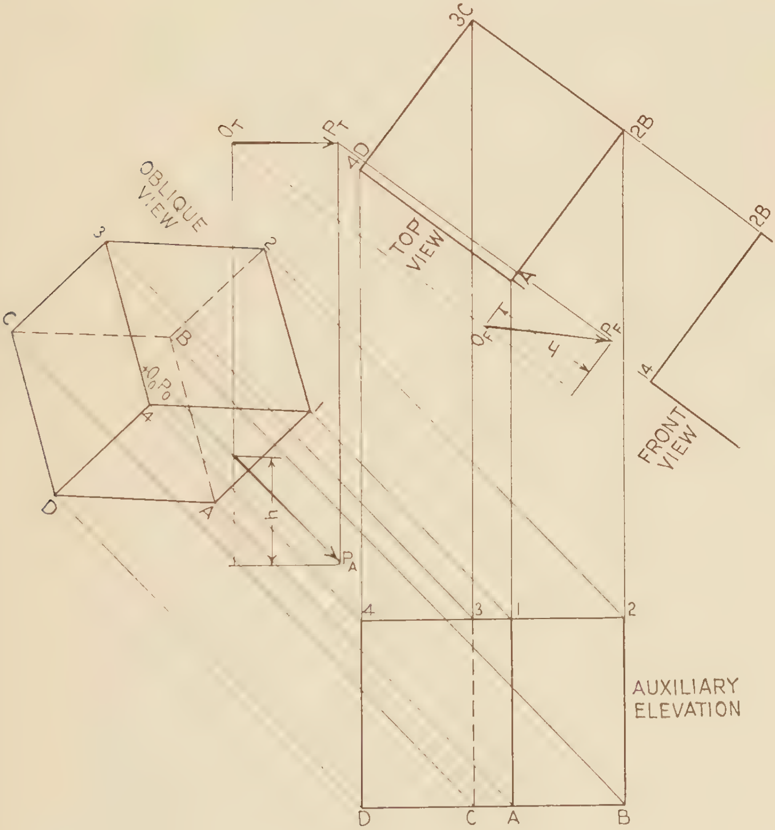


Fig. 19.—Oriented drawing of cube.

### 36. Problems.

Group 7. Oblique Views Taken in Specified Directions.

**37. Object in Oblique Position.**—Take as an example the following problem: A4,2½,6 B617 is the axis of a right hexagonal prism. The diagonals of the hexagonal bases are 2 inches long, and one diagonal of each hexagon is horizontal. Draw the front, top, and left-side views, and the necessary auxiliary and oblique views of the prism.

The front and top views of the axis of the prism are shown in Fig. 20. The axis is an oblique line. The simplest views of the prism should first be visualized as clearly as possible. An end view of the prism, taken in a direction parallel to the axis, will show the true shape and size of the hexagonal ends of the prism. Since the end view shows the characteristic shape of the prism, it should be drawn first. In the drawing, the end view will be an oblique view.

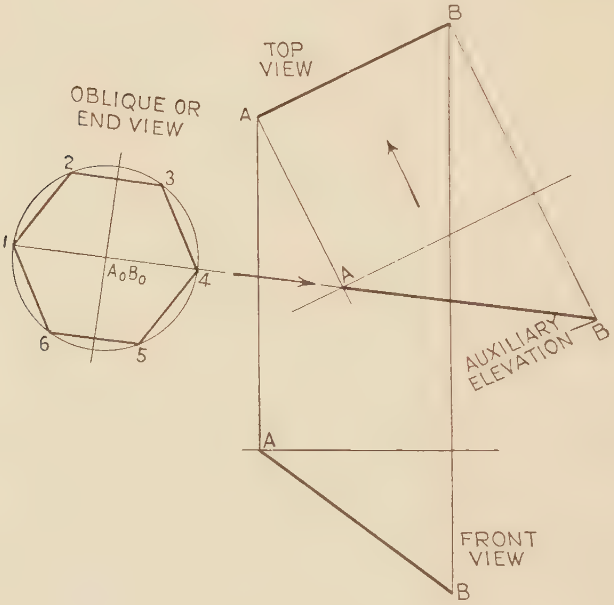


Fig. 20.—Oblique or end view of prism.

Before the oblique view can be drawn, it is necessary to draw an auxiliary elevation of the axis A B taken in a direction perpendicular to its top view. This has been done in Fig. 20. In the auxiliary elevation, the point A appears above B the same distance that it is above B in the front view. Also, in the auxiliary elevation, the points A and B are equidistant from the observer.

Locate next the oblique or end view $A_0$ $B_0$ of the axis. This is taken in the direction of the arrow that is parallel to the auxiliary elevation of the axis. The observer is looking at the end of the axis of the prism. To him, the points A and B appear to coincide. In this end view the nearer hexagon, centered at A, should now be drawn. Since the diagonals of the hexagon are to be 2 inches long, the corners of the hexagon will be on the 2 inch circle centered at $A_0$. The diameter 1 4 of the circle is horizontal. This can be determined by visualizing this line in the auxiliary elevation and in the top view. The other corners of the hexagon are now located on the circle, and then the hexagon is drawn in the oblique view. This hexagon represents both ends of the prism, and the corners of the hexagon represent the end views of the six, parallel, straight-line edges of the prism. Only the corners of the upper hexagon have been numbered.

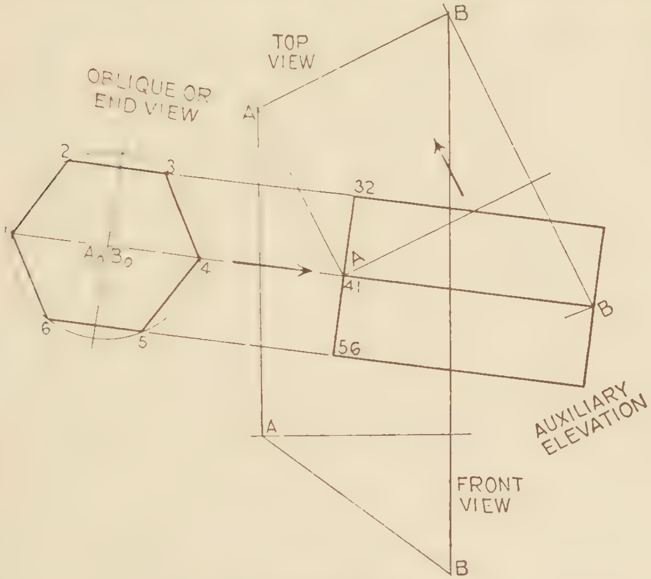


Fig. 21.—Auxiliary view of prism.

The auxiliary elevation of the prism, Fig. 21, is now completed by drawing the edge views of the hexagonal ends perpendicular to the axis of the prism, and then drawing the edges of the prism parallel to the axis. The auxiliary elevation and the oblique view of the prism are the simplest views of the prism. They may be considered as front and top views by orienting the drawing so that the hexagonal bases are horizontal. The simplest, most characteristic views of an object are always drawn first; and then the more complicated views are drawn.

The rays leading to the top view from the corners of the prism, as seen in the auxiliary elevation, may now be drawn. To locate

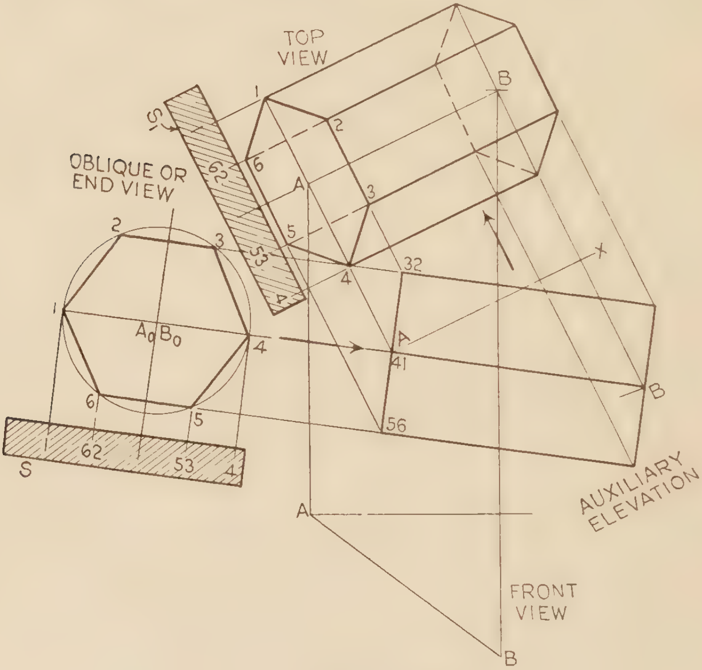


Fig. 22.—Top view of prism.

the points in the top view, mark on a paper scale S, shown in Fig. 22, the horizontal distance that each point is to one side or the other of the axis. Transfer this scale to the top view in a position perpendicular to the axis, and locate each point at its proper horizontal distance from the axis, and on the ray in which it is seen. Letter each point as soon as its location is determined. Complete the top view by drawing the necessary solid and dotted lines.

The corners in the front view, as shown in Fig. 23, may now be found by locating each point on its own vertical ray and at its proper level. Levels may be determined from the auxiliary elevation. Another paper scale may be used for transferring these levels from the auxiliary elevation to the front view. The line A X, in each of these views, may be used as a horizontal base line or plane from which elevations may be measured.

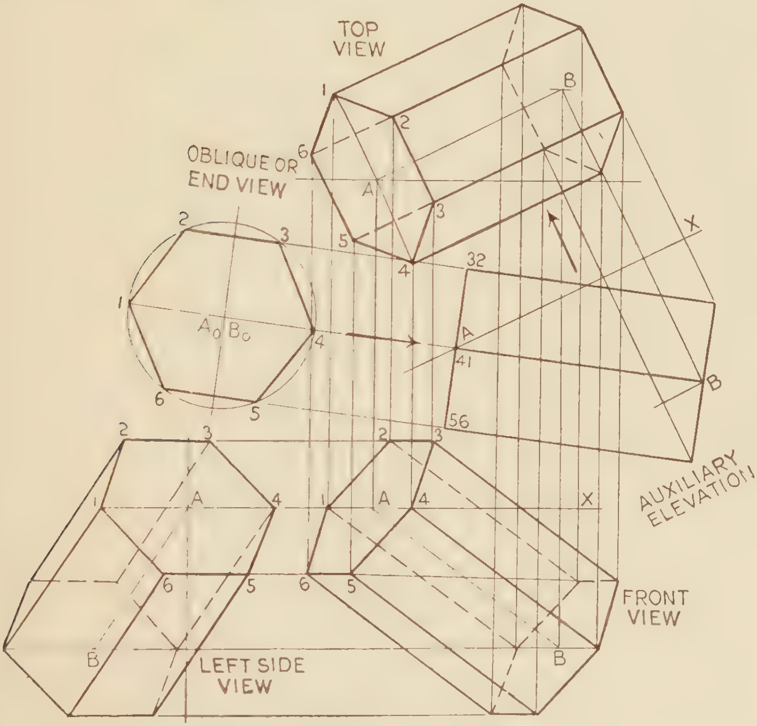


Fig. 23.—Completed views of prism.

The points in the side view are then located by transferring the elevations from the front view. The distance that each point is in front of or behind another point is determined from the top view and transferred to the side view by means of a paper scale. When all views are completed, check to see that the prism does not appear distorted in any view, and be sure that the correct lines are dotted. The complete solution of the problem is shown in Fig. 23.

**38. Problems.**

Group 8. Objects in Oblique Positions.

## CHAPTER V

## ANALYSIS OF STRUCTURES

**39.** The experience gained in drawing principal, auxiliary, and oblique views is a preparation for the more exacting task of analyzing structures into their geometrical elements, and of solving problems related to these elements. Without an understanding of these relations, the engineer is often likely to have difficulty in thinking about the details of structures, and in representing these details in drawings. The principles involved have definite applications to practical work. Solving the problems gives experience in applying the principles.

A problem in the geometry of engineering drawing usually specifies some simple structure or object, or group of geometrical elements: a triangle, a cube, a group of points or lines, a pyramid, a cylinder, a screw thread, a curved line, an unusual surface, etc. These may all be considered as parts of more complicated structures.

**40. Statement of Problem.**—The statement of a problem should specify exactly and distinguish between the data that are given and the results that are required. The distinction as to what is given and what is required should be very clear before the solution of the problem is attempted. All of the items in the statement should be kept in mind while solving the problem, so that each item will be readily available as needed.

**41. Solving the Problem.**—To solve a problem in the geometry of engineering drawing, four steps are necessary:

1. Visualize the structure or object specified.
2. Analyze the structure into its geometrical elements.
3. Determine the geometrical relations of the elements of the structure.
4. Represent the structure by means of drawings in which the elements of the structure are shown in their true geometrical relations.

**42. Checking and Reviewing.**—After the problem has been solved, check and review the solution. Make sure that the requirements of the problem have been fully met, that the structure has been correctly represented, and that it does not appear distorted in any view.

**43. Visualizing the Structure.**—Visualizing the specified object is the first step leading to the solution of the problem. After reading and understanding the statement of a problem, the student should visualize or imagine, as clearly as he is able, the structure or object specified. He then has something on which to base the solution of the problem. Sketches and models are an aid in forming a mental image, but they should not be used habitually.

The engineer who designs a structure must first obtain a more or less complete mental image of the proposed structure. If the engineer in his imagination sees very clearly the structure as a whole, if he sees the relation of each part to the other parts, if he sees the result of all internal or external conditions that affect the structure or its operation favorably or unfavorably—if he clearly sees all of these things—then he has a complete mental image of the structure. Few are able to visualize as thoroughly as this. It is an ideal to strive for. Thorough visualization saves much time and produces a structure better adapted to its purpose. The ability to visualize, to imagine, mentally to construct an object, is a valuable asset to the engineer. The structure must be visualized before it can be drawn, and must again be visualized by the reader of the drawing.

**44. Analysis of Structures.**—The second step in solving a problem is to analyze the structure into its geometrical elements.

Complicated structures are analyzed as made up of simpler structures, and these in turn may be analyzed into their geometrical elements. Even the most complicated structures may be analyzed into simple elements; and by combining simple elements the most complicated structures can be designed and built.

When a person visualizes or looks at a structure, he sees: the surfaces that bound the structure, the lines that determine the edges and surfaces of the structure, and the points located at the corners and on the lines of the structure. Points, lines, and surfaces are the basic geometrical elements of structures.

A *point* indicates location only, and is assumed to have no dimensions. A point may be represented on paper by a dot or by a very small circle.

A *line* has length, but not width or thickness. It has one dimension only, and may be straight or curved. It may be

**Geometrical Elements of Structures**

- **Points**
- **Lines**
  - Straight
  - Curved
    - Single-curved
      - Circle
      - Ellipse
      - Parabola
      - Hyperbola
      - Spirals
      - Involute
      - Cycloids
    - Double-curved
      - Helix
- **Surfaces**
  - Ruled
    - Plane
      - Prism
      - Pyramid
      - Polyhedron
    - Single-curved
      - Cone
      - Cylinder
      - Convolute
    - Warped
      - Hyperbolic paraboloid
      - Conoid
      - Helicoid
      - Hyperboloid of revolution
  - Double-curved
    - Sphere
    - Ellipsoid
    - Paraboloid
    - Hyperboloid of two nappes
    - Annular Torus
    - Surfaces of revolution
    - Serpentine

Chart: Geometrical elements of structures.

represented on paper by a straight or a curved line. A line is assumed to be generated by the motion of a point. The law which governs the motion of the generating point determines the nature of the line.

A *surface* has two dimensions only. It has area but not thickness. A surface may be flat or curved, and may be represented on paper by drawing points and lines of the surface. A surface is assumed to be generated by the motion of a straight or curved line called the "generatrix" The law which governs the motion of the generatrix determines the nature of the surface.

The chart on page 36 names and classifies the geometrical elements of structures. In the chart the commonly used types of lines and surfaces and a few of the uncommon types are named. The definitions of the various terms used in the chart will be stated as needed.

The chart should be read as follows: The basic geometrical elements of structures are points, lines, and surfaces. Points have no subclassification. Lines may be either straight or curved. Straight lines have no subclassification. Curved lines are either single-curved or double-curved. The circle, the ellipse, etc., are single-curved lines. The helix is a double-curved line. Surfaces are either ruled or double-curved. Ruled surfaces are either plane, single-curved, or warped. Various examples of each type are given. Double-curved surfaces of various kinds are named.

**45. Geometrical Relations of Elements.**—The third step in solving a problem in the geometry of engineering drawing is to determine the geometrical relations existing between the elements of the structure. These geometrical relations are generally of a simple nature, but they must be determined before the drawing of the structure can be made.

When considering the geometrical relations between the points of a structure, we are concerned only with the relative position of one point to another. The questions to be answered are: Is one point above or below, to the right of or to the left of, in front of or behind another point? What is the distance in each case?

A point is geometrically related to a line in that it may be on the line, or any distance from the line, or that it may have some particular relation to the line, as, for example, the relation of the center to the circle, or of the focus to the parabola.

Geometrically, a point may be on a surface, or any distance from the surface, or it may have some special relation to the surface, as the relation of the center of a sphere to the surface of the sphere.

The ordinary geometrical relations of one straight line to another are considered in the following questions: Do the lines intersect? Are the lines parallel or perpendicular? What is the angle between the lines?

Also the consideration of the direction of a single line is often necessary: Is the line horizontal, vertical, oblique, etc.? Is it seen in its true length? If the line is curved, what is its position, and is it seen in its true shape?

The usual questions regarding the geometrical relations of a line and a surface are as follows: Does the line lie wholly in the surface? Is the line parallel or perpendicular to the surface? Does the line have some particular relation to the surface, as, for example, the relation of the axis of a cylinder to the surface of the cylinder?

The geometrical relations of one surface to another suggest many questions, some of which are here stated: Are the surfaces parallel or perpendicular? Do the surfaces intersect, and, if so, what is the line of intersection? Or, when considering a single plane, is it horizontal, vertical, oblique, etc.?

**46. Representation of Structures.**—The fourth step in solving a problem is to represent the structure by means of drawings in which the elements of the structure are shown in their true geometrical relations. This requires a knowledge of how to represent each element, and also how to show it in its correct geometrical relations to the other elements of the structure. The methods used will be described in this and the following chapters. Each element will be taken up in the order in which it appears in the chart that classifies the elements of structures.

**47. Representation of Points.**—A single point, P for example, that has no relation to any other point, may be represented by a dot or a very small circle placed anywhere on the drawing paper and lettered P.

When two or more points are to be represented, it is first necessary to know the geometrical relations existing between these points. For example, assume that, starting with a point A, the point B is 5 feet to the right of A, 1 foot above A, and 2 feet in front of A, and that the point C is 3 feet to the left of A, 4 feet below A, and 2 feet behind A.

The problem is to represent the relative positions of the three points A, B, and C, in a drawing. At least two views will be necessary. Assume that the front and top views are to be drawn. These may be drawn to any suitable scale. As a convenience in measuring the relative locations of the points, coordinately ruled paper may be used, as shown in Fig. 24. Each square is here taken as representing a distance of 1 foot. To distinguish between the front and top views of a point, it is convenient to add, to the letter that names the point, the subscript F to indicate the front view and the subscript T to indicate the top view.

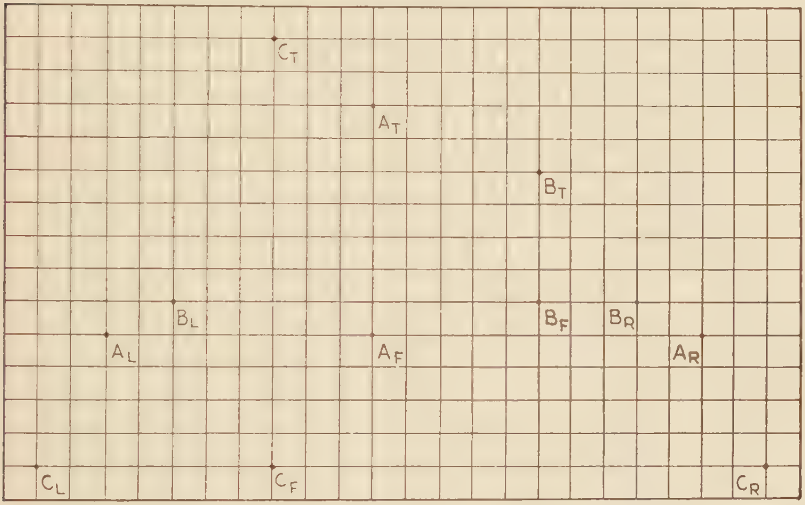


Fig. 24.—Representation of points.

First locate the front view of A at some point and mark it $A_F$, Fig. 24. Some distance directly above the front view of A locate its top view and mark it $A_T$. Then, locate $B_F$ 5 feet to the right of, and 1 foot above $A_F$. Next, locate $B_T$ 5 feet to the right of, and 2 feet in front of $A_T$. Locate $C_F$ 3 feet to the left of, and 4 feet below $A_F$. Finally, locate $C_T$ 3 feet to the left of, and 2 feet behind $A_T$. The top view of each point will be directly above its front view. The relative position of each point to the other two points is correctly represented.

In Fig. 24 the right-side view and the left-side view of the points A, B, and C are also shown. The different views should agree as to the relative locations of the three points. The top view shows that B is 4 feet in front of C, and both side views should show the same relation. A reversed side view would show B behind C. If two views disagree in any detail, then at least one of the views is wrong. In Fig. 25 the front and top views disagree as to the distance that E is to the right of D. In the figure, D is correctly represented, and E is incorrectly represented. Either the front view or the top view of E has not been placed on the ray in which it is seen.

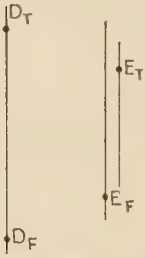


Fig. 25.—Correct and incorrect views.]

**48. Representation of Lines.**—Lines may be either straight or curved. A single line may be represented by drawing a straight or curved line anywhere on the drawing paper. When several lines are to be shown in their correct geometrical relations, it is necessary to draw two or more views of each line. These views are determined by locating the views of two or more points of each line.

**49. Representation of Surfaces.**—A surface may be represented by drawing various points and lines of the surface. A plane, for example, may be represented and located by showing the location of three points that lie in the plane. Likewise, a triangle, or a point and a line, or two intersecting or parallel lines, may serve to represent and locate a plane.

## CHAPTER VI

# STRAIGHT LINES

**50.** Lines are basic geometrical elements of structures. A line may be considered as generated by the motion of a point. The law that governs the motion of the generating point determines the nature of the line. There are two general types of lines—straight and curved.

A straight line is generated by a point that moves in one direction. A straight line appears straight from every point of view.

**51. Location of a Straight Line.**—The location of a straight line may be fixed by specifying the location of two points of the line, or by specifying the location of one point of the line and stating the direction of the line.

**52. Representation of Straight Lines.**—If the front and top views of a straight line are to be drawn, locate first the front and top views of two points of the line, for example, the points E and F in Fig. 26. Next, draw the front view of the line through the front views of the points and then draw the top view of the line through the top views of the points.

The reader of the drawing in Fig. 26 should note that the front view shows that the point E on the line E F is to be the left of, and above F, and that the top view shows that E is to the left of, and in front of F. The line E F, then, is an oblique line extending to the left, upward and forward from the point F. In the drawing, the exact relation of E to F or of F to E may be measured.

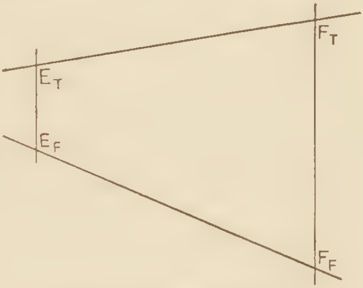


Fig. 26.—Straight line.

In Fig. 26 the line is shown as extending beyond the points E and F. This indicates that the line is not limited in length. If the line had been shown as terminating at the points E and F, the line might then be considered as limited in length, and having its ends in the points E and F.

**53. Problems of the Straight Line.**—Straight-line problems involve: drawing a line in a specified direction, finding the true length of a line, drawing intersecting lines, drawing parallel lines, drawing perpendicular lines, finding the distance from a point to a line, and finding the angle between two lines.

**54. Direction of a Line.**—The direction of a straight line may be stated as horizontal, vertical, frontal, etc.

In a *horizontal line* all points are at the same level. No point of a horizontal line is above or below the other points. All points in a horizontal line are equidistant from the observer of the top view. A number of lines may be horizontal but they need not be parallel; neither is it necessary that they be at the same level.

In a *frontal line* no point is in front of or behind the other points. All points in a frontal line are equidistant from the observer of the front view. A frontal line is not necessarily located in the front face of an object.

In a *profile line* no point is to the right or to the left of the other points. All points in a profile line are equidistant from the observer of a side view.

In a *vertical line* each point is directly above or directly below every other point. In any single structure all vertical lines are assumed to be parallel. Since the earth is a sphere, vertical lines that are far apart are not parallel. The top view is an end view of a vertical line.

In an *oblique line* each point is above or below, and to the right or left of, and in front of or behind every other point. No two points of an oblique line are equidistant from the observer of the principal views. An oblique line is neither horizontal, frontal, nor profile.

Vertical and horizontal lines are independent of the location of the station point of the observer of a structure. Frontal and profile lines are dependent upon the location of the station point of the observer.

A vertical line is a special position of a frontal and a profile line. A line that is both horizontal and frontal may be called horizontal-frontal. A line that is both horizontal and profile may be called horizontal-profile.

A triangle, a square, a hexagon, a circle, a plane, etc., may also be specified as horizontal, frontal, profile, or oblique.

**55. Use of Subscripts.**—In any drawing, each view of the structure occupies a certain position relative to the other views, so that it is not usually necessary to indicate the names of the views.

When solving problems, however, it is generally advisable to mark each view of a point with a subscript. Subscripts should always be used when views overlap. The subscripts F, T, R, L, A, and O, may be used respectively for front, top, right-side, left-side, auxiliary, and oblique views. X, Y, and Z, or 1, 2, and 3 may be used when more than one auxiliary view is drawn. Care should be used in making subscripts so that they will be neither too large nor unreadable. The name of each point that is mentioned in the statement of a problem or that is used in solving the problem should always be lettered in every view in which the point is shown. When a series of points is to be marked, it is often convenient to use numbers instead of letters for the names of the points.

**56. Problems.**

Group 9. Drawing Lines in Specified Directions.

**57. Normal View of a Line.**—Any view of a line in which all points of the line are equidistant from the observer may be called a *normal view* of the line.

**58. True Length of a Straight Line.**—One of the commonest problems in the design of a structure is to determine the actual or true lengths of the lines of the structure. A straight line appears in its true length when all points of the line are equidistant from the observer. This statement is true because the distance between the parallel lines of sight leading to the ends of the line is equal to the true length of the line. In such a view the lines of sight are perpendicular to the line itself.

Every normal view of a line shows the true length of the line. Therefore, to determine the true length of a line, it is necessary to draw a normal view of the line. In any view that is not a normal view of a line, the line appears shorter than it really is, because the perpendicular distance between two rays is shorter than any oblique distance between these rays.

The top view is a normal view of a horizontal line and shows the true length of that line. The front view is a normal view of a frontal line and shows the true length of the frontal line. Either side view is a normal view of a profile line and shows the true length of that line. A view that shows the true shape of a triangle, a circle, a square, or other plane figure is a normal view of that figure.

**59. True Length of an Oblique Line.**—Neither the front, top, nor side view is a normal view of an oblique line. In all of these views the oblique line will appear shorter than its true length. To obtain the true length of an oblique line, it is necessary to draw a normal view of that line. A normal view of an oblique line is an auxiliary view taken in a direction perpendicular to the oblique line. In such a view all points of the oblique line will be equidistant from the observer and the line will be seen in its true length.

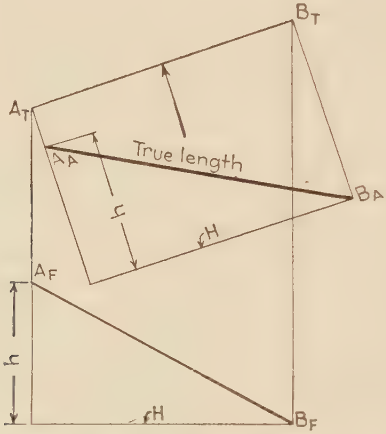


Fig. 27.—True length of line.

In Fig. 27 are shown the front and top views of an oblique line A B. Neither of these views shows the true length of the line. To obtain the true length of A B, draw a normal view. This view may be an auxiliary elevation taken in a direction perpendicular to the top view of A B. The horizontal arrow perpendicular to the top view of A B indicates this direction. Since the observer is here looking in a horizontal direction at the line, he will be able to see how far one end of the line is above the other end. The front view shows that *h* is the distance that A is above B. In the normal view, draw A above B the same distance, *h*, and connect $A_A$ and $B_A$ by a straight line. This is the true length of the line A B. Since required data are to be drawn in broad lines, this normal view is drawn as a broad line, so that it will stand out from the lines used in solving the problem. This finishes the solution of the problem. The light lines marked H are horizontal base lines or planes by means of which the difference in elevation of the points A and B is measured.

When the normal view of the line is drawn as an auxiliary elevation, this view may be taken from either side of the top view of the line. Figure 28 shows the normal view of the line A B taken from the rear. This view is best read by turning the book up-side-down so that the arrow points directly away from the reader. In this position the normal view temporarily becomes a front view, in which the point A is seen to be above the point B. This agrees with the information to be found in the original front view. An auxiliary elevation should never be drawn up-side-down, so that it reads in a direction contrary to that in which the observer is looking. Rays extend from the observer to the object, but never beyond the object. Figures 27 and 28 are both correct.

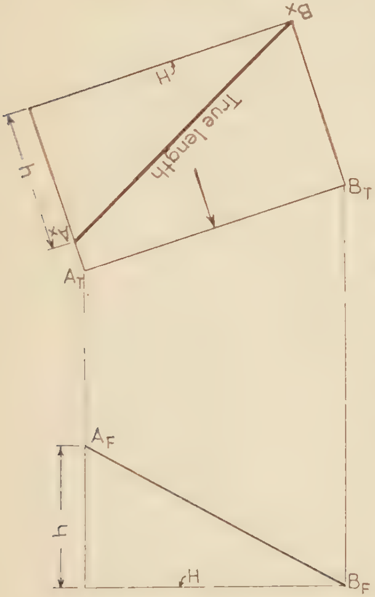


FIG. 28.—True length of line.

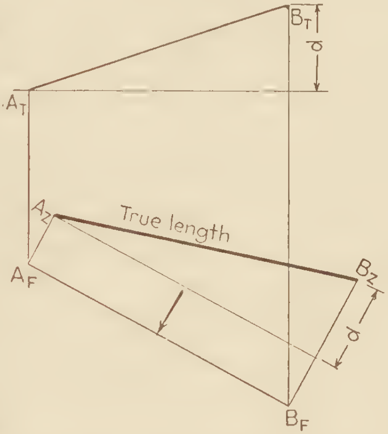


FIG. 29.—True length of line.

The true length of the line A B can be determined as readily by drawing a normal right-auxiliary view, as is shown in Fig. 29. This normal view is taken in a frontal direction perpendicular to the front view of the line. This view shows the distance that B is behind A, and the depth *d* in the normal view is accordingly made equal to the depth *d* in the top view. As usual, the required length is drawn heavy. The light weight base line is a frontal line used to determine the distance that B is behind A in the top view and again used to mark off the same distance in the auxiliary view.

**60. Problems.**

Group 10. Finding the True Lengths of Lines.
Group 11. Measuring Given Distances on Oblique Lines.
Group 12. True Lengths of Lines—Miscellaneous Problems.

**61. Calculating the Lengths of Lines.**—When it is necessary to obtain the length of a line, judgment must be used to decide whether or not the length of the line can be determined with sufficient accuracy by graphical means. If proper tools and a proper drawing surface are used, and if the work is very carefully done, drawings should not vary more than two or three hundredths of an inch from the correct measurement. If a structure is large, the error in the drawing will be multiplied in the structure as many times as the structure is larger than the drawing. In small structures that must be very accurate, certain dimensions must always be calculated.

As an example of the above, suppose that the distance between centers of the rivet holes in a diagonal brace is about 12 feet and the drawing is made to a scale of $\frac{3}{4}$ inch = 1 foot. The drawing will be one-sixteenth of the size of the diagonal brace. An error of 0.01 inch in the drawing will produce an error of 0.16 inch in the center to center distance between rivet holes. The maximum allowable error would be less than this. In this case a small scale drawing could not be used without calculating the distance between rivet holes.

The distance between two points whose coordinates are known may be calculated as follows: Subtract each coordinate of one of the points from the similar coordinate of the other point. These three differences represent the distances between the two points, measured in three mutually perpendicular directions. To calculate the true length of the line, find the square root of the sum of the squares of the above differences.

Take, for example, the points A137 and B625.

$$\begin{array}{lll} 6 - 1 = 5 & 2 - 3 = -1 & 5 - 7 = -2 \\ 5^2 = 25 & -1^2 = 1 & -2^2 = 4 \end{array}$$

$$\sqrt{25 + 1 + 4} = 5.4772$$

Prove that the geometry of the above method of calculating the true length of an oblique line is exactly similar to the geometry of finding the true length of an oblique line by means of a normal view.

**62. Problems.**

Group 13. Calculating the Lengths of Lines.

**63. Intersecting Lines.**—If a point is on a line, then the line passes through the point. Each view of the point must be on the corresponding view of the line; and, of course, the top view of a point must be directly above its front view, and the side view must be at the same level as the front view. That is, each view of the point must be on the ray in which it is seen. In Fig. 30 the point O is on the line A B, while the point X is not on the line A B. The point X is a short distance directly behind the line A B.

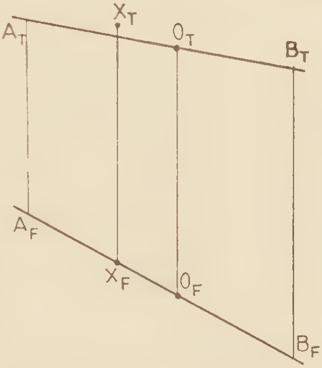


Fig. 30.—Point on line.

Two lines intersect if they pass through a common point. Another way of saying this is: Two lines intersect if it is possible to find a point that is on both of the lines. In Fig. 31 the lines

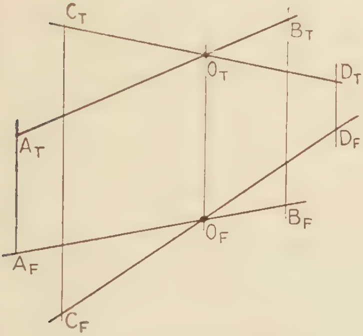


Fig. 31.—Intersecting lines.

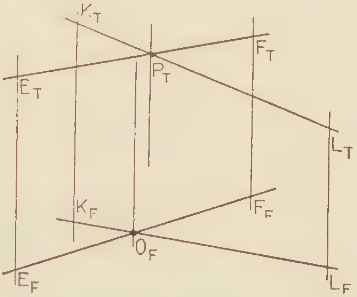


Fig. 32.—Lines not intersecting.

A B and C D intersect at the point O. Here, the top view of O is on the top views of both lines, the front view of O is on the front views of both lines, and the top and front views of O are in the same vertical. The lines A B and C D pass through the common point O; hence they intersect.

The lines E F and K L in Fig. 32 do not intersect. The apparent crossing point at $P_T$ in the top view is not directly above the apparent crossing point $O_F$ in the front view; hence these lines do not pass through a common point.

**64. Problems.**

Group 14. Drawing a Line to Intersect a Given Line.
Group 15. Do the Given Lines Intersect?
Group 16. Drawing a Line to Intersect Two Given Lines.
Group 17. Locating Points at Intersections.

**65. Parallel Lines.**—Two lines are parallel if the distance between the lines is constant, or if the lines extend in the same direction. Two lines are parallel if each is parallel to a third line.

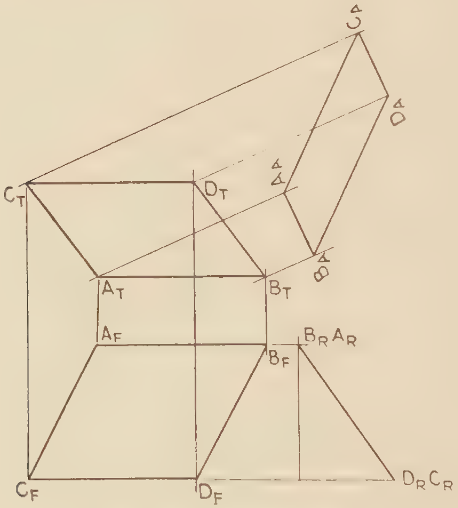


FIG. 33.—Parallel lines.

When considering parallel lines, it is necessary to distinguish between a condition that determines the direction of the line and a condition that determines the location of the line. If a line is to be parallel to another line, its direction, but not its location, is determined. If a line is to be drawn through a given point and parallel to a given line, both its direction and its location are determined. If a line is to be parallel to a given plane, its direction is limited in part, but its location is not limited.

The method of drawing parallel lines in space may be determined with the aid of Fig. 33. In this figure any two horizontal-frontal lines A B and C D are first drawn equal in length. These two lines are parallel because they extend in the same direction from right to left only. Now, draw the front and top views of the lines A C and B D. Since the lines A B and C D are equal and parallel, the figure A B D C is a parallelogram. The opposite sides of the parallelogram are parallel; therefore, the lines A C and B D are parallel in space. In the front view, the lines A B and C D appear parallel and equal in length; hence the front view of the parallelogram appears as a parallelogram. Therefore, in the front view the parallel lines A C and B D appear parallel. For similar reasons the top views of the parallel lines A C and B D appear parallel.

In Fig. 33 the side views of the parallel lines A C and B D coincide. If this condition leaves the reader in doubt as to whether parallel lines appear parallel in the side view, he can readily prove that parallel lines will appear parallel in the side view and in the front view by drawing vertical the lines A B and C D of Fig. 33. Also in any auxiliary view—the auxiliary elevation in Fig. 33, for example—it may be readily proved that parallel lines appear parallel. Shortening or lengthening a line will not affect its direction.

**66. Parallel Line Principles.**—From the above arguments the following general principle may be deduced:

**Parallel lines appear parallel in every view.**

Hence, lines that are known to be parallel must be drawn parallel in every view.

The inverse statement—of interest to the reader of the drawing—namely, that lines are parallel if they appear parallel in every view, is generally true. The exception to be noted is that two lines, whether parallel or not, will always appear to be parallel in any view that is taken in a direction parallel to a plane that is parallel to the two lines. For example, profile lines always appear parallel in the front and top views. A side view is necessary to determine whether or not profile lines are parallel. Also, it is impossible to tell from a single view whether or not lines are parallel. Horizontal lines, for example, always appear parallel, in the front view, while the top view may show them not parallel. To determine whether or not lines are parallel, it is necessary to refer to at least two views, and even then care must be taken to be sure that the exception stated above does not apply.

**67. Problems.**

Group 18. Are the Given Lines Parallel?
Group 19. Drawing One Line Parallel to Another.
Group 20. Drawing Parallel Line Objects.

**68. Perpendicular Lines.**—Since the lines of structures are so frequently perpendicular to each other, it is important to know how to draw one line perpendicular to another, and also how to tell whether two lines that are represented in a drawing are perpendicular or not.

Two lines are mutually perpendicular if the direction of one of the lines is at right angles to the direction of the other line. Two lines may be perpendicular to each other whether they intersect or not. In the problems, perpendicular lines are to be taken as intersecting unless otherwise stated.

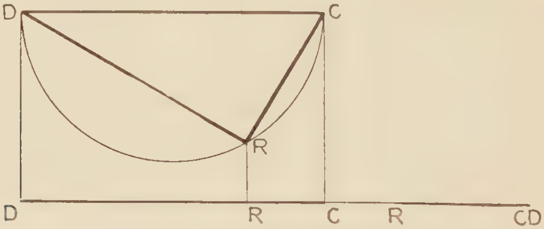


Fig. 34.—Perpendicular lines.

In Fig. 34 are shown three views of a horizontal right-angle triangle D R C. The top view is a normal view of all three sides of the triangle; therefore, in this view each side of the triangle is shown in its true length, and the true shape of the triangle is seen. The angle at R is a right angle and the sides R C and R D are mutually perpendicular. This last statement may be checked by means of a circle drawn on the hypotenuse D C as a diameter. If the corner R is on the circumference of this circle, the angle at R is a right angle. The sides of the triangle are drawn heavy since they are here of greater interest to the reader than is the hypotenuse. In the front view the right angle appears as a straight angle and in the side view as a zero angle. From the top view of Fig. 34 may be drawn the conclusion: *Two lines that are perpendicular appear perpendicular in any view that is a normal view of both of the lines.*

In Fig. 35 the triangle D R C is again shown; but in this view the triangle is rotated upward, without changing its shape, by using the hypotenuse as an axis or hinge. The corners, C and D, being on the axis, do not move. As the triangle is rotated, the corner R moves in a circle from R to $R_1$ and on to $R_2$, as shown in the three views. In the front view the angle at R gradually changes from a straight angle to a right angle, and in the top view this angle gradually changes from a right angle to a straight angle. In all intermediate positions, at $R_1$ for example, the angle between the lines does not appear as a right angle. In this position the sides $R_1$ C and $R_1$ D are oblique lines that do not appear perpendicular in either view. The actual angle between these lines, however, is a right angle. From either the front or the top view of Fig. 35 the following conclusion may be reached: *Per-*

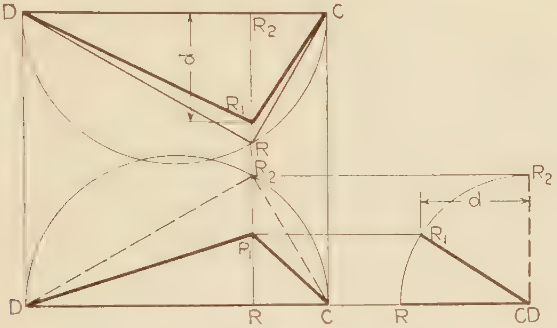


FIG. 35.—Perpendicular lines.

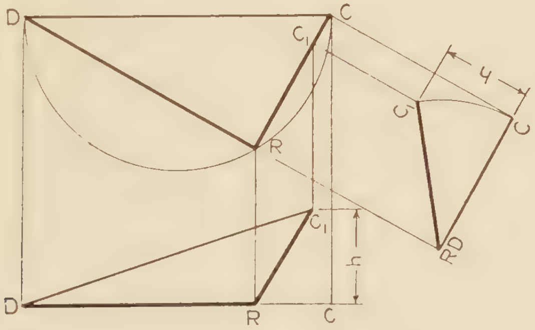


FIG. 36.—Perpendicular lines.

*pendicular lines do not appear perpendicular in any view that is an oblique view of both of the lines.*

In Fig. 36 the right-angle triangle D R C is again shown. In this figure the side D R of the triangle is taken as an axis or hinge, and the triangle rotated upward until the corner C moves to an intermediate position $C_1$. The corners D and R do not move. Since the axis D R is horizontal, the corner C moves in a vertical circle, as shown in the auxiliary elevation. In the top view, the edge view of this circle is seen. The point C here appears to move along the line C R. It has no motion parallel to the axis. The triangle D R $C_1$ remains a right-angle triangle, and in the top view the angle at R continues to appear as a right angle.

The top view of Fig. 36 is a normal view of the line R D, but not of the line R $C_1$, and in this view these lines appear perpendicular. From the top view the conclusion may be reached: *Two lines that are perpendicular appear perpendicular in any view that is a normal view of one of the lines.* In Fig. 36 the front view is not a normal view of either the line R $C_1$ or R D; hence, in this view these lines do not appear perpendicular.

**69. Perpendicular Line Principles.**—A correct understanding of the general principle involved in drawing perpendicular lines is very important. A summation of the principles developed above may be stated as follows:

**Two lines that are perpendicular appear perpendicular only in a view that is a normal view of one or both of the lines.**

This principle is useful when perpendicular lines are to be drawn. It indicates that these lines must first be drawn in some view that is a normal view of at least one of the lines.

Frequently, one of the principal views will be a normal view of one or both of the lines. If one or both of two perpendicular lines is horizontal, the lines will appear perpendicular in the top view; if frontal, they will appear perpendicular in the front view; if profile, they will appear perpendicular in a side view. Oblique lines that are perpendicular will not appear perpendicular in any of the principal views.

The following inverse statement is useful to the reader of drawings.

*If two lines appear perpendicular in any view, the lines are perpendicular only if the view is a normal view of one or both of the lines.*

**70. Problems.**

Group 21. Are the Given Lines Perpendicular?
Group 22. Horizontal and Frontal Lines Perpendicular to Oblique Lines.
Group 23. Profile Lines Perpendicular to Oblique Lines.

**71. Drawing Oblique Lines Perpendicular.**—As an example, take the following statement of a problem: Draw the line R B perpendicular to the line R D. Since it is possible to draw an infinite number of lines through the point R and perpendicular to the line R D, it is necessary to limit the solution of this problem to a single line by stating some limiting condition, for example, the location of one of the views of the line R B. In the problem stated above, it is assumed that both views of R D are given but that only the top view of the line R B is given and that its front view is to be found.

The solution of the problem is carried out in Fig. 37. In this figure, after the given data have been located, the various lines and points have been numbered consecutively in the order in which they are drawn or located. These numbers should aid the student in reviewing the method of solving this problem.

To draw the line R B perpendicular to the line R D, it is necessary to obtain an auxiliary view that is a normal view of one of these lines. This must be a normal view of the line R D because its location is known. In Fig. 37 draw a normal auxiliary elevation of the line R D. This is a normal view taken in a direction perpendicular to the top view of the line R D as indicated by the arrow. The lines R D and R B will appear perpendicular in this view. After the normal view $R_A$ $D_A$ is found, draw the line 10 through R and perpendicular to R D. This is the line R B. The point B will be located at 11 on the ray 1 in which it is seen. In the normal view, B is found to be located a distance "*a*" below the point D. In the front view, B is located the same distance "*a*" below the point D, at the level 13 and on the ray 14. The front view of R B is next drawn. All views of the line R B are drawn heavy, and the solution of the problem is completed. It should be noted that the perpendicular lines do not appear perpendicular in either the front or the top views.

It must be definitely understood that the auxiliary view in Fig. 37 is a normal view of the line R D only. In the normal view, R and D are equidistant from the observer, but B is a greater distance, as may be seen in the top view. In the normal view the true length of the line R D is seen, but the line R B does not here appear in its true length. The student who misinterprets a view causes himself much trouble.

If the problem as stated above is now changed, so that the front view of R B is given and the top view is to be found, the same lines would be drawn as in Fig. 37, but the lines would be drawn in a different order.

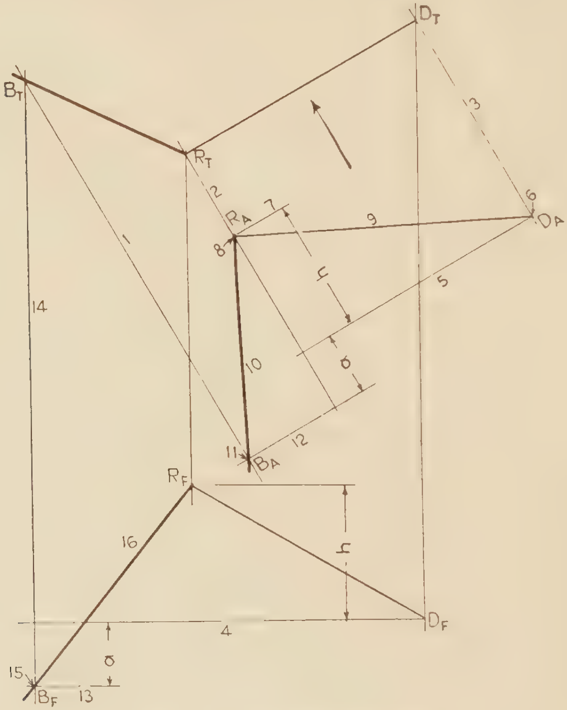


Fig. 37.—Perpendicular lines.

A normal right-auxiliary view of the line R D, taken in a direction perpendicular to the front view of the line, would serve as readily to solve the problem as does the normal elevation.

**72. Checking Perpendicular Lines.**—If two oblique lines are given, and it is required to find out whether or not they are perpendicular, proceed as follows: In Fig. 38 the intersecting lines V C and V A are given. To determine whether or not the angle at V is a right angle, draw a view that is a normal auxiliary elevation of either one of the lines. V A is here chosen. In the auxiliary elevation take one point on each line, as A and C, and on this line as a diameter draw a circle. If V falls on the circle, the angle C V A is a right angle. In Fig. 38, V falls outside the circle; hence, the angle is not a right angle, and the lines V C and V A are not perpendicular. The true size of the angle at V is not here shown.

**73. Distance from a Point to a Line.**—The distance from a point to a line is measured on the line that passes through the given point and that is perpendicular to and intersects the given line. To solve the problem: Draw through the given point a line that is perpendicular to, and intersects, the given line; find where this perpendicular intersects the given line; the true length of the straight line connecting the point of intersection with the given point is the required distance.

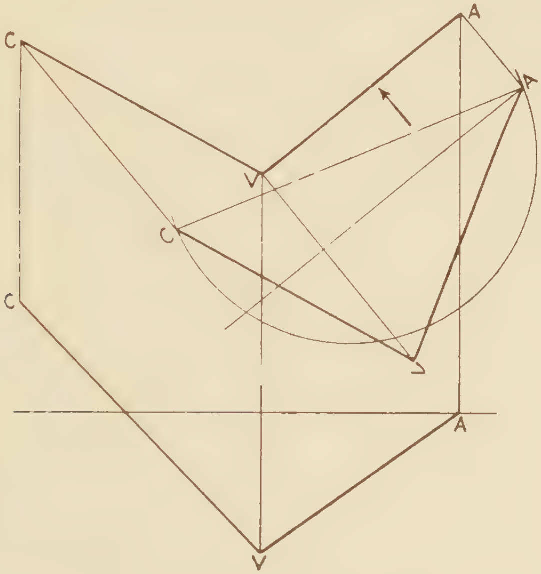


FIG. 38.—Test for perpendicular lines.

**74. Problems.**

Group 24. Drawing Oblique Lines Perpendicular.
Group 25. Figures Having Right Angles.
Group 26. Drawing One Line Perpendicular to Another and Intersecting a Third Line.
Group 27. Distance from a Point to a Line.

**75. To Draw a Line Perpendicular to Two Lines.**—*Problem:* To draw a line A D perpendicular to the two given lines A B and A C. The given lines A B and A C are taken as shown in Fig. 39. The line A D is to be found. The point D is used to give a name to

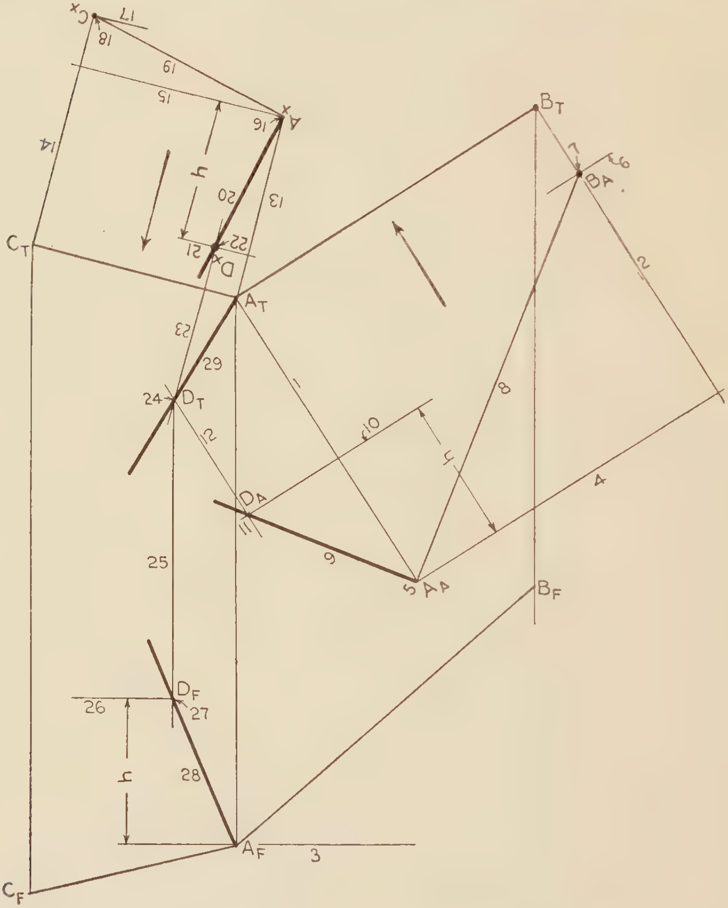


Fig. 39.—To draw one line perpendicular to two lines.

the line A D, and may be located at any point on this line. The lines A B and A C may or may not be perpendicular; this is here of no concern.

To solve this problem it is necessary to draw a normal view of each of the given lines. After the given data are located, twenty-nine different steps are necessary to solve the problem. These are consecutively numbered in Fig. 39, in the order in which they should be drawn.

Draw first a normal auxiliary elevation of the line A B.

1 and 2 are rays drawn parallel to the arrow which is perpendicular to the top view of A B.
3 is a horizontal base line through the front view of A.
4 is the same base line in the normal view.
5 is the location of $A_A$.
6 is above 4 the same distance that $B_F$ is above 3.
7 locates $B_A$ on its ray 2 and at the level 6.
8 is the normal view of the line A B.
9 is drawn perpendicular to $A_A$ $B_A$. This is the line A D.
10 is drawn parallel to 4 at any convenient distance *h* above or below 4. Since the dimension *h* is to be used three times in solving this problem, it saves time to make *h* some known distance such as 1 inch, 1½ inches, or 2 inches.
11 locates $D_A$ where 9 and 10 meet. $D_A$ $A_A$ is the auxiliary view of the required line A D. In this view it appears perpendicular to A B. $D_A$ may be located anywhere on the perpendicular 9 by varying the distance *h*. It is used to give a name to the line A D, as required by the statement of the problem.
12, parallel to 1 and 2, is the ray in which the top view of the point D must be located.

In order to proceed with the solution of the problem, it is now necessary to draw a normal view of the line A C. This is taken as an auxiliary elevation perpendicular to the top view of A C, and is drawn from the rear in order to keep the views from overlapping.

13 and 14 are the rays in which the points A and C are seen.
15 is a horizontal base line drawn any distance from the top view of A C, and is assumed to be at the same level as base lines 3 and 4.
16 locates $A_X$ on its ray 13 and at the proper level 15.
17 locates the level of $C_X$ below $A_X$ the same distance that $C_F$ is below $A_F$.
18 locates $C_X$ on its ray and at the proper level 17.
19 is the normal view of the line A C.
20 is drawn perpendicular to $A_X$ $C_X$. This is the required line A D.
21 is a level line drawn above 15 the same distance that 10 is above 4. This is the level of the point D.
22 locates $D_X$ on 20 and 21. $A_X$ $D_X$ is a second auxiliary view of the required line A D.
23, parallel to 13 and 14, is the ray in which the top view of D will be seen.
24 locates $D_T$ at the crossing point of the rays 12 and 23, in each of which the top view of D is to be seen.
25 is the ray in which the front view of D is to be seen.
26 is a level line that is the same distance *h* above the base line 3, as the level 10 is above 4, and the level 21 is above 15.

27 locates the front view of the point D at the level 26 and on the ray 25. The distance *h* that D is above A in the front view thus agrees with the difference in elevation of these points as shown in the two auxiliary views.
28 is the front view of the required line A D.
29 is the top view of the required line A D. The front, top, and auxiliary views of the line A D are drawn heavy.

The solution of the problem is now complete.

**76. Problems.**

Group 28. Drawing One Line Perpendicular to Two Lines.
Group 29. Objects Having Edges or Axes at Right Angles.

**77. Common Perpendicular to Two Lines.**—The solution of this problem is useful, whenever it is necessary or desirable to connect two non-intersecting and non-parallel straight-line parts by means of another straight-line part, in order to meet either or both of the requirements that the connecting part is to have the shortest possible length or is to be perpendicular to each of the first two parts. Examples of such cases are: Connecting two pipes by means of a third pipe, when using right-angled pipe fittings; determining the least diameter of skew gears to drive one shaft from another; locating the shortest tunnel to connect two tunnels.

It is possible to draw but one line perpendicular to two lines that are neither intersecting nor parallel. The common perpendicular will be the shortest distance between the two lines.

The following problem may be taken as a specific example: Find O P, the common perpendicular to the two lines A538 B815 and E406 F718. The required line O P is to be perpendicular to each of the given lines, and will intersect them respectively at O and P. The given data are shown and the solution is carried out in Fig. 40.

Since the required line is to be perpendicular to each of the given lines, to solve the problem it will be necessary to draw a normal view of one of the given lines, and then to draw a view that is a normal view of the other given line or of the required line.

Draw first a normal auxiliary elevation of the line A B and in this same view show the line E F. This view is taken in a horizontal direction parallel to the arrow that is perpendicular to the top view of the line A B in Fig. 40.

Draw the rays 1, 2, 3, and 4 parallel to the arrow.

Draw the base line 5 and take this as the level of the point $B_x$ in the auxiliary elevation.

$F_x$ is at the same level as $B_x$ and on the ray 3.

Locate 6 and 7, the levels of $A_x$ and $E_x$, to agree with their relative levels in the front view.

Draw 8 and 9, the auxiliary views of the lines A B and E F. The auxiliary view is a normal view of the line A B. The required line O P will appear perpendicular to A B in this view, but, as yet, it cannot be drawn because its location is still unknown.

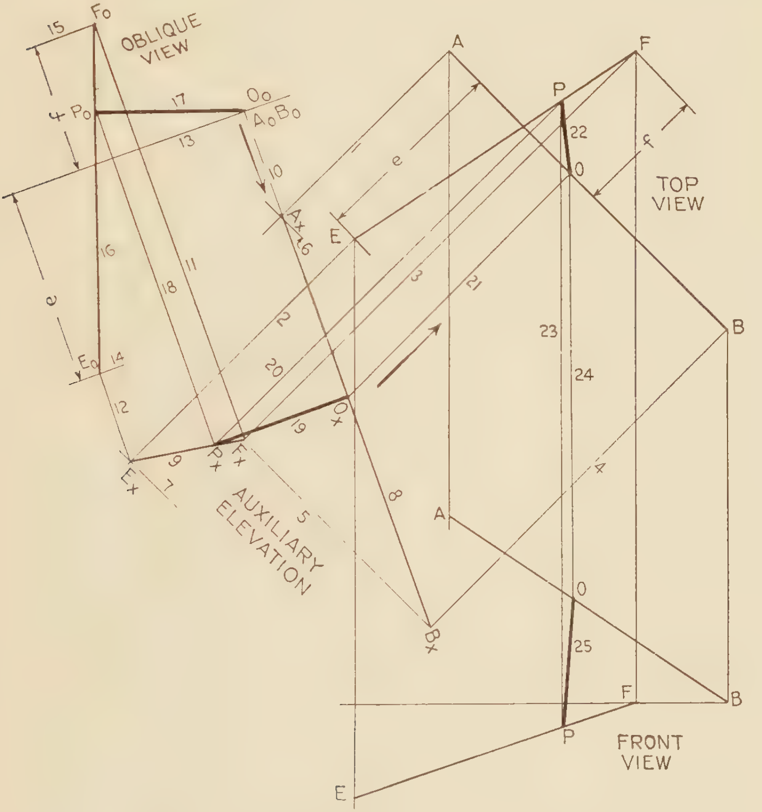


Fig. 40.—Common perpendicular to two lines.

Draw next an oblique view taken in a direction parallel to the auxiliary view of the line A B as indicated by the second arrow. This oblique view will be an end view of the line A B.

Draw the rays 10, 11, and 12 parallel to the arrow.

Draw the base line 13, and locate $A_o$ and $B_o$ on 13 and 10.

Locate the lines 14 and 15 at the distances *e* and *f* from 13, as determined from the top view, and letter the points $E_o$ and $F_o$.

Draw 16, the oblique view of the line E F.

The oblique view will be a normal view of the required line O P, because this view is an end view of the line A B to which O P is perpendicular. Since the oblique view is a normal view of the line O P, and since O P is also to be perpendicular to E F, O P will appear perpendicular to E F, in this view.

Locate $O_o$ on $A_o$ $B_o$, and through $O_o$ draw 17 perpendicular to $E_o$ $F_o$. This is the required perpendicular O P.

Locate $P_o$ at the intersection of 17 and $E_o$ $F_o$.

Draw the ray 18 and locate $P_x$ on $E_x$ $F_x$.

The location of $O_x$ is somewhere on the auxiliary view of the line A B; but this location cannot be determined from the oblique view. Since O P is perpendicular to A B, and since the auxiliary view is a normal view of the line A B, in this view O P will appear perpendicular to A B.

Through $P_x$ draw 19 perpendicular to $A_x$ $B_x$.

Locate $O_x$ on 19 and on $A_x$ $B_x$.

$O_x$ $P_x$ is the auxiliary view of the common perpendicular.

Draw rays 20 and 21, and locate the top views of P and O on the top views, respectively, of E F and A B.

Draw 22, the top view of the required common perpendicular.

Draw the rays 23 and 24, and locate the front views of P and O, each on its proper line.

Draw 25, the front view of the required common perpendicular.

See that the line O P is drawn as a heavy line in each view. This completes the solution of the problem.

As a check on the accuracy of the work, the distance between the top view of P and the top view of A B should be equal to the distance that $P_O$ is from the base line 13; and the relative elevations of the points O and P in the front view and in the auxiliary elevation should be the same.

If the shortest distance between the lines A B and E F is required, it may be measured from $O_O$ to $P_O$ in the oblique view which is a normal view of the line O P.

**78. Problems.**

Group 30. Common Perpendicular to Two Lines.
Group 31. Straight-line Questions.

**79. Angle between Two Lines.**—It is possible to find the angle between two lines, A B and A C, for example, by drawing the views of the triangle A B C, and then determining the true length of each side of the triangle and drawing the triangle in its true shape. A more direct plan is explained in the chapter on planes.

**80. Summary.**—Objects are represented and problems are solved by drawing principal, auxiliary, and oblique views. In each of the principal views, two of the three rectangular dimensions of the object may be measured. In an auxiliary view, one of the rectangular dimensions may be measured; and in an oblique view, none. Parallel lines appear parallel in every view. Perpendicular lines appear perpendicular only in a view that is a normal view of one or both of the lines.

## CHAPTER VII

## CURVED LINES

**81.** The edges of structures and objects are lines. A line may be either straight or curved. A straight line is generated by a point that moves in one direction.

**82. Curved Lines.**—A curved line is generated by a point that moves in a constantly changing direction. The exact nature of each curved line is determined by the law that governs the motion of its generating point.

Curved lines may be considered as made up of short, straight-line elements, infinitesimal in length. The generating point is conceived as moving an infinitesimal distance in one direction, and then changing its direction and again moving an infinitesimal distance. No two consecutive elements of a curved line lie in the same direction. A tangent to a curved line at any point has the same direction as the element of the curve at that point.

**83. Types of Curved Lines.**—There are two classes of curved lines—single-curved and double-curved. A chart that gives the classification of lines is shown on page 36.

**84. Single-curved Lines.**—All elements of a single-curved line lie in the same plane. Single-curved lines are also called plane curves. The most frequently used single-curved lines are the circle, the ellipse, the parabola, the hyperbola, the involute, and several kinds of spirals and cycloids. The circle, ellipse, parabola, and hyperbola are called the conic sections, since they may be derived by taking plane sections of a right circular cone.

**85. Double-curved Lines.**—No three consecutive elements of a double-curved line lie in the same plane. The most frequently used double-curved lines are the cylindrical helix and the conical helix.

**86. The Circle.**—A circle is a single-curved line generated by a point which moves so that its distance from a fixed point, called a center, is constant. The constant distance is the radius. The word "circle" is also used to designate the plane area within the circle. A circle is specified by stating its radius or diameter, the location of its center, and its axis or plane. The size of a circle should generally be specified by stating its diameter. The tangent to a circle is perpendicular to the radius at the point of tangency.

Figure 41 shows the front, top, and right-side views of a frontal circle. The front view is here a normal view of this circle and shows its true shape. The top and right-side views are edge views of the circle. The straight line, that represents the circle in an edge view, should be visualized as a circle having points both on the side near the observer and on the far side. Although the circle is a very familiar figure, there is danger that it will not be correctly visualized when it is seen as a straight line. A circle may appear in a view as a circle, an ellipse, or a straight line.

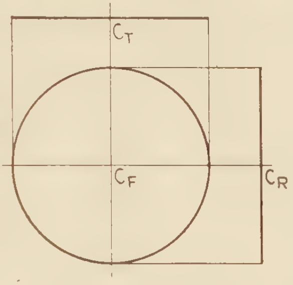


Fig. 41.—The circle.

The straight line that passes through the center of a circle and is perpendicular to the plane of the circle may be called the axis of the circle. The axis is the center line of the shaft on which the circle would rotate when running true on its shaft.

**87. Inclined Circle.**—A circle having an inclined axis A B is shown in Fig. 42. The circle is first drawn in the auxiliary view on the center $C_A$. The setting of the compass used in drawing this circle should be retained, as it will soon again be needed. The front view is next drawn. This is an edge view of the circle.

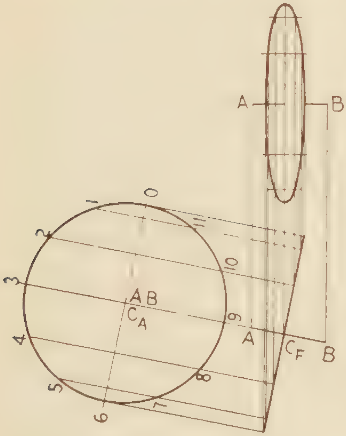


Fig. 42.—Inclined circle.

Before attempting to draw the top view, a number of points, not less than twelve, should be located on the circle in the auxiliary view. These should be evenly spaced around the circle. Draw first the horizontal and frontal diameters 3 9 and 0 6. Using as centers the points last named, locate the remaining points by means of the compass, set to the radius of the circle.

The points are consecutively numbered and are then transferred, by means of rays, to the front view.

Next, draw the rays leading to the top view. Locate each point in the top view on the ray in which it is seen and at the proper distance in front of or behind the axis. This distance may be determined from the auxiliary view. The distances may be transferred from the auxiliary view to the top view by means of the compass or a paper scale. As an aid in transferring points, it should be noted that the points 2 and 4 are the same distance behind the axis that the points 10 and 8 are in front of the axis. Three settings of the compass will transfer all twelve points. The points in the top view must be carefully located if the resulting ellipse is to have the proper shape. If greater accuracy is desired, the circle should be divided into twenty-four equal parts. After the points in the top view are located, the curve may be drawn through the points. This should be done with care, by drawing the curve freehand or with the aid of a French curve.

In all drawings, wherever circles appear as circles, it is nearly always best to draw the circles first. This method of procedure will greatly aid in keeping symmetrical the different views of an object, and also will save time and work by considerably decreasing the number of times that the diameter of the circle must be measured in the different views.

**88. Circle in Oblique Position.**—Figure 43 shows the views of a circle in an oblique position. The point C is the center of the circle, and A C is the axis of the circle. Starting with the front and top views of the axis, a normal view of the axis is next drawn. This is here taken as a normal auxiliary elevation of the axis. The oblique or end view of the axis $A_0C_0$ is now located. In this view the true shape of the circle will be seen, and here the circle of the desired diameter is drawn. The horizontal diameter 9 3 and the diameter 0 6 perpendicular to it are drawn. Twelve or more points equally spaced are then located on the circle, and numbered, and these points are transferred by means of rays to the auxiliary elevation of the circle. The last-named view is an edge view of the circle. In it the circle appears perpendicular to the axis A C.

The rays leading from the auxiliary elevation to the top view are now drawn, and the top view of each point is located on its own ray, at the correct distance from the axis, and on the proper side of the axis. These distances may be transferred from the oblique end view to the top view by means of a paper scale or a compass. Care must be taken that points located in front of the axis in the end view are not placed behind the axis in the top view.

The rays leading from the top view to the front view are now drawn, and each point in the front view is located at its proper level, as determined from the auxiliary elevation. Each view of every point should be lettered as soon as it is located. The front and top elliptical views of the circle are now drawn and the solution of the problem is finished.

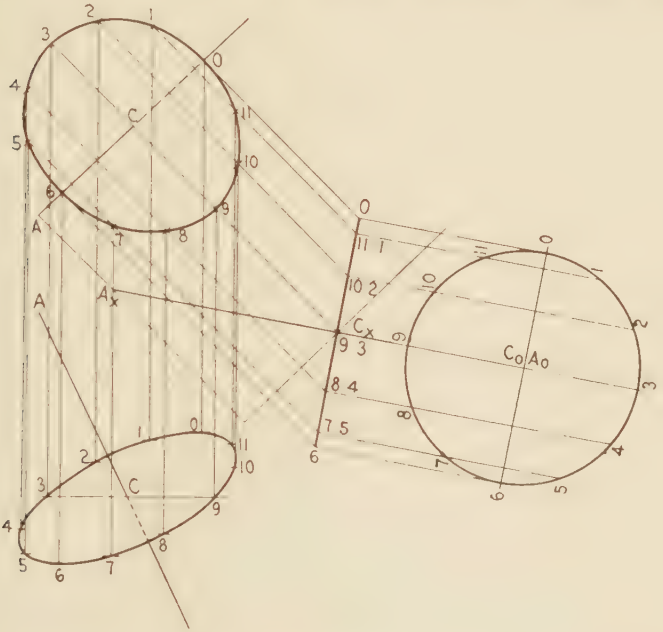


Fig. 43.—Circle in oblique position.

In each view where the circle appears as an ellipse, it should be noted that the minor axis of the ellipse coincides with the axis of the circle. In visualizing this, however, it should be realized that the minor axis represents a diameter of the circle that is perpendicular to the axis, as are all diameters of the circle. The major axis is, of course, always perpendicular to the minor axis. It should also be noted in each view in which the circle appears as an ellipse, that the diameter which is the major or minor axis in one view is neither the major nor the minor axis in the other view.

The diameter that is seen in its true length as the major axis in the front view, for example, is a frontal line and must so appear in the top view.

### 89. Problems.

Group 32. The Circle.

**90. The Ellipse.**—An ellipse is a single-curved line generated by a point which moves so that the sum of its distances from two fixed points, called foci, is constant.

Elliptical-shaped structures are not very common, but the ellipse often appears in drawings as a view of a circle. An ellipse is shown in Fig. 44. The longest diameter A B is the major axis, and the shortest diameter C D is the minor axis. Each axis is a perpendicular bisector of the other axis. If the ellipse is horizontal, the top view will be a normal view of the ellipse and will show its true shape, and the front view will be an edge view of the ellipse as is shown in the figure.

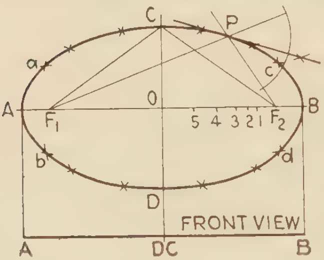


FIG. 44.—The ellipse.

One focus of the ellipse is located at $F_1$, and the other at $F_2$. The point P is taken as any point on the ellipse. The definition of the ellipse requires that as P moves around the curve, the sum $P F_1 + P F_2$ shall remain constant. This constant can be represented by a straight line of fixed length. Since the point B is on the curve, the sum $B F_1 + B F_2$ must be constant; and since $B F_2 = A F_1$, this constant is equal to A B, the major axis of the ellipse. Also, since C is a point on the curve, $C F_1 = C F_2 =$ one-half of the major axis. Knowing this, it is possible to locate the foci with the aid of a compass, if the major and minor axes are known.

Given the definition of the ellipse, and the major and minor axes—or either axis and the distance between the foci—to draw the ellipse, proceed as follows: Locate the foci. Next, on the major axis, as shown in Fig. 44, locate a number of points, as 1, 2, 3, 4, and 5. Set one compass to the length B 1, and another compass to the remaining length of the major axis A 1. The sum of these lengths is equal to the length of the major axis. Next, center one compass on one focus and center the other compass on the other focus. Swing the leads of the compasses near each other, and with each compass mark a short arc where the two circles cross. Locate a second crossing point on the other side of the major axis. Now change each compass to the other focus, and locate two more crossing points. Four points as shown at *a*, *b*, *c*, and *d* in Fig. 44 are thus found. Locate as many points of the ellipse as are needed, by setting the compasses to the other divisions 2, 3, 4, etc., marked on the major axis. Points may be located on the ellipse by using one compass instead of two, but this is much slower. To produce accurate work, the lead of the compass must be kept very sharp.

After a sufficient number of points is found, draw the curve of the ellipse. Curves may first be sketched in freehand in a rather fine line, providing the work is very carefully done. Sketch the curve around the ends of the axes, not up to the ends. The ends of an ellipse are not pointed. The French curve should then be used to finish the penciling. The tangent to the ellipse at the point P bisects the exterior angle between the focal radii, as is shown in Fig. 44.

**91. The Pin and String Ellipse.**—A rough ellipse may be drawn by setting a pin or stake at each focus and a third pin or stake at some point of the curve. Around these three pins or stakes is stretched an inelastic cord. The third pin or stake will trace an ellipse if it is now moved within the loop of the cord, providing the cord is kept taut. This ellipse is called the "gardener's ellipse." A fairly accurate large ellipse may be drawn by this method if care is used in carrying out the details. It is not a practical plan for accurate work on the drafting board.

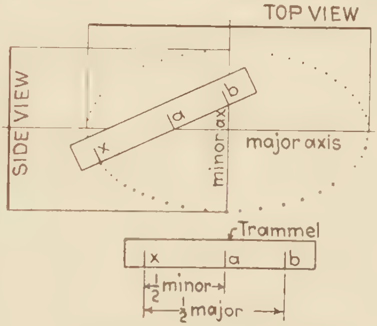


FIG. 45.—Ellipse, trammel method.

**92. Ellipse by Trammel Method.**—When the major and minor axes of an ellipse are known, the draftsman often uses a strip of paper called a trammel for locating points on the curve. As shown in Fig. 45, a mark is placed on the straight edge of the trammel at a point *x*, and two other marks, *a* and *b*, are made at distances equal, respectively, to one-half the minor axis and one-half the major axis from the point *x*. The trammel is then placed so that the point *a* is on the major axis and the point *b* is on the minor axis of the ellipse. In this position the point $x$ will indicate the location of a point on the ellipse. This point should be marked on the drawing. The trammel should be moved to new positions and the process repeated until a sufficient number of points is found. The points should be marked lightly and not indented into the paper with the lead pencil. Enough points should be located so that each point will check the location of the adjacent points. The points on the curve should be located closer together as the curve becomes sharper.

The trammel method of locating points on an ellipse is the quickest method, and it is accurate if the work is carefully done. The axes should be clean-cut lines drawn with a sharp pencil, and the marks on the trammel should be sharp and should extend to the edge of the trammel.

The ellipsograph is an instrument used for drawing ellipses and for cutting elliptical shapes in glass and in metal. The design of the ellipsograph is usually based on the trammel principle, explained above. Elliptical shapes may be turned in the lathe or cut in the milling machine by using a chuck based on the same principle. Methods of drawing approximate ellipses are described in engineering drawing textbooks.

### 93. Problems.

Group 33. The Ellipse.

**94. The Parabola.**—A parabola is a single-curved line generated by a point which moves so that its distance from a fixed point, called a focus, is constantly equal to its distance from a fixed straight line, called a directrix.

Figure 46 shows a parabola. F is the focus and C D is the directrix. The definition of the parabola requires that the point P on the curve shall move so that P F = P X; that is, the focal radii are equal. P X is perpendicular to the directrix.

To draw the parabola: Locate the focus and directrix. Draw a number of straight lines, 1, 2, 3, etc., parallel to the directrix. Set the compass to the distance that one of these lines is from the directrix and, using the focus as a center, describe two short arcs across this line. The two points thus located are points on the curve. In a similar manner, locate two points on each of the lines that is parallel to the directrix. The vertex V of the curve is halfway between the focus and the directrix. Locate a sufficient number of points, and then pencil the curve according to the plan given in Art. 90 for penciling the ellipse. The tangent to the curve at the point P is the bisector of the angle F P X.

The parabola appears in structures of the beam type that are designed to carry the required load and at the same time to use the least amount of material possible in the beam. Some arches and road sections are also parabolic in shape. Since, in Fig. 46, P F and P K make equal angles with the tangent, a reflector that is parabolic in shape and has a light located at F, will throw a pencil of light, because it will reflect all rays of light parallel to the axis. The parabola also appears frequently in the development of the theory of engineering problems.

**95. Problems.**

Group 34. The Parabola.

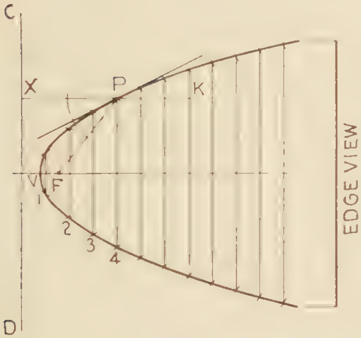


FIG. 46.—The parabola.

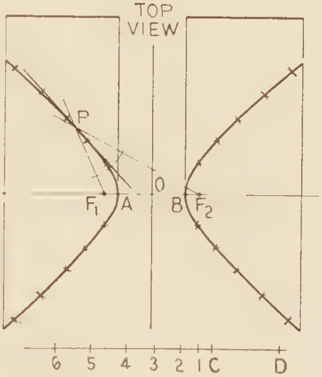


FIG. 47.—The hyperbola.

**96. The Hyperbola.**—An hyperbola is a single-curved line generated by a point that moves so that the difference of its distances from two fixed points, called foci, remains constant. This definition should be compared with that of the ellipse.

An hyperbola is shown in Fig. 47. $F_1$ and $F_2$ are the foci or fixed points. The point P, on the curve, moves so that the difference of the distances P $F_1$ and P $F_2$ remains constant. This constant may be expressed as a line of a certain length.

To draw the hyperbola, take a straight measuring line, as shown in the lower part of Fig. 47, and mark off the common difference C D. Locate several points, numbered 1 to 6, on this line. Set one compass to the distance D 3, for example, and set another compass to the distance C 3. The difference in these distances is C D. Now center one compass at $F_1$ and the other at $F_2$, and draw short arcs that cross at two points. These points are on the curve. Interchange the compasses on the foci and locate two more points. Reset the compasses to a new numbered point on the measuring line and repeat the above operation. When a sufficient number of points is found, draw the hyperbola. An hyperbola has two separate branches. The tangent to the curve, at any point P, is the bisector of the angle between the focal radii P $F_1$ and P $F_2$.

The hyperbola is the curve that represents the relation between the volume and the pressure of a perfect gas. The hyperbola appears in only a few structures, some of which are described in the chapter on warped surfaces.

**97. Problems.**

Group 35. The Hyperbola.

**98. Spirals.**—Of the various spirals, the spiral of Archimedes is most frequently used. The spiral of Archimedes is a single-curved line generated by a point that revolves at a uniform rate about a fixed point, called the pole, and at the same time moves at a uniform rate toward or away from the pole. In other types of spirals the generating point moves at a variable rate.

The Archimedean spiral is used in the threads of scroll chucks and similar devices. Here the thread is cut into a plane disk and travels uniformly away from the center as it travels around the center. The outlines of cams, designed to convert uniform rotary motion into uniform reciprocating motion, are Archimedean spirals.

In Fig. 48 is shown an Archimedean spiral. The pole is at P. The distance that the generating point travels away from the pole in one turn is the lead or pitch of the spiral. In the figure a right-hand spiral is shown.

To draw the Archimedean spiral, it is first necessary to locate the pole and to determine the lead and hand of the spiral. A paper scale S, as shown in Fig. 48, provides a convenient means of locating various positions of the generating point. The lead of the spiral is marked on the scale and is divided into any convenient number of equal parts. Not less than twelve or sixteen divisions should be made. If more than one turn of the spiral is to be drawn, other points, as A and B, should be marked on the scale at the distance indicated, but these distances should not be divided into smaller divisions. The space around the pole is divided into equal sectors: by drawing a circle of any convenient radius, with the pole as its center; dividing the circle into the same number of equal parts into which the lead was divided on the paper scale; and drawing the radii of the circle through these division points. The radii should be extended as far as is necessary.

Points on the spiral are located by first assuming that the generating point is at the pole, and that it is beginning to move away from the pole on some chosen radius, P 0, for example. The generating point now moves one-twelfth of a revolution around the pole and one-twelfth of the lead away from the pole. This point may be located on the radius P 1 by means of the scale, and at the same time a second point on the next turn of the spiral may also be located on the same radius. The scale should be placed with the first one-twelfth division at the pole, and the zero point 0 and the points A and B on the radius P 1. The locations of the three points last named are marked on the radius, and indicate points on three turns of the spiral. The scale is next moved to the radius P 2; the second one-twelfth division point on the scale is set at the pole; and one or more points of the spiral are marked on this radius. This process is continued around the circle until points have been located on each radius. The curve may now be drawn. Instead of a paper scale, a draftsman's scale may be used, providing it carries a division equal to

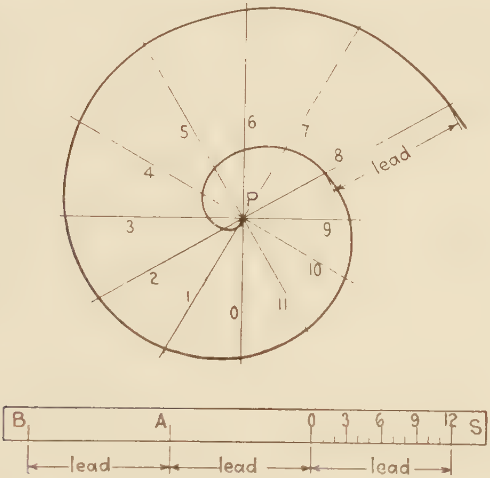


FIG. 48.—The Archimedean spiral.

the lead of the spiral. Approximate methods of drawing spirals may be found in textbooks on engineering drawing.

**99. Problems.**

Group 36. The Archimedean Spiral.

**100. The Involute.**—The involute is a single-curved line generated by a fixed point in a straight line which rolls on a circle. The straight line is tangent to the circle and must roll without slipping. An involute may be generated by fixing a marking point in a thread that is wound around a cylinder, and keeping the thread tight as it unwinds from the cylinder; but this method should not be used for accurate work.

The involute is used as the curve to which gear teeth are shaped so that the teeth will roll and not slide on each other. A few other curves are used for this purpose, but the involute system of gearing is for most purposes supplanting the other systems.

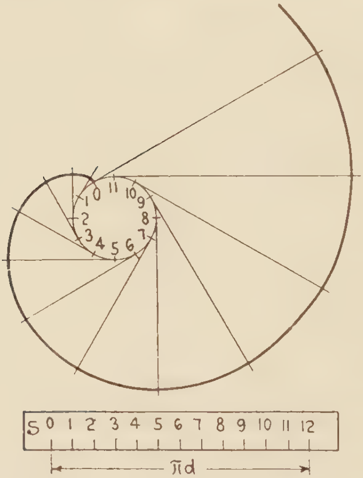


FIG. 49.—The involute.

An involute is shown in Fig. 49. Before drawing the involute, the size of the director circle and the hand of the involute must be determined. The circle is divided into twelve or more equal parts, and a tangent extending in one direction is drawn at each of these points. The circumference of the director circle is calculated. A scale S is made by marking the length of this circumference on a straight line, and then dividing this length into the same number of equal parts as the number into which the circle is divided. In the figure, the circle has been divided into twelve equal parts. Since any line and any arc may be bisected, it may be better to use sixteen or thirty-two parts.

The generating point starts at some point 0 on the circle. The tangent rolls until it is tangent at the point 1. The generating point is now on the tangent, and one-twelfth of the circumference from the tangent point 1. This is the distance that the tangent has unrolled from the circle. This distance is measured with the scale, and the location of the generating point is marked on the tangent. The tangent now rolls to 2, and by means of the scale a point on the curve is marked two-twelfths of the circumference from 2. This process is carried on for as many turns of the involute as are needed.

It should be noted that the curve is perpendicular to the tangent, because the generating point is rotating around the point of tangency as an instantaneous center. This information aids the draftsman when he draws the curve. The mistake should not be made of extending the tangents in the wrong direction from the point of tangency.

**101. Problems.**

Group 37. The Involute.

**102. The Cycloid.**—A cycloid is a single-curved line generated by a fixed point on a circle that rolls on a straight line. Compare this definition with that of the involute. The cycloid is the path traced by a point on the rim of a rolling wheel. This curve is also used to a limited extent for the outlines of gear teeth.

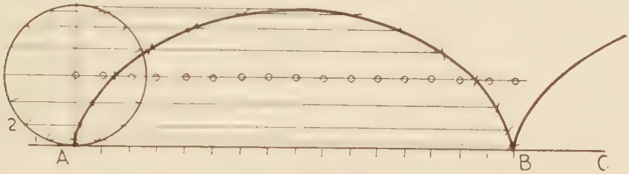


FIG. 50.—The cycloid.

In Fig. 50 a cycloid is shown. Before drawing the cycloid, it is necessary to determine the diameter of the rolling circle. The circumference of the rolling circle is then calculated and marked off on the straight-line director. This distance, indicated as A B in the figure, is divided into any number of equal parts. Since any line may be bisected, it is desirable to divide this length into sixteen or thirty-two equal parts. The rolling circle is divided into the same number of equal parts. The rolling circle is shown at the left of the figure, and the generating point is taken at the lowest point on the circle. The circle then rolls to the right along the straight line and temporarily is brought to a stop when the center is above the first division point. The generating point is now at a higher level, as indicated by the point 1 on the circle, and is distant from its new center a distance equal to the radius of the rolling circle. This point is readily located by a short arc crossing the level line through 1. The circle now rolls to the next division, and another point is found at the level 2, and so on. After the points are found, the curve is drawn.

### 103. Problems.

Group 38. The Cycloid.

**104. The Epicycloid.**—The epicycloid is a single-curved line generated by a fixed point on a circle that rolls on the outside of another circle. This curve is used to a limited extent for the outline of gear teeth.

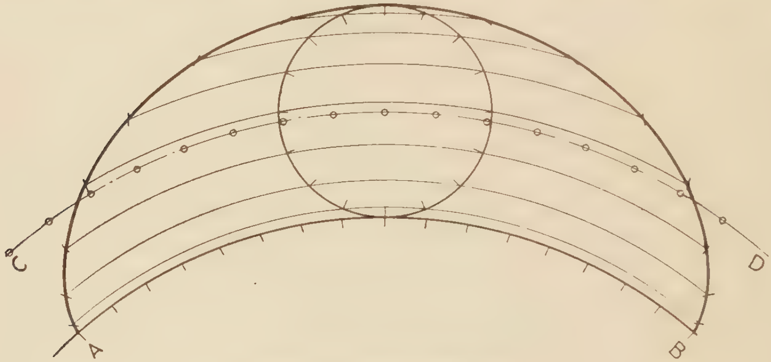


Fig. 51.—The epicycloid.

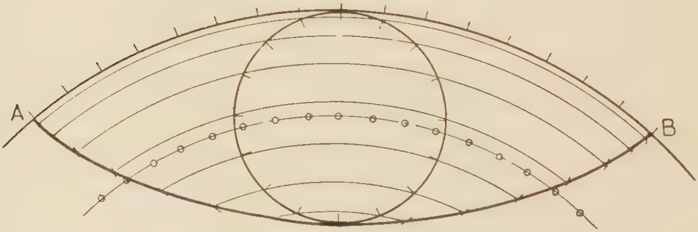


Fig. 52.—The hypocycloid.

Before starting to draw the epicycloid, it is necessary to determine the diameters of the fixed director circle and of the rolling circle. The circumference of the rolling circle is then calculated, and this distance is marked off, as shown from A to B in Fig. 51, on the arc of the director circle. This distance may be measured by stepping off the distance around the arc, or by calculating the angle between the radii of the fixed circle. The method to be used depends upon the accuracy desired. The arc of the director circle and the circumference of the rolling circle should now be divided into the same number of equal parts. Since it is always possible to bisect an arc, this number should be sixteen or thirty-two. Eight divisions are not usually sufficient. The greater the number of divisions the more nearly accurate the work is likely to be.

As the generating circle rolls on the director circle, each division point on the generating circle will in turn come into contact with the corresponding division point on the fixed arc, and the center of the generating circle will move along the arc C D. The locations of the center when the division points are in contact should be determined by marking the points where each radius through the division points on the arc A B meets the arc C D. The top point of the rolling circle in its central position is taken as the generating or marking point. As the circle rolls and the different division points come into contact, the generating point will be located successively on the arcs that have been drawn concentric with the arc A B, at a distance from the successive centers equal to the radius of the rolling circle. After these points are located, the curve may be drawn.

**105. The Hypocycloid.**—The hypocycloid is a single-curved line generated by a fixed point on a circle that rolls on the inside of another circle. This curve also is used, to a limited extent, for the outline of gear teeth.

The hypocycloid is shown in Fig. 52. The method of drawing it is similar to the method of drawing the epicycloid.

**106. Problems.**

Group 39. Epicycloids and Hypocycloids.

**107. Double-curved Lines.**—In a double-curved line no three consecutive elements lie in the same plane. Lines of this type are also called space curves. A double-curved line cannot be completely represented in a single view. At least two views must be drawn. A double-curved line will appear curved from every point of view. It is impossible to obtain a normal view of such a line. The most commonly used double-curved lines are the cylindrical helix, or simply the helix, and the conical or tapering helix.

**108. Problems.**

Group 40. Double-curved Lines.

**109. The Helix.**—The helix is a double-curved line generated by a point that revolves around an axis at a uniform rate, and at the same time moves parallel to the axis at a uniform rate. A helix may be right hand or left hand; that is, it may be generated by a point that may revolve in either direction around the axis or that may move in either direction parallel to the axis. The distance that the generating point travels parallel to the axis in one turn or revolution is called the lead or pitch of the helix. It is possible to have a helix of variable lead.

The curve of the helix is very generally used in structures: in screw threads, in springs, in "spiral" stairways, and in conveyers. Its use in connection with the structures just named is explained in the chapter on warped surfaces. The helix should not be confused with the spiral.

Before drawing the helix, it is necessary to determine the diameter, hand, and lead of the helix. In Fig. 53 the cylinder of the helix is first drawn. As the generating point travels around this cylinder, it also travels along the cylinder. The circle is divided into any number of equal parts, not less than twelve or sixteen. The rays extending from these division points to the front view may now be drawn.

In the end view, the generating point appears to move around the cylinder as many times as there are turns in the helix. When the generating point makes one turn in the end view, it travels parallel to the axis in the front view a distance equal to the lead. When the generating point makes one-sixteenth of a turn in the end view, it travels parallel to the axis in the front view a distance equal to one-sixteenth of the lead.

To locate points on the helix, a scale, as shown at S, may be made. On this scale the lead is marked off as many times as there are turns in the helix, and the lead at one end of the scale is divided into as many equal divisions as were found for the circle. One end of the cylinder may be used as a base line for measuring; the top end C, for example. Starting with any point P on the cylinder, the scale is placed with its zero point at P on the base line and with its edge parallel to the axis in the front view, and each lead point, 0, A, and B, is marked on the ray. The scale is now moved to the next ray, and then parallel to that ray until the next division point 1 of the scale is on the base line. The lead points are then marked on this ray. As the scale is moved from one ray to the next, the scale is moved one-sixteenth of the lead parallel to the axis. This operation is continued from one ray to the next, until an entire revolution has been completed.

The turns of the helix may now be drawn, but care must be taken to make dotted the parts of the curve that are at the rear of the cylinder. This will depend upon whether the helix is right hand or left hand. The helix may be considered simply as a curved line, independent of a cylinder, in which case there will be no hidden part. The end view of the helix is a circle. As an aid in drawing the helix accurately, it should be noted that the curvature of the helix is reversed in the middle, and that it cannot come to a sharp point at the sides.

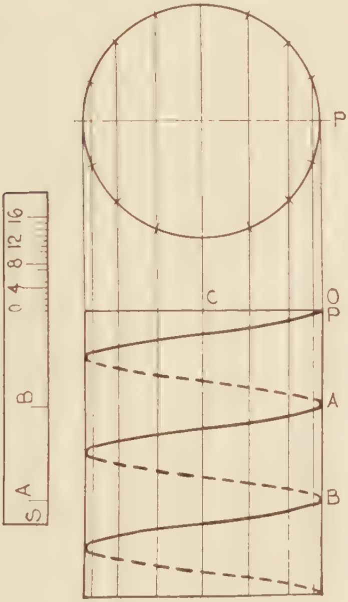


Fig. 53.—The helix.

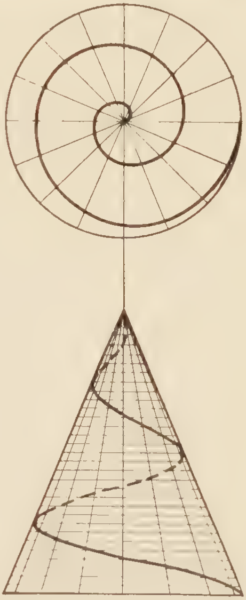


Fig. 54.—The conical helix.

**110. Problems.**

Group 41. The Helix.

**111. The Conical Helix.**—The conical helix is a double-curved line generated by a point that revolves around an axis at a uniform rate, and at the same time moves at a uniform rate along a line that intersects the axis. It is a helix on a cone.

To draw a conical helix, it is first necessary to determine the lead and hand of the helix, and the taper of the cone. The lead should be measured parallel to the axis of the cone. The method of locating points on the conical helix is similar to the method used for the cylindrical helix, except that here a scale does not save time. Instead, it is probably better to rule the divisions of the lead across the front view of the cone, as in Fig. 54. In the top, or end view, the conical helix appears as a spiral of Archimedes.

**112. Problems.**

Group 42. The Conical Helix.

## CHAPTER VIII

## PLANES

**113.** The classification of surfaces is shown in the chart on page 36. Surfaces are classified either as ruled or double-curved. A ruled surface is generated by a moving straight line called the "generatrix." A plane is a ruled surface generated by a moving straight line that constantly touches two intersecting straight lines, called "directrices."

A plane face of a structure is never a true plane. The allowable variation from a true plane depends upon the use of the surface. Surface plates and the flat faces of master gages are made by a lapping process, with a variation from a true plane of not more than one one-hundred-thousandth of an inch. With greater care, they can be made to vary less than one-millionth of an inch from a true plane. The accuracy of these planes is tested by optical methods.

**114. Representation of Planes.**—All structures are bounded by surfaces. The plane surfaces or faces of a structure are usually indicated in drawings by showing the lines and points of the surface. These lines and points are generally edges and corners of the structure. A plane surface of a structure is limited in extent by the lines that bound it. These lines may be straight, and form a triangle, square, or other polygon, or they may be curved, and form a circle, ellipse, or other single-curved line.

The location of a plane may be definitely specified by stating the location of three points that lie in the plane. These three points, however, must not lie in the same straight line. Two intersecting lines, two parallel lines, a point and a line, a triangle, may also serve to locate a plane. Certain planes may be specified by stating the location of one or two points in the plane, and, in addition, stating the direction of the plane, as horizontal, vertical, frontal, etc. A plane has the same name as have the points or lines that specify its location. The extent of a plane is, in theory, not limited; and when solving problems, the lines of a plane may be extended as far as is necessary.

**115. Plane Problems.**—In this chapter are explained the methods of solving problems that involve the consideration of the geometrical relations of a point and a plane, of a line and a plane, and of two planes. The method of finding the angle between two lines is also explained here, since the solution of this problem involves drawing edge views and normal views of planes.

**116. Oblique Plane.**—In an oblique plane no point is directly above or below, in front of or behind, or to the right or left of any other point of the plane. The oblique plane Q S T in Fig. 55 is definitely specified by showing the location of two views of each of the points Q, S, and T. The reader of the drawing is aided in visualizing the plane by drawing the views of the triangle Q S T. The lines of the triangle may be extended beyond the corners of the triangle to indicate that the plane is not limited to the area within the triangle.

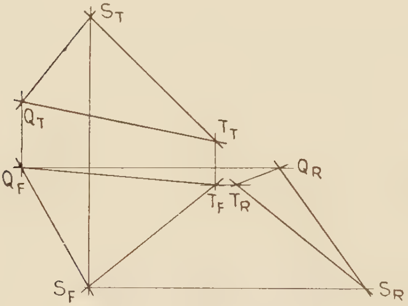


Fig. 55.—Oblique plane.

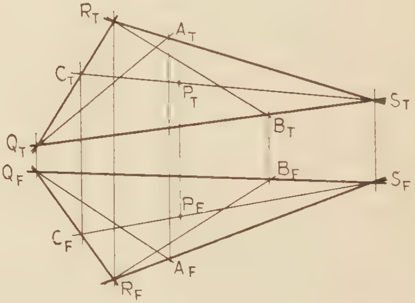


Fig. 56.—Points and lines in a plane.

**117. Points and Lines in a Plane.**—If any two points in a plane are connected by a straight line, every point on that line will lie in the plane. An oblique plane Q R S is shown in Fig. 56. The points Q, R, and S are in the plane. All points of the lines Q R, Q S, and R S are in this plane. For example, the point A on the line R S, the point B on the line Q S, and the point C on the line Q R are in the plane Q R S. The lines Q A, R B, and S C are in the plane. The line B C is also in the plane. Every point on these lines is in the plane. The point P, for example, on the line S C is in the plane. It is thus possible to locate innumerable points and lines in any plane.

**118. Horizontal and Frontal Lines of Planes.**—In Fig. 57 a horizontal line A H and a frontal line A F have been located in a given plane A B C, which may now be replaced by the plane

A F H. The line A H is a horizontal line. Its top view indicates the direction in which the plane extends. If, now, the reader of the drawing also observes that the point F is below the line A H, he can readily visualize the approximate slope and position of the plane in the top view. Similarly, the plane may be visualized in the front view, when it is observed that the line A F is frontal and that H is in front of F. It requires less effort to visualize a plane that is represented by horizontal and frontal lines instead of by oblique lines. In any given plane, all horizontal lines are parallel and all frontal lines are parallel. It is usually sufficient to visualize a plane in one view only.

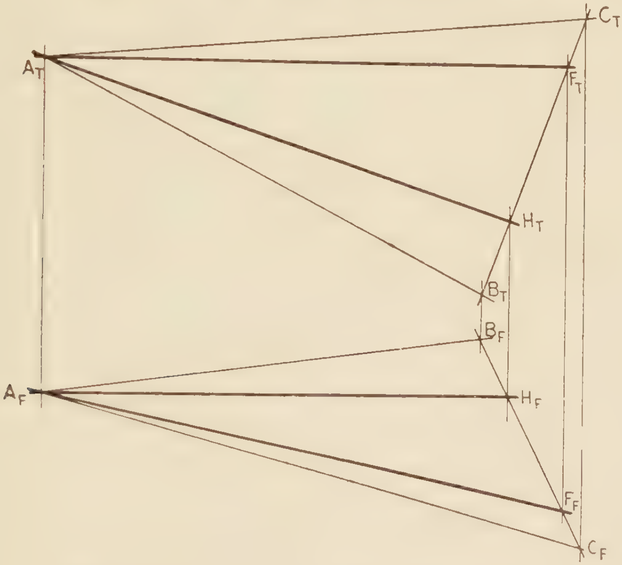


Fig. 57.—Horizontal and frontal lines of a plane.

**119. Problems.**

Group 43. Points and Lines in Planes.

**120. Direction of a Plane.** When the edge view of a plane is seen in any view, this plane may be called a "receding" plane in that view, since it extends directly away from the observer of the view. Special names may be given to certain receding planes, as "horizontal," "frontal," "profile," "vertical." In a normal view of a plane, all points of the plane are equidistant from the observer.

In a *horizontal plane* no point is above or below any other point. All points of a horizontal plane are at the same level. The front and side views are edge views, and the top view is a normal view of a horizontal plane.

In a *frontal plane* no point is in front of or behind any other point. The top and side views are edge views, and the front view is a normal view of a frontal plane.

In a *profile plane* no point is to the right of, or to the left of any other point. The front and top views are edge views, and either side view is a normal view of a profile plane.

In a *vertical plane* points may be located above or below any point in the plane. Any plane containing a vertical line is vertical. The top view is an edge view of a vertical plane. Frontal and profile planes are vertical planes in a special position.

**121. Edge View of a Plane.**—In the principal views of a structure, the plane faces of the structure usually appear as an edge view of the plane or as a normal view of the plane. The edge view of a plane is seen in any view that is taken in a direction parallel to any line of the plane. In an edge view of a plane, the plane appears as a straight line, but it should be visualized as a plane in which are located points at varying distances from the observer. Edge views are often necessary when solving problems relating to lines and planes.

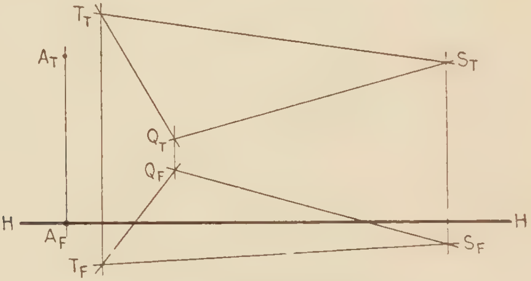


Fig. 58.—Horizontal plane.

In Fig. 58 are shown the front and top views of an oblique plane Q S T, and the edge view of a horizontal plane through the point A. The edge view may be marked H H in the front view, to indicate to the reader of the drawing that it is the edge view of a horizontal plane. In the top view this plane is not indicated, excepting as certain points or lines in the horizontal plane may appear in the solution of some problem. Other receding planes may be similarly indicated in any view that shows the edge view of the plane.

**122. Edge View of an Oblique Plane.**—An edge view of an oblique plane may be obtained by drawing an auxiliary view taken in a direction parallel to either a horizontal, a frontal, or a profile line of the plane. The edge view of an oblique plane may also be seen in an oblique view that is taken in a direction parallel to any oblique line of the plane.

To obtain the edge view of the plane R S T, Fig. 59: The front view of a horizontal line R P is first drawn. The top view of the point P is located on S T, and then the top view of the line R P is drawn. The line R P is a horizontal line that lies in the given oblique plane. The edge view of this plane will be seen in an auxiliary elevation taken in a direction parallel to the top view of R P, as indicated by the arrow. The points R, S, T, and P are located in this view, at the levels obtained from the front view, and the edge view of the plane is drawn. Here, the plane R S T appears as a straight line, but it should be visualized as a plane. Reference to the top view shows that, in the edge view, the points R, S, T, and P are at different distances from the observer.

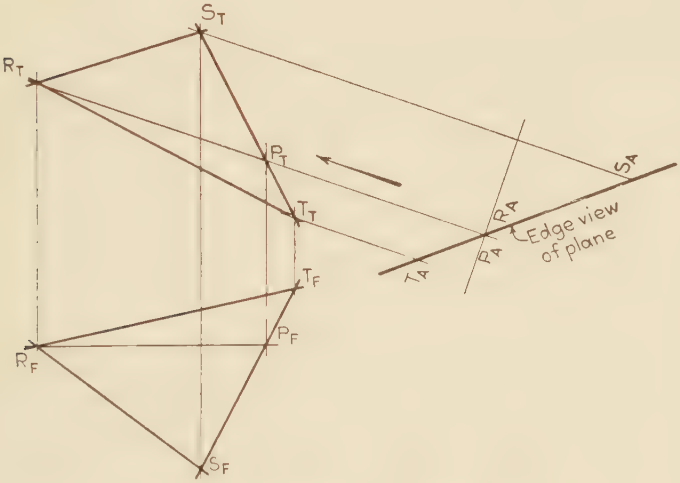


FIG. 59.—Edge view of an oblique plane.

An auxiliary elevation that shows the edge view of a plane shows the slope, or dip, of the plane; by which is meant the angle that the plane makes with the horizontal. Two points are sufficient to determine the edge view of a plane, but three points should be located to check the accuracy of the work.

**123. Normal View of a Plane.**—In a normal view of a plane all points of the plane are equidistant from the observer, and any figure in this plane is seen in its true shape. The normal view of a plane is taken in a direction perpendicular to the plane.

To draw the normal view of the oblique plane A B C, Fig. 60, locate a horizontal line C X of the given plane, and draw the edge view $A_A$ $B_A$ $C_A$ of the plane taken in the direction X C. A normal view of the given plane may now be had by drawing a view taken in the direction of the arrow that is perpendicular to the edge view of the plane. In this normal view the true shape of the triangle A B C is shown.

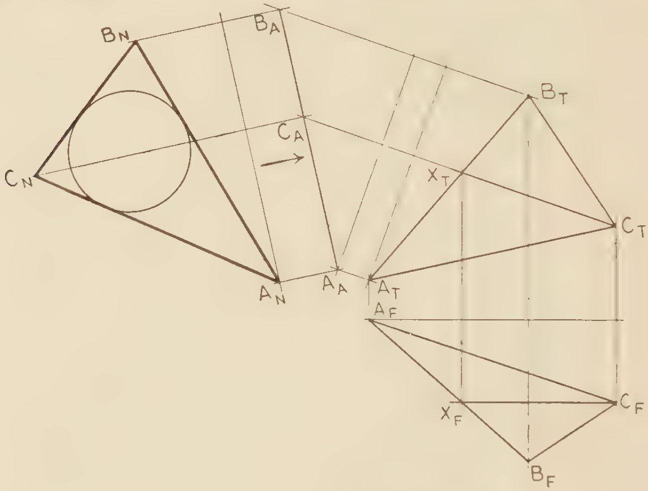


Fig. 60.—Normal view of plane.

Any figure that lies in this plane may now be drawn in its true shape in the normal view, and then transferred back through the auxiliary elevation to the top and front views. For example, to draw the circle that is inscribed in the triangle A B C, it is first necessary to obtain a normal view of the plane of the triangle, and then in this normal view to draw the triangle and the inscribed circle, as shown in Fig. 60. A number of equally spaced points should now be marked on the circle and these points transferred through the auxiliary elevation back to the top and front views.

**124. Problems.**

Group 44. Plane Figures.

**125. Parallel Relations of Line and Plane.**—To draw a line parallel to a given plane, it is necessary only to draw it parallel to any line of the given plane. Unless some other condition limits the required line, the number of possible solutions is infinite.

To pass a plane through a given point and parallel to a given line, it is necessary to draw through the given point a line parallel to the given line. The line last drawn is a line of the required plane. If no other condition limits the plane, the number of solutions is infinite.

To pass a plane through a given point and parallel to a given plane, draw through the given point two lines that are parallel to any two lines of the given plane. The two lines last drawn will definitely determine the location of the required plane.

**126. Problems.**

Group 45. Parallel Planes.

**127. Line Perpendicular to a Plane.** *Problem:* To draw a line O P through a given point O and perpendicular to a given plane Q S T, Fig. 61. The required line will be perpendicular to every line of the given plane. Its direction is completely determined by drawing it perpendicular to any two lines of the plane. This might be done by the method used in Chapter VI for drawing one

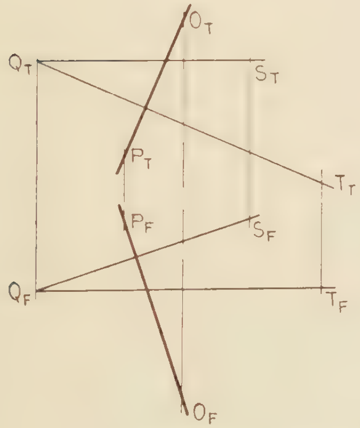


FIG. 61.—Line perpendicular to a plane.

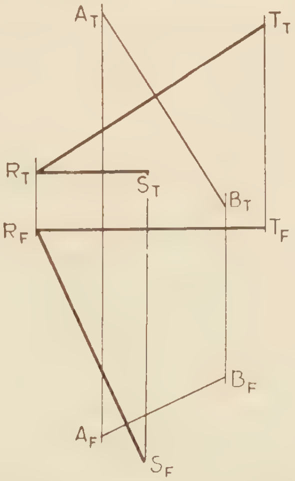


FIG. 62.—Plane perpendicular to a line.

line perpendicular to two oblique lines. There is, however, a much easier way.

The plane in Fig. 61 is represented by a horizontal and a frontal line of the plane. Since the top view is a normal view of the horizontal line Q T of the plane, in this view the required line may be drawn through the top view of O and perpendicular to the top view of the line Q T. Also, since the front view is a normal view of the line Q S, the front view of the required line may be drawn through the front view of the point O and perpendicular to the front view of the line Q S. To give a name to the line, the point P is now located anywhere on the line O P, and the solution of the problem is completed.

In case the given lines of the plane are not horizontal and frontal, it is necessary first to draw a horizontal and a frontal line of the plane and then to proceed as above.

### 128. Problems.

Group 46. Line Perpendicular to a Plane.

**129. Plane Perpendicular to a Line.**—*Problem:* To pass the plane R S T through the given point R and perpendicular to the given line A B. Through the given point R, Fig. 62, draw a horizontal line R T and a frontal line R S perpendicular to the given line A B. R S T is the required plane.

### 130. Problems.

Group 47. Plane Perpendicular to a Line.

**131. Plane Perpendicular to a Plane.**—*Problem:* To pass the plane A B C through the given line A B and perpendicular to the given plane D E F. Through any point of the given line, A, for example, draw a line A C perpendicular to the given plane, by using the method of drawing a line perpendicular to a plane. The lines A B and A C will determine the required plane.

### 132. Problems.

Group 48. Plane Perpendicular to a Plane.

**133. Intersection of a Line and a Plane. First Method.**—A line intersects a plane at some point. Two methods of finding the intersection of a line and a plane are here explained. The method first described should, as a rule, be used when it is desired to find the intersection of only one or two lines with an oblique plane. The second method, described in Art. 135, should be used when finding the intersection of one or more lines with any receding plane, or when finding the intersection of a number of lines with an oblique plane.

To find the intersection of a line and a plane by the first method: Draw a line that lies in the given plane and that at the same time lies in a receding plane containing the given line. The line last drawn intersects the given line at a point which is on the given line and in the given plane, and hence is the required point of intersection. The details of the solution are explained in the paragraph that follows.

*Problem:* Locate the point P where the line E F intersects the plane Q R S. In Fig. 63 the vertical plane V V is used as a receding plane containing the given line E F. In this vertical plane is to be located a line, 1 2, that also is in the given plane. Locate the top view of the point 1 in the vertical plane V and on the top view of the line Q R of the given plane. Next, locate the front view of 1 in its ray and on the front view of the line Q R. Similarly, locate the top and front views of a point 2 in the vertical plane V and on some other line, for example Q S, of the given plane. Draw the front and top views of the line 1 2. The line 1 2 lies in the given plane Q R S, and every point on this line is in the given plane. The lines 1 2 and E F lie in the vertical

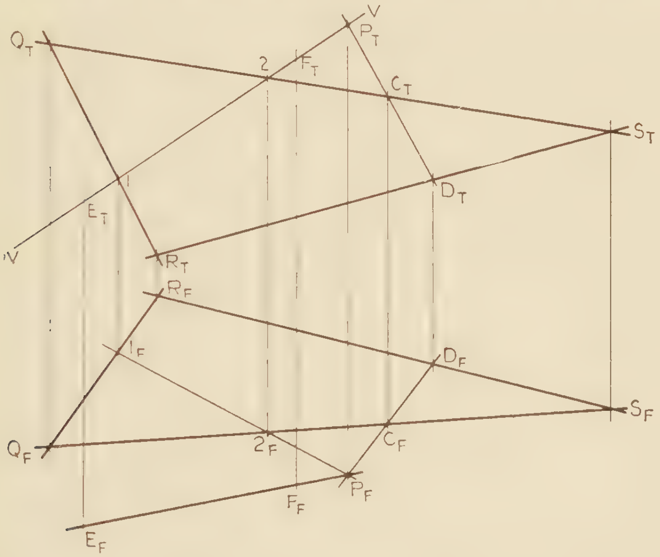


FIG. 63.—Intersection of a line and an oblique plane.

plane V, and, since they lie in the same plane, they will intersect at some point P. In the front view the lines E F and 1 2 intersect at $P_F$. The top view of P is on its ray and on the coinciding top views of the lines E F and 1 2. The point P is in the given plane, since it is on the line 1 2. It is also on the given line. The point P is, therefore, the required intersection of the given line with the given plane, and satisfies the requirements of the problem. The problem may be solved as readily by passing a receding plane through the front view of the given line.

To check the solution: Draw any straight line P C D in the top view, Fig. 63. The top views of the points C and D are located on the top views of the lines of the given plane. Locate the front views of the points C and D, each on its own line of the given plane. Draw the front view of the straight line C D. It will pass through the front view of P if the point P is in the plane. Check also to see that P is on the line E F.

**134. Problems.**

Group 49. Intersection of a Line and a Plane.

**135. Intersection of a Line and a Plane. Second Method.**—To find the intersection of a line with a plane that is receding in any view: In the view that shows the edge view of the plane, locate the point in which the line is seen to pierce the plane; and then locate the other views of this point on the other views of the given line. In Fig. 64 the given plane J K L is receding in the front view. In this view the given line is seen to intersect the given plane at the point O. Locate the top view of O on its ray and on the top view of A B, and the problem is solved.

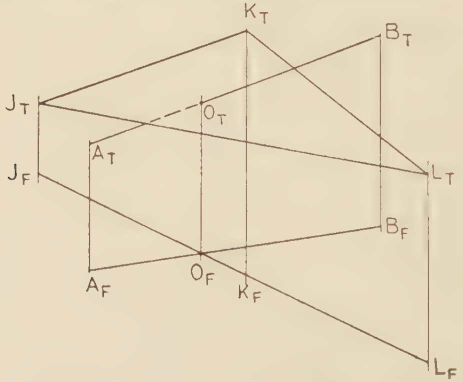


Fig. 64.—Intersection of a line and a receding plane.

When it is necessary to find the intersections of a number of lines with a plane, it is generally best to draw an edge view of the plane and then to find the intersections as explained in the preceding paragraph. For example: Find the intersection of the tetrahedron A B C D with the plane Q R S, Fig. 65. Either three or four of the six lines of the tetrahedron will intersect the plane; unless the plane should miss the tetrahedron entirely. Draw a horizontal line Q H of the plane. In the direction of this line, as indicated by the arrow, draw an auxiliary elevation of the plane and tetrahedron. In this view the edge of the plane is shown, and three lines of the tretrahedron are seen to intersect the plane. The three points of intersection are marked 1, 2, and 3. The top views of these points are now located, each on its proper line, and then the front views are located. The part of the tetrahedron that is above the plane is here drawn in heavy lines, so as to distinguish it from the remainder that is below the plane.

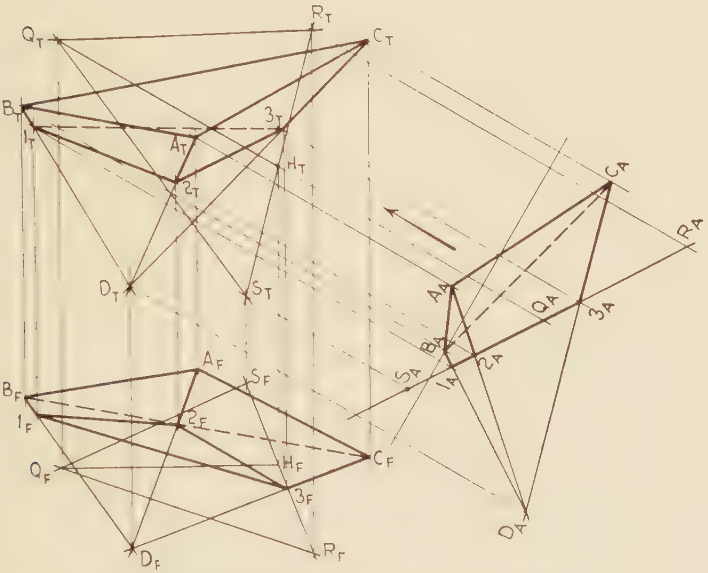


FIG. 65.—Intersections of several lines and a plane.

In Fig. 65, the front views of the points 1, 2, and 3 are found by extending rays from the top views of the points. The elevations of these points in the front view should be checked against the elevations that were found in the auxiliary elevation. Unless the work has been very carefully done, these views will not agree. This check is a test of the quality of the workmanship.

It should be recognized that the edge views of planes are often seen in drawings, and that the intersections of lines with these planes are readily found. The end view of a hexagonal prism, for example, shows edge views of six planes. Also, the end view of a cylinder may be considered as the edge view of a plane that has been curved. In the end view of the cylinder the intersections of a line with the curved surface are seen.

**136. Problems.**

Group 50. Intersections of Several Lines and a Plane.

**137. Distance from a Point to a Line.**—The distance from a point to a line may be found as follows: Pass a plane through the point perpendicular to the line. Find the point of intersection of the line and plane. The distance from the point last found to the given point is the required distance.

**138. Problems.**

Group 51. Distance from a Point to a Line.

**139. Distance from a Point to a Plane.**—To find the distance from a point to a plane: Draw a line through the given point perpendicular to the plane. Find the point of intersection of this perpendicular with the plane. The true length of the line connecting the point of intersection with the given point is the required distance. The distance from a point to a plane may also be seen in any view that shows the edge view of the plane.

**140. Problems.**

Group 52. Distance from a Point to a Plane.

**141. Intersection of Planes.**—The intersection of two planes is a straight line. It may be found by locating two points that lie in both planes and then drawing the required straight-line intersection through these points. To locate one point that lies in both planes, find the point of intersection of a line of either plane with the other plane. In a similar manner locate a second point. The solution is based on the problem of finding the intersection of a line and a plane, Art. 133.

*Problem:* To find A B, the intersection of the planes Q R S and J K L, Fig. 66. First, find the intersection A of any line Q R, for example, of one plane with the other plane. In the front view take a receding plane through the line Q R. Locate 1 in this plane and on the line J K, in both the front and top views. Locate 2 in the same receding plane and on the line J L, in both views. In the top view the line 1 2 intersects the line Q R at A, and the front view of A is on its ray, on 1 2, and on Q R. The point A is common to both of the given planes.

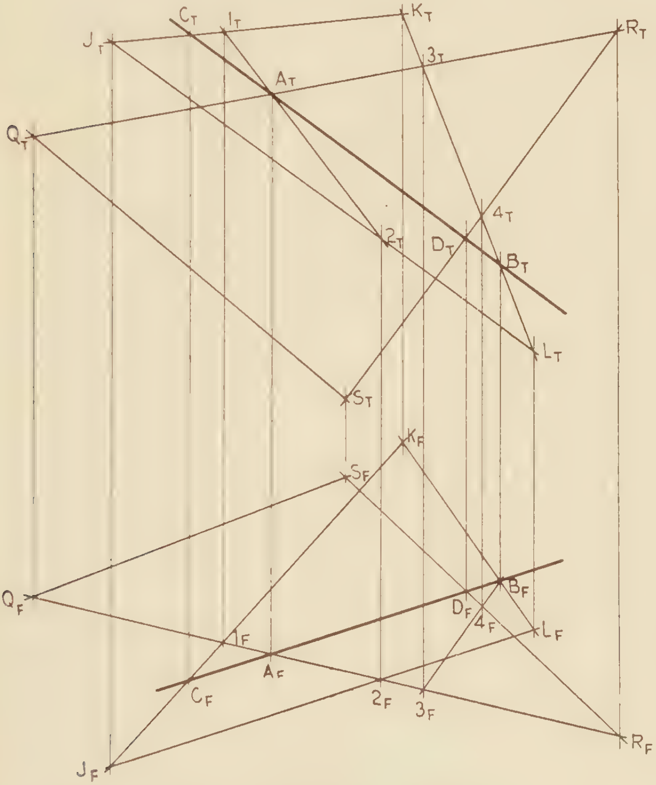


Fig. 66.—Intersection of planes.

Choose some other line of either plane, K L, for example, and find its intersection with the plane Q R S. In the top view locate the point 3 in the vertical plane containing K L and on the line Q R. The front view of 3 is on the front view of Q R. Similarly, locate the top and front views of the point 4 on the line R S and in the vertical plane containing K L. In the front view the line 3 4 intersects the line K L at B, and the top view of B is on the top views of 3 4 and K L. The point B is common to both of the given planes. Since the points A and B are common to both planes, the required intersection is now drawn through these points.

To check the accuracy of the solution: In the top view of Fig. 66 locate C at the intersection of A B and J K. Do the same in the front view. If, now, the front and top views of the point C are in the same vertical ray, the point C is on the lines J K and A B; and, since B is on K L, the line A B is in the plane J K L. Also locate the front and top views of the point D on the lines A B and R S. If the front and top views of D are now in the same vertical ray, the point D is on the lines R S and A B; and, since A is on Q R, the line A B is also in the plane Q R S. If the checks show that any two points of the line A B lie in each of the given planes, the solution is correct. The solution will not check accurately unless the drawing has been carefully made.

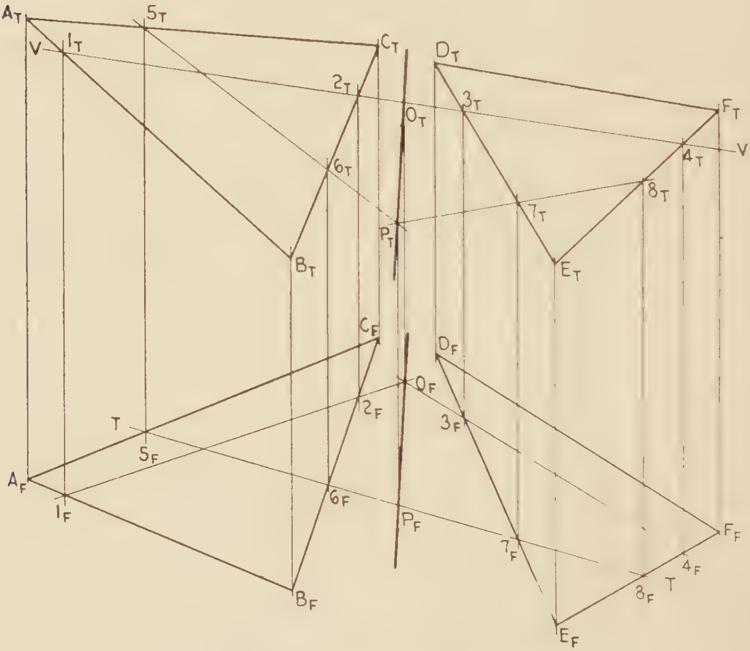


FIG. 67.—Intersection of planes.

At times the given data are such that it is difficult or impossible to find the intersection of two planes by means of the method just stated. The following plan may then be used. To find the intersection of the planes A B C and D E F, Fig. 67: Choose any vertical plane, as indicated by V V in the top view. This plane intersects the plane A B C in the line 1 2, and the plane D E F in the line 3 4. The lines 1 2 and 3 4 intersect at the point O. The point last found is in both of the required planes, and hence is a point on the intersection of the two planes. Take another receding plane T T, and in a similar manner locate the point P. The required intersection is the straight line O P.

**142. Problems.**

Group 53. Intersection of Planes.

**143. Angle between Two Lines.**—A normal view of a plane containing two intersecting lines will show the true size of the angle between the lines. *Problem:* To find the true size of the angle B A C, Fig. 68. Draw a horizontal line C D of the plane A B C, and then draw an auxiliary elevation that shows the edge view of this plane taken in the direction C D. Draw now an oblique view that is a normal view of the plane A B C taken in the direction of the arrow that is perpendicular to the edge view of the plane. In this normal view the true size of the angle is seen at $B_O A_O C_O$, and is indicated by the arc and arrowheads.

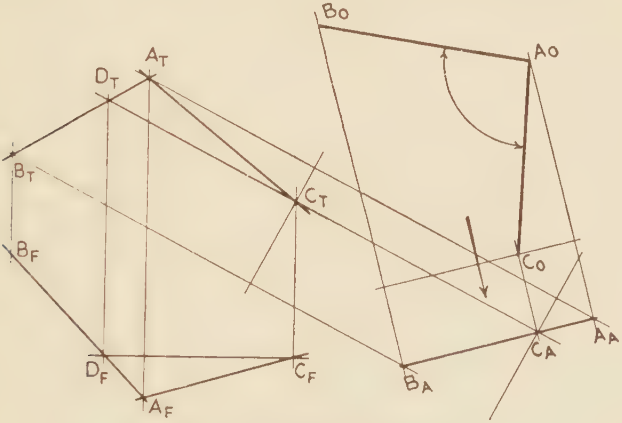


FIG. 68.—Angle between two lines.

The bisector of the angle between two oblique lines does not, in any principal view, appear to bisect the angle. Why is this so?

**144. Problems.**

Group 54. Angle between Lines.

**145. Angle between a Line and a Plane.**—To determine the angle between a line and a plane, draw any line intersecting the given line and perpendicular to the given plane. The angle between the perpendicular line and the given line is the complement of the required angle.

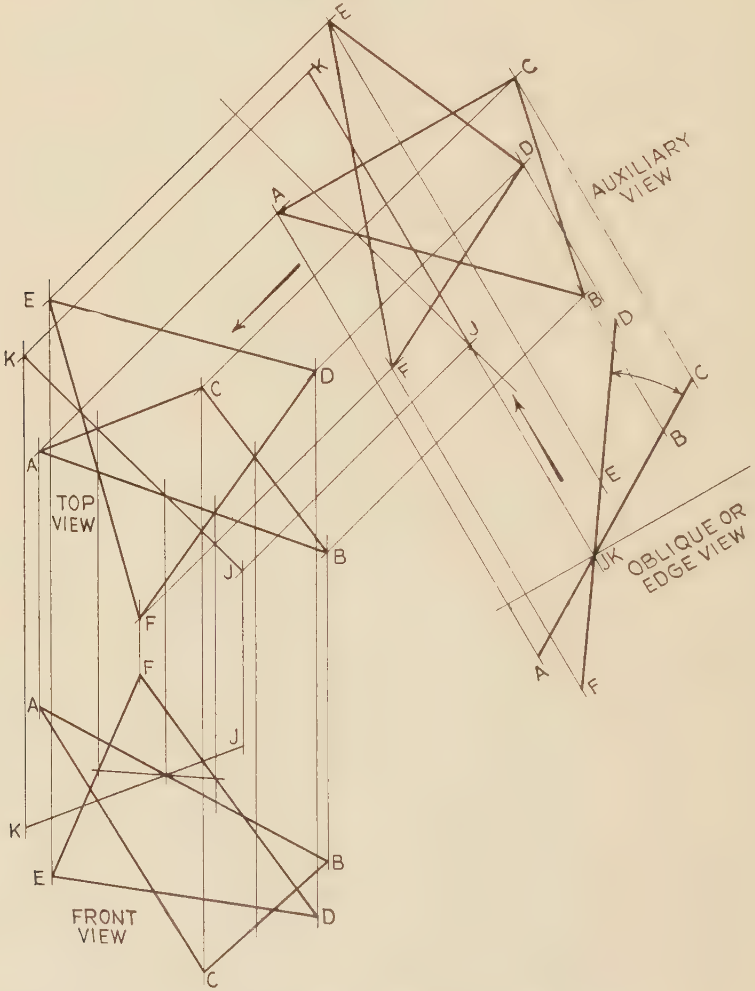


FIG. 69.—Angle between two planes.

**146. Angle between Two Planes.**—The angle between two planes is called a dihedral angle. The planes are the faces of the dihedral angle, and the straight-line intersection of the faces is the edge of the dihedral angle. The true size of the dihedral angle may be seen in a view that shows the edge views of both planes. Such a view is taken in a direction parallel to the straight-line edge of the dihedral angle.

*Problem:* To determine the dihedral angle between the planes A B C and D E F, Fig. 69. Starting with the front and top views of three points of each plane, find J K, the intersection of the two planes. The line J K is the edge of the dihedral angle. Draw next an auxiliary view that is a normal view of the line J K, and in this view show the three points of each of the given planes. Draw finally an oblique view taken in the direction J K indicated by the arrow. In this view are shown the end view of the line J K, the edge views of the planes, and the true size of the dihedral angle, as indicated by the arc and arrowheads. Either the angle indicated or its supplement may be used as a measure of the dihedral angle.

This problem may be solved with less work by showing, in the auxiliary and oblique views, only one point of each plane in addition to the line of intersection. It is desirable, however, to show in these views at least two points of each plane, in addition to the line of intersection, as a check on the accuracy of the work.

Frequently the intersection of two planes is a frontal, horizontal, or profile line, in which case the end view of the line, the edge views of the planes, and the size of the dihedral angle may be obtained in an auxiliary view. Also, in drawings of structures it is quite commonly the case that the intersection of the plane faces of the dihedral angle is shown as a line of the structure, and this line is available in solving the problem.

**147. Polyhedral Angles.**—A polyhedral angle is determined by three or more planes that meet at a point called the vertex. The planes form the faces of the polyhedral angle, and the straight-line intersections of the planes are the edges of the angle. A polyhedral angle bounded by three planes is a trihedral angle; a polyhedral angle bounded by four planes is a tetrahedral angle, etc. The angle between any two edges or between any two adjacent faces may be determined by the methods previously explained.

**148. Applications of Angle Problems.**—When the angle between two lines or two planes of a structure is an oblique angle, it is often necessary to represent this angle in the views of the structure and in some view to show the true size of the angle.

This is commonly the case in oblique structures and in oblique parts of rectangular structures; for example, in skew bridges, rafters of buildings, oblique braces, tunnels and shafts, crystals, parts of machines, etc.

### 149. Problems.

Group 55. Angle between Planes.

**150. Timbered Structures.**—When a wooden framework is to be built, the framer usually cuts the timbers to fit the structure, before lifting them into place. In Fig. 70 a hip roof is shown as an illustration of a framed structure.

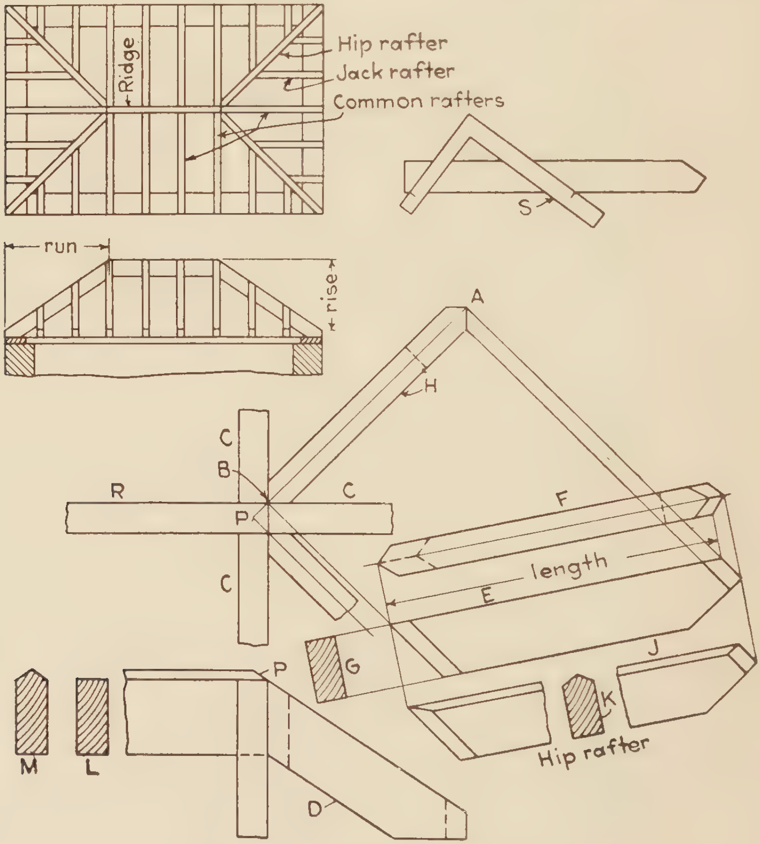


Fig. 70.—Framing of hip roof.

In the front and top views, showing the assembled roof frame, the names of the different types of rafters are indicated. The rise and run of the roof are also indicated. The slope of the roof is equal to the rise divided by the run. This is equal to the tangent of the angle that the plane of the roof makes with the horizontal. The term "pitch" as applied to roofs is equal to the rise of the roof divided by the total width of the roof. The slope of all parts of the hip roof is generally made uniform, so that the common rafters can all be made alike and the jack rafters can be made in pairs, right hand and left hand.

In the lower part of Fig. 70 are shown, to a larger scale, the details of the common and hip rafters, and one method of forming the joints at the peak of the roof. The center lines of the ridge, two hip rafters, and three common rafters meet at P. The ridge may be left rectangular as shown at L; or it may be beveled as shown at M, in case the timbers are very large, or in case a better finished job is desired.

The common rafter is cut as shown at D. The angle to which the rafter is cut is measured by the framer by means of a steel square, as indicated at S in the upper right-hand corner of the figure. The 12-inch mark on the body of the square, corresponding to the run, and the mark on the tongue of the square, indicating the corresponding rise, are placed on one edge of the timber, and a mark is scribed along the tongue of the square. The timber is cut to this mark. The length of the rafter is determined from tables of figures given on the face of the square, or by measuring diagonally with the square.

The face view of the hip rafter is shown at E, the edge view at F, and the end view at G. The upper edge of this rafter may be beveled to fit exactly the theoretical shape of the roof, as shown in the face view at J and in the end view at K. The views E and J are auxiliary elevations; G and K are end or oblique views; the edge view F is an oblique view. The length of the hip rafter E is equal to the square root of: the square of the rise, plus twice the square, of the run minus one-half the thickness of the common rafter. This is the true length of the line A B. The angles to which the hip rafter must be cut are marked on the timber, and the length of the rafter is determined by the framer with the aid of his steel square and the tables thereon. In the drawing, the angles and length to which the hip rafter is cut are determined, as usual, by maintaining the same relative elevations of points in the front view and in the auxiliary elevation.

A very short jack rafter is shown in Fig. 71. The top view is at 1. At 2 is the front view that is also a face view of the rafter. In this view the slope of the rafter is the same as the slope of the roof. The edge view of the jack rafter is shown at 3, and the end view and size of the timber at 4. To the draftsman, the view 3 is a right-auxiliary view, as is also the view 4. The angles of the cuts, as shown in the face and edge views, are determined by the framer, as usual, with the aid of his steel square.

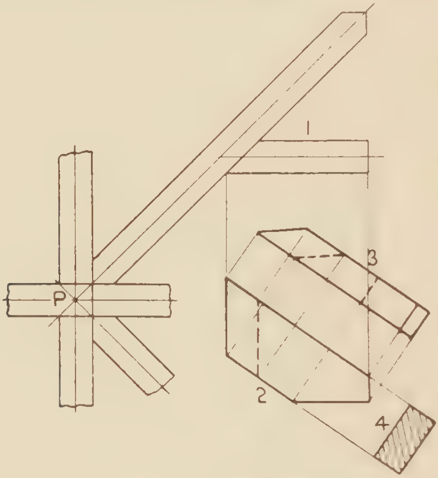


Fig. 71.—Jack rafter.

In Figs. 70 and 71, the rafters are represented in the drawings by means of points and lines; the angles between the lines have been determined by obtaining normal views of the planes of the lines; the angles between pairs of planes have been determined by obtaining views taken in the directions of the intersections of the planes. The theory of the geometry of engineering drawing thus finds continual applications in practice.

**151. Problems.**

Group 56. Timber Framing.

Group 57. Miscellaneous Problems.

# CHAPTER IX

# INTERSECTION AND DEVELOPMENT OF SURFACES

## INTERSECTIONS

**152.** A structure is limited by the surfaces that bound it. Each surface is, in turn, limited by the straight or curved lines in which it intersects another surface of the structure. These lines of intersection determine the edges of the structure, and are shown in drawings to describe the structure. If any of the lines are not shown, the structure is not completely described, and, as a result, the drawing has an unfinished appearance. In addition to the lines of intersection, the outlines of the curved surfaces of structures are shown in drawings.

Since structures are described in drawings mainly by showing the intersections of the surfaces, it is necessary to know how to find the intersection of any possible combination of surfaces. This may be done by using the methods explained in this chapter.

**153. Finding Intersections.**—Two surfaces may intersect in one or more lines. A line of intersection may be straight, single-curved, or double-curved, or it may be some combination of these. If the line of intersection is straight, it is necessary to find only two points of the line in order to determine the line. If the line of intersection is curved, it is usually necessary to locate a series of closely spaced points of the line to determine the line accurately. Obviously, a line of intersection cannot appear outside the limits of either surface.

Three methods of finding intersections are commonly used. A general description and an example of each method are given below. When solving a problem, judgment, dictated by experience, will determine which method it is best to use. Often it is expedient to use several methods in solving a single problem.

**154. First Method.**—To find the intersection of two surfaces: Pass a cutting plane through the two surfaces. The cutting plane will intersect each surface in a straight or a curved line. These lines will intersect at one or more points that are common to both surfaces and, hence, are points on the required intersection. Repeat the above operation until a sufficient number of points is found. Then draw the line of intersection.

This method is generally used when both surfaces are curved, but it is also useful when both surfaces are planes. Judgment must be used in choosing the direction in which cutting planes are to be passed through the two surfaces. Since only straight lines or circles may be readily drawn, it is practically necessary to pass cutting planes so that they cut from the given surfaces either straight lines or circles. If circles are cut from a surface, the cutting plane should usually be frontal, horizontal, or profile, in order that the circle will not appear as an ellipse in any of the principal views. Also, whenever it is possible to pass cutting planes in several directions, it is expedient to choose the direction that will give the greatest number of points for each plane that is passed.

**155. Second Method.**—To find the intersection of two surfaces when one of the surfaces is a plane: Obtain a view that is an edge view of the given plane. In this view, the points, where the lines of the other given surface intersect the plane, are at once seen. These points are then transferred to the other views, and the line of intersection is drawn.

The second method is particularly useful when several lines intersect a plane, or when one or both of the given surfaces is a prism or a cylinder. An end view of the prism or cylinder will immediately show the location of all points of intersection with the lines of the other given surface.

**156. Third Method.**—To find the intersection of two surfaces when both surfaces are plane faced: Find the intersection of each straight line of each surface with the plane faces of the other surface by using the method for finding the intersection of a line and a plane that is explained in Art. 133.

This method is useful when finding the intersection of one or two lines and a plane, for example, when finding the intersection of two pyramids, two prisms, or a pyramid and a prism.

**157. Example of First Method.**—This example is discussed at considerable length to show the need for giving consideration to many details. Preliminary thinking and planning expedite the solution of a problem.

Find the intersection of the sphere and the cylinder, shown in Fig. 72, by passing cutting planes. First, determine the best direction for the cutting planes. Considering the sphere: Any cutting plane will cut a circle from the sphere; hence, so far as the sphere is concerned, the cutting planes may be passed through in any direction. The cutting plane should, however, be horizontal, frontal, or profile; otherwise the circle would appear as an ellipse and would be difficult to draw. Considering the cylinder: A horizontal plane will cut a circle from the cylinder, while a frontal or profile plane will cut two straight lines from the cylinder. So far as the cylinder itself is concerned, the cutting plane may be horizontal, frontal, or profile.

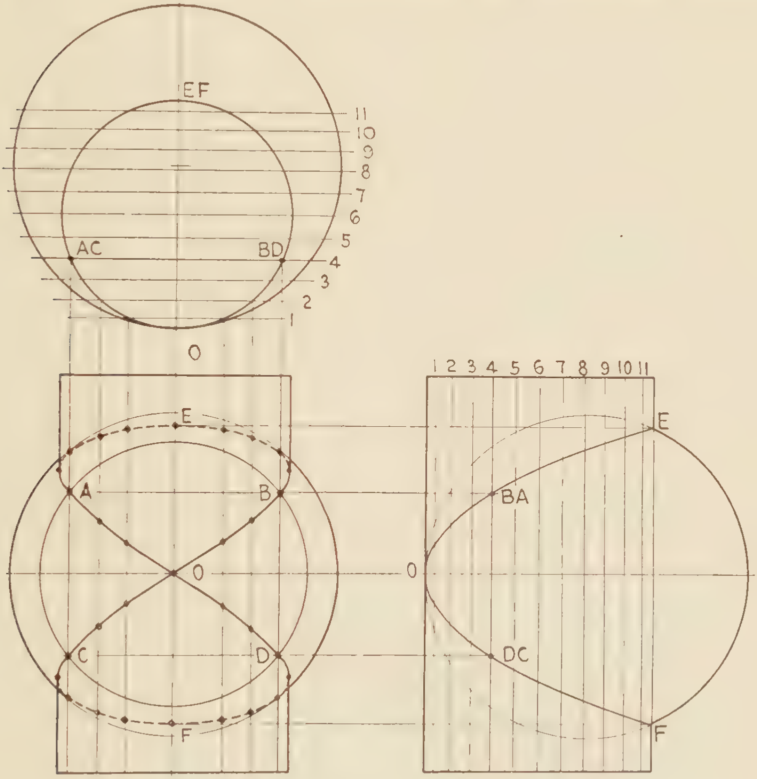


FIG. 72.—Intersection of phere and cylinder.

Considering both surfaces: If horizontal cutting planes are used, each plane will cut a circle from the sphere and a circle from the cylinder. These circles will intersect at two points. If frontal or profile cutting planes are used, each plane will cut a circle from the sphere and two straight lines from the cylinder.

The two straight lines will cross the circle at four points. The above considerations show that, in finding the intersection of the sphere and the cylinder of Fig. 72, it is better to use either frontal or profile cutting planes, since each plane will determine four points on the required intersection, and the work will proceed twice as fast as it would if horizontal cutting planes were used.

In Fig. 72 a frontal cutting plane is shown in the top and side views at 4. In the front view are shown the circle that is cut from the sphere, and the two straight lines that are cut from the cylinder by the plane 4. The straight lines and the circle intersect at the points A, B, C, and D. These are points on the required intersection. The points of intersection are indicated by a small circle drawn around each of the four points. The top and side views of the points are then located, each on its own ray and on the top and side views of the straight lines and circle. The operation is repeated by taking other frontal cutting planes, as shown in the figure. Enough cutting planes should be used to determine accurately the line of intersection. Cutting planes should usually be more closely spaced as the curve of intersection becomes sharper.

In the problem just explained, for each frontal cutting plane that is passed in front of the axis of the cylinder, another plane should be passed an equal distance behind the axis. This will make it necessary to draw only one-half as many lines of the cylinder in the front view.

Several points may be found by inspection. The point O is directly in front of the center of the sphere. The points E and F in the rear of the cylinder and on the profile great circle of the sphere are located first in the side view and then in the front view. It should be noted that the frontal cutting plane which passes through the axis of the cylinder determines the four points where the line of intersection passes from the front to the back of the cylinder.

After a sufficient number of points is found, the curve of intersection should be penciled. Care should be used to indicate by dotted lines the parts of the curve that are invisible, and to draw very fine, or erase, the lines that disappear when the two surfaces are considered as one structure.

In Fig. 73 the intersection of the sphere and the cylinder has been found by passing horizontal cutting planes. The horizontal plane H cuts a circle from the sphere and a circle from the cylinder. In the top view the two circles intersect at the points A and B. The front views of these points are next located. By passing other horizontal planes more points may be found.

For solving this problem, the disadvantage that horizontal cutting planes have as compared to frontal cutting planes is partially overcome by the fact that in Fig. 73 the circle that is cut from the cylinder needs to be drawn but once. The disadvantages of this method are that it is slower, and that the intersections of some of the circles cannot be located accurately, because the circles cross at such sharp angles. At times, it is convenient to pass cutting planes in more than one direction. Experience will aid in choosing the best plan.

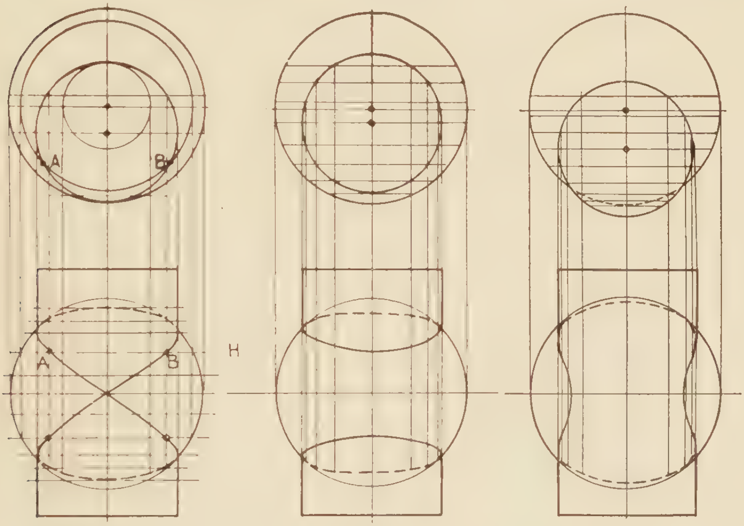


Fig. 73. Fig. 74. Fig. 75.

Figs. 73 to 75.—Intersection of sphere and cylinder.

If the position of the cylinder is changed slightly with relation to the sphere, the line of intersection will change materially, as is shown in Figs. 74 and 75.

**158. Example of Second Method.**—Find the intersection of the pyramid and the plane of Fig. 76, by means of an edge view of the plane. The edge view, taken in a direction parallel to the horizontal line R S of the plane, shows immediately which of the eight edges of the pyramid pass through the given plane. The four points of intersection thus found are transferred from the auxiliary elevation to the top view, and from the top view to the front view. The straight-line intersections connecting the points are then drawn. Check to see that each point in the front view has the same relative elevation that it has in the auxiliary elevation. The cut section is cross-hatched to give it the appearance of having been cut off. This completes the solution of the problem.

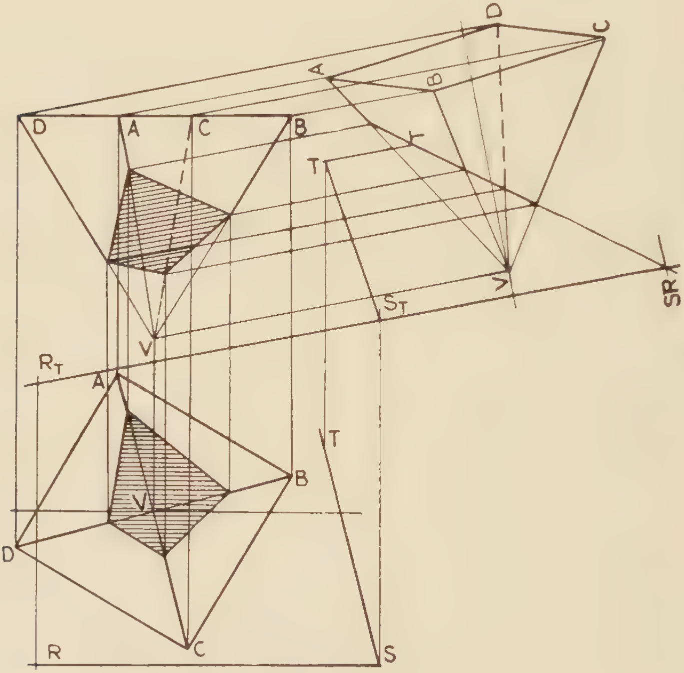


Fig. 76.—Intersection of pyramid and plane.

A similar method may be used for determining the intersection of the cylinder and the cone of Fig. 77. The side view is here an edge view of the curved surface of the cylinder. In this view, the point of intersection of each straight-line element of the cone with the surface of the cylinder is at once apparent. The points of intersection are next located in the front view, each on its ray and on its straight-line element of the cone. The top views of these points are then located from the front views. The intersection is a double-curved line. To save work, the circular base of

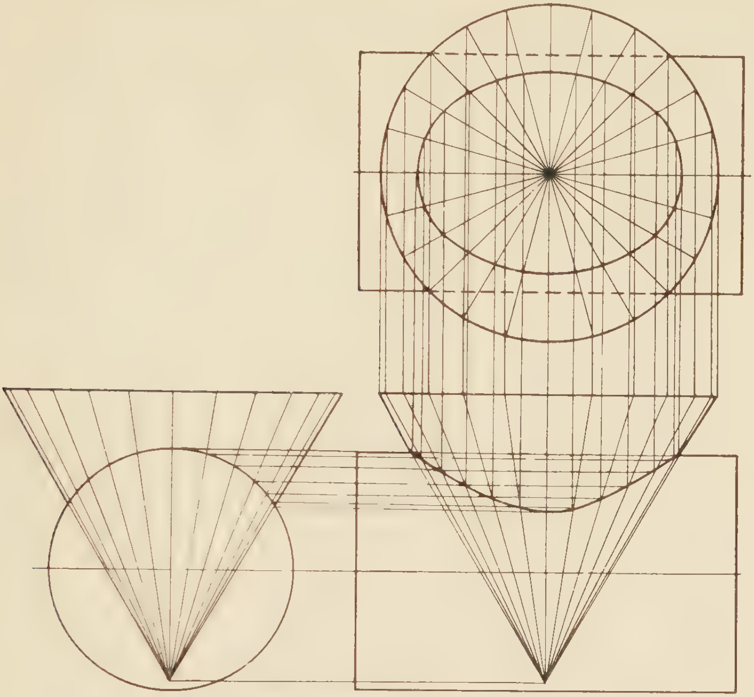


FIG. 77.—Intersection of cylinder and cone.

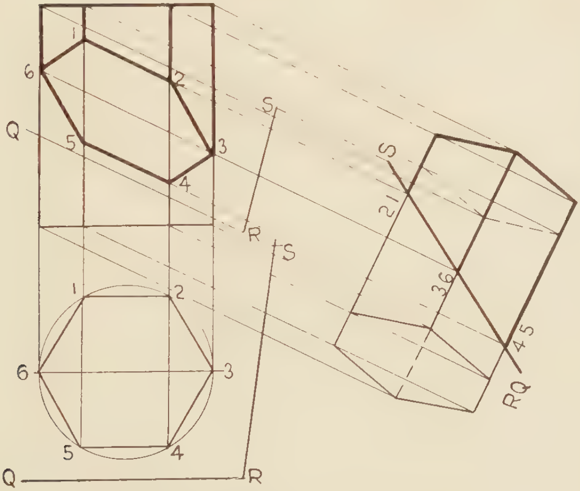


FIG. 78.—Intersection of prism and plane.

the cone should be symmetrically divided into equal parts so that in the front and side views each line of the drawing will represent two straight lines of the cone.

To illustrate the need for using judgment as to which is the quickest method for finding the intersection of two surfaces, note that the intersection of the hexagonal prism and the plane Q R S of Fig. 78 has been found by obtaining the edge view of the given plane. This problem may, however, be solved with less work by passing three cutting planes through, respectively, the upper, middle and lower edges of the prism, Fig. 79. These cutting planes intersect the given plane in the parallel lines 1 2, 6 3, and 5 4, and these, in turn, determine the six points of intersection. To solve this problem it is not necessary to draw an auxiliary elevation.

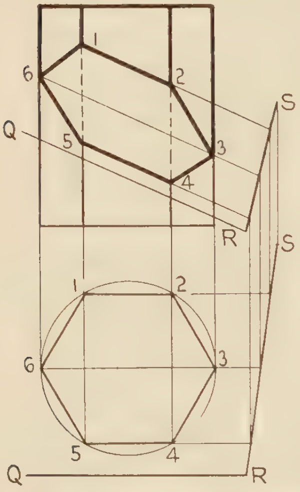


Fig. 79.—Intersection of prism and plane.

**159. Example of Third Method.**—Find the intersection of the two pyramids shown in Fig. 80. Each of the eight edges of either pyramid may intersect one or two of the five plane faces of the other pyramid. To solve this problem it is theoretically necessary to find the intersection of a line and a plane eighty times. The amount of work may be materially reduced by visualizing the pyramids and deciding which lines are likely to intersect which planes.

To find whether or not the line O 3 of one pyramid intersects the plane face V D C of the other pyramid, proceed as follows: In the front view locate the point K on the line V C and directly in front of or behind the line O 3. Now locate the top view of K on the top view of the line V C. Next, locate in the front view point L on the line V D and directly in front of, or behind, the the line O 3. Then locate the top view of L on the top view of the line V D . Draw the views of the line K L. In the top view, the line K L will intersect the line O 3 at the point P. In the front view, P is on the lines O 3 and K L. The point P is on the line O 3 and in the plane V D C, and, hence, is a required point of intersection. This process is repeated with as many lines and planes as are necessary fully to determine the intersection of the two given pyramids. The method of finding the intersection of a line and a plane, here used, is fully explained in Art. 133. To use this method with facility it should be understood, not simply memorized.

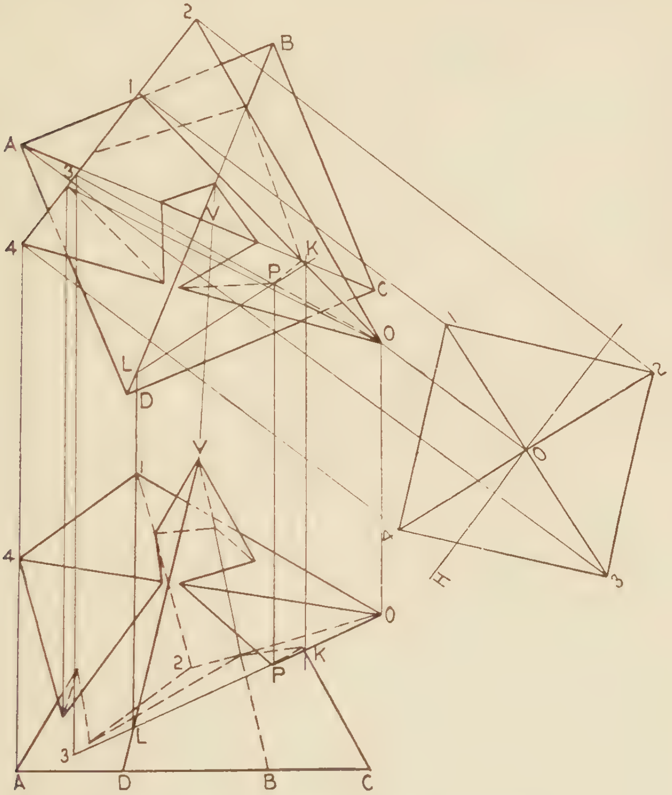


FIG. 80.—Intersection of two pyramids.

## 160. Problems.

Group 58. Miscellaneous Intersections.

### DEVELOPMENTS

**161.** A great variety of articles of manufacture is made from sheet metals. The production of these articles has rapidly increased in recent years. Sheet-metal articles are usually lighter in weight, stronger, and less expensive to manufacture, than are similar articles made from castings. The sheet metals commonly used are iron, steel, tin plate, brass, copper, aluminum, nickel silver, etc. Other sheet materials, as paper and fiber, are also extensively used.

**162. Use of Developments.**—The flat sheet of metal from which an article is to be made is regarded as a plane. The shape to which the sheet material is cut, while in the flat, is called the development, pattern, stretchout, or blank. After the article is designed, it is necessary to develop the surface so as to obtain the development or pattern for the blank from which the piece is to be made. If accurate work is desired, the development must be very carefully determined.

**163. Practical Considerations.**—The blank is cut to the proper shape from the specified sheet material and is then folded, curved, or stretched until it assumes the desired form. The blank is usually folded by means of a machine called a brake, curved by means of rolls, or stretched by means of dies or by hammering. Dies are also used quite commonly for all three operations.

After the blank is folded or curved, the edges that meet are joined by seaming, welding, riveting, soldering, etc. In a drawing, the net development is usually shown by solid lines, and the allowance for seams, if indicated at all, is shown by means of a dotted line outside the boundary of the development. The seam should generally be on the shortest line. When possible, the development should be symmetrical.

When drawing patterns for articles that are to be made from sheet metal, the thickness of the metal need not be taken into account if the metal is 20 gage (0.0375 inch) or thinner. When metal thicker than 20 gage is to be used, it is necessary to make allowance for the shrinkage of the metal when it is bent or curved. This allowance is usually taken as three or four times the thickness of the metal. It should not be added to one end of the pattern, but distributed over the entire pattern. The allowance for thick-gage metal may also be determined by showing the thickness of the metal in the drawing, and then making the development for the surface that is halfway between the two surfaces of the sheet metal.

To transfer the lines of a development to sheet metal, lay the paper pattern on the metal and hold it down with weights. Points on the lines of the pattern may now be pricked through the paper into the metal with a prick punch. The lines are than scratched on the metal with an awl or marked with a pencil.

**164. Surfaces That May be Developed.**—Sheet material, when curved, bends or folds along a straight line; hence, only ruled surfaces may be developed. Of the ruled surfaces, plane-faced and single-curved surfaces may be developed; but warped surfaces cannot be developed, since the consecutive straight-line elements of warped surfaces do not intersect. An article that is to be made simply by folding or curving sheet metal must be designed so that its surfaces are either plane faced or single-curved. That is, the article must be composed of prisms, pyramids, polyhedrons, cones, cylinders, or convolutes. Other surfaces can only be approximately developed. The methods of developing prisms and pyramids are explained in this chapter. The methods of developing cones, cylinders, and convolutes are explained in the next chapter. The reasons why a given surface may or may not be developed are stated in the articles dealing with that surface.

**165. Surfaces That Cannot Be Developed.**—Warped surfaces and double-curved surfaces cannot be developed. These surfaces cannot be formed from sheet metals except by stretching the metal with the use of dies. Since dies are expensive, these surfaces should not ordinarily be used in the design of large articles that are to be made from sheet metal, or small sheet-metal articles that are to be made in small quantities without dies. Warped surfaces may be approximated in sheet-metal work by dividing the surface into triangles. Double-curved surfaces may be approximated in shape by designing them so that they may be constructed from cones and cylinders.

**166. Model Making.**—Much may be gained by making models of the objects specified in the problems. Model making helps to fix the ideas of the practical use of developments and the methods of obtaining developments. The maker of a model that fits well at the joints and that has a well-finished appearance derives a considerable amount of satisfaction from the experience. After a model is made, its relation to the drawing should be studied and checked. A model is a great aid in visualizing the object. Models may be made any multiple of the size shown in the drawing, by multiplying each line of the development by a chosen factor. Small models should be made from a stiff paper, like cream detail drawing paper. Large models should be made from very stiff paper or sheet metal. An allowance for the seam or lap should be made on the development. Good joints may be made by leaving rectangular tabs along the seam, and interlocking these, much as the fingers of the two hands may be made to interlock. The lapped joint in paper models should be finished by gluing the tabs.

Very satisfactory joints in paper models may also be made by fastening the joints with gummed-paper strips, in which case no allowance for seams is made on the development. Paper models should be stiffened by adding the ends of prisms, pyramids, cones, and cylinders. Models may be given an attractive finish by painting them with aluminum paint.

**167. Finding the True Lengths of Lines.**—To develop a surface it is necessary to find the true lengths of the straight lines of the surface. The true lengths of the straight-line elements of ruled surfaces may always be found by obtaining a normal view of each line. This method is, however, not always the most expedient. Several practical methods are explained below. Each of these methods should be understood, not simply memorized. Judgment will determine the best method to use in a given problem.

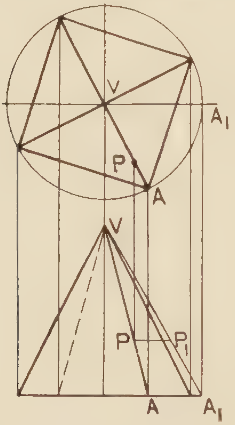


FIG. 81.—True lengths of lines of a right pyramid or cone.

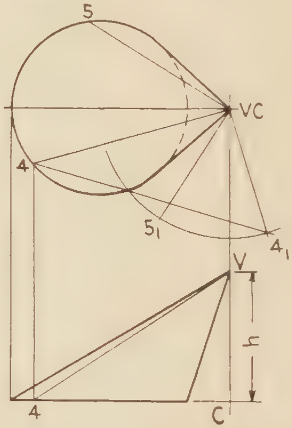


FIG. 82.—True lengths of lines of an oblique cone or pyramid.

**168.** To find the true lengths of the oblique lines of a *right pyramid or cone:* The simple views of a right pyramid or cone are those taken in directions respectively parallel and perpendicular to the axis, as shown in Fig. 81. To find the true length of the oblique edge V A, imagine that the pyramid is rotated about its axis. The point A will move in the horizontal circle that is shown in the top view. Stop the rotation when the point A is at $A_1$. In this position the edge V A is a frontal line, and its true length is V $A_1$ in the front view. Any point P on the edge V A will rotate with the line to the point $P_1$, and the true distance from V to P is seen at V $P_1$. The same method may be used to find the true distance from the vertex of a right cone to any point on the surface of the cone.

**169.** To find the true lengths of the oblique lines of an *oblique pyramid or cone:* In Fig. 82 are shown two views of an oblique cone. To obtain the true length of an element V 4, for example: In the plane of the base of the cone draw a circular arc with center at C, directly below the vertex, and with a radius equal to the altitude $h$ of the cone. Draw the straight line C $4_1$ perpendicular to the top view of the element V 4. The distance from 4 to $4_1$ is the required true length of the element V 4. This is so, because the true length of a line is equal to the hypotenuse of a right-angle triangle, of which one leg is equal to the length of the top view of the line, and the other leg is equal to the distance that one end of the line is above the other end of the line. This latter distance is a fixed distance for all elements of an oblique cone or pyramid; hence, but one construction circle is drawn in the top view. The true length of V 5 is 5 $5_1$. There is no need of drawing the line 5 $5_1$. This distance may be transferred to the development with the aid of the compass. The true lengths of the edges of oblique pyramids may be found in a similar manner.

**170.** To find the true lengths of the straight lines of a *right or oblique prism or cylinder:* Here it is only necessary to obtain an end view of the prism or cylinder, and a view taken in a direction perpendicular to its axis. The latter view is a normal view of all straight lines parallel to the axis of the surface. In the cylinder of Fig. 83 the top view is a normal view of all the straight lines of the cylindrical surface. In the normal view the true length of the line A B is seen, as is also the true distance from A to E, as well as from B to E.

**171.** To find the true lengths of the oblique lines of an *oblique prism or cylinder:* In Fig. 84 an oblique prism having parallel horizontal ends is taken as an example. The distance between the bases is $h$. Since all points in the upper base are the same distance above all points in the lower base, it is possible to find the true lengths of the oblique edges and diagonals of the faces of the prism by means of the following construction: Draw any two perpendicular lines X Y and X T. Make the distance X T equal to *h*, the altitude of the prism. To find the true length of the edge B 2, for example, make the distance X P equal to the length of the top view of B 2. The true length of B 2 may now be measured from P to T. The true length of B 2 is here the hypotenuse of a right-angle triangle, of which one leg is the length of the top view of B 2, and the other leg is the distance that B is above 2 as seen in the front view.

Similarly, the true length of the diagonal 4 A is found by making X L, in the construction diagram, equal to the length of the top view of 4 A. The true length of 4 A is equal to T L.

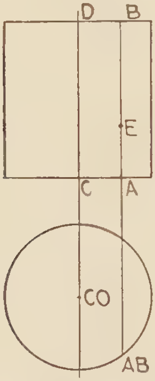


Fig. 83.—True lengths of lines of a right cylinder or prism.

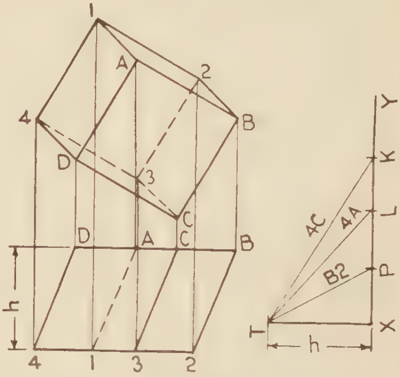


Fig. 84.—True lengths of lines of an oblique prism or cylinder.

The true length of 4 C has also been found. There is no need of drawing the diagonals in the construction diagram. This method may also be used to find the true lengths of the elements of oblique cylinders. Note that this and the preceding article offer two methods for accomplishing the same result.

**172. Development of Right Pyramid.**—The surface of the hexagonal right pyramid of Fig. 85 consists of six equal isosceles triangles. The true length of the base of each triangle may be seen in the top view at 1 2, 2 3, etc., and the true length of the equal sides of the triangles may be seen in the front view at V 1 and V 4. The fan-shaped pattern or development of the pyramid is made by drawing these triangles side by side. If the bottom of the sheet-metal pyramid is to be closed, the hexagonal base should also be shown in the development.

**173. Development of Frustum of Pyramid.**—In the auxiliary elevation of Fig. 85, the top of the pyramid has been cut off by an inclined plane. To develop the frustum of the pyramid, the general procedure is as follows: First, obtain the fan-shaped development of the entire pyramid as explained above. Next, determine the point of intersection of each edge of the pyramid with the inclined plane, and locate this point in the development. As an example, take the edge V 5. In the auxiliary elevation V 5 is seen to intersect the inclined plane at the point E. Obtain the true length of the line V E by using the method explained in Art. 168. The true length of V E is V $E_1$. This distance, V $E_1$, is laid off on the development at V $E_2$ on the line V 5. Other points on the development are similarly found, and the development is completed by drawing the straight lines connecting the two points in each face. The development of the frustum is shown by the heavy outline. The folding lines are also indicated.

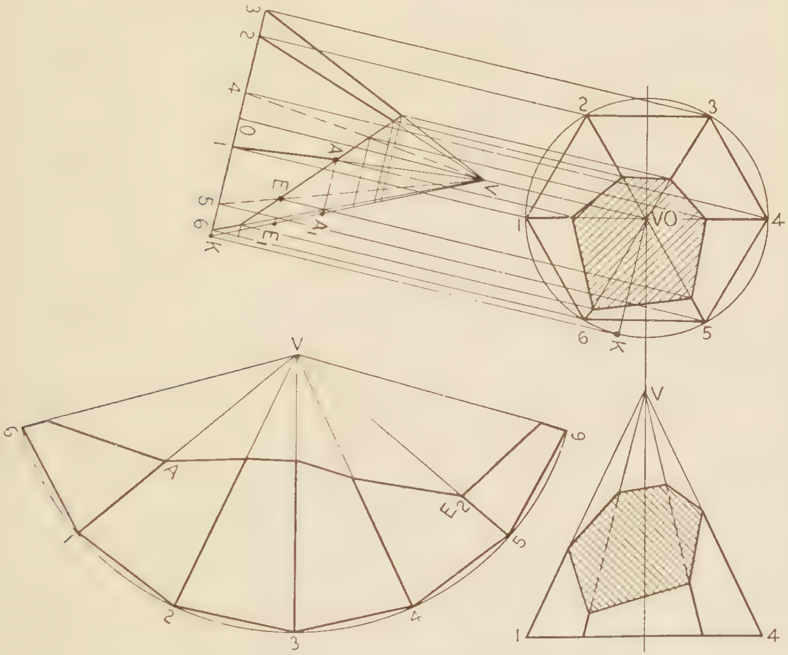


FIG. 85.—Development of right pyramid.

**174. Practical Details.**—It should be noted that if the development of the frustum were now cut out of sheet metal or stiff paper, it would be possible to fold it so that either one side or the other of the development would be inside the pyramid. Folding the development one way would make what might be called a right-hand frustum, and folding it the other way would make a left-hand frustum. The two would not be alike. Developments should be drawn so that the side nearest the draftsman is the inner side.

Making an accurate development requires patience and careful workmanship. One of the common mistakes is to leave off one section of a development, for example, one face of a pyramid or one section of the surface of a cone. A hexagonal pyramid has six edges that pass through the vertex, but on the development there are seven. Two of these join together at the seam to form one edge. The joining lines should be checked to make sure that they are equal in length.

If possible, the development should be made symmetrical, and the seam or cut should generally be on the shortest line. When an article is made from several developments, care is usually taken to see that the pattern is cut so that three seams do not come together at any one point.

**175. Problems.**

Group 59. Right Pyramids.

**176. Development of Oblique Pyramid.**—The oblique pyramid shown in Fig. 86 is symmetrically divided by the frontal plane that passes through the vertex. The surface of the pyramid is formed from three pairs of similar triangles. The pattern will be symmetrical, and it should be cut on the shortest edge V 0.

Start the development by drawing the longest line V 3, since it is the middle line of the symmetrical development. Its true length is seen in the front view. V 3 is the common edge of the two triangles V 3 2. These triangles are drawn by finding the true length of each side and then locating the vertex 2 of the triangle with the aid of the compass.

The true length of the line 3 2 is seen in the top view. The true length, B 2, of the line V 2 should be found by using a construction circle as described in Art. 169. One compass is set to the length B 2, and is then centered at V in the development, and another compass is set to the length 3 2 and centered at 3 in the development. The arcs drawn by these compasses will cross at 2 and 2 on either side of the line V 3. The triangles

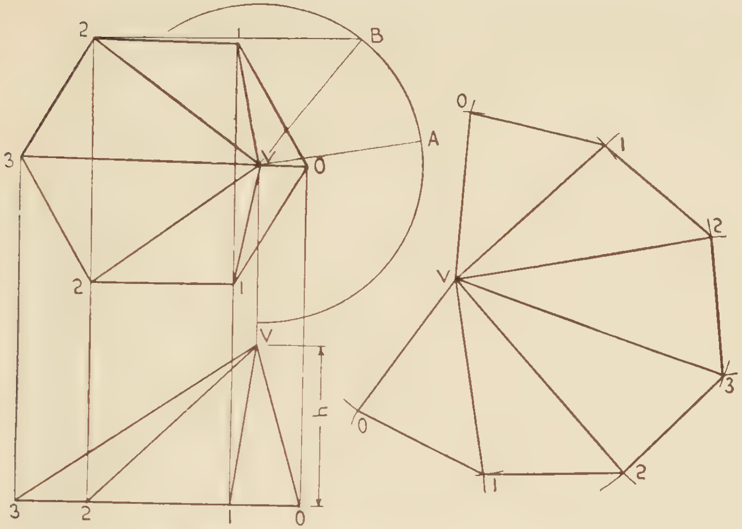


FIG. 86.—Development of oblique pyramid.

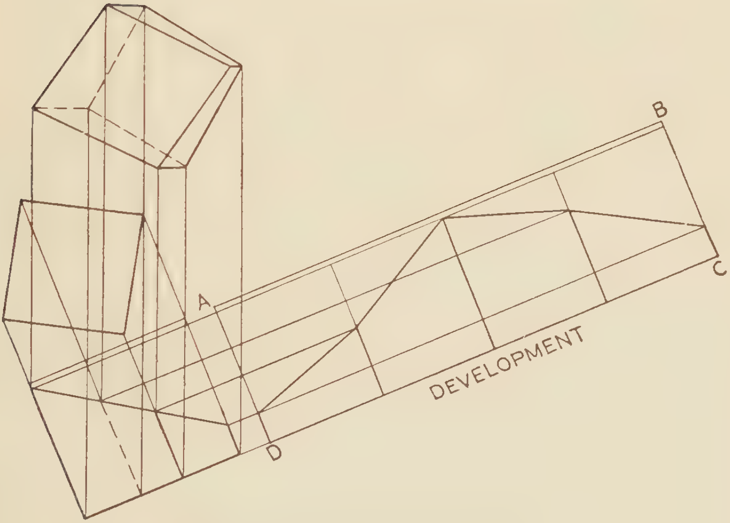


FIG. 87.—Development of right prism.

V 3 2 are now drawn. The true lengths of the sides of the triangles V 2 1 and V 2 0 are found in turn, and these triangles are added to the development as shown in Fig. 86. When the boundary lines and the folding lines of the development are drawn, the development is finished.

**177. Problems.**

Group 60. Oblique Pyramids.

**178. Development of Right Prism.**—In Fig. 87 is shown the frustum of a right prism that is tipped to the left. The development of the entire prism is a rectangle A B C D. The length A B is equal to the perimeter of the square that is shown in the end view of the prism. The folding lines divide the development into four equal rectangles. The development is placed on a slant, in line with the front view, to aid the draftsman in transferring the true lengths from the front view to the development. The development of the frustum is shown in heavy lines. The pattern is not symmetrical. Check to determine whether or not the near face of the pattern is the inner side. Were the upper face of the prism to be considered as a part of the development, it would be necessary to obtain a normal view of this quadrilateral and to attach it to the development.

**179. Problems.**

Group 61. Right Prisms.

**180. Development of Oblique Prism.**—The inclined faces of the oblique prism that is shown in Fig. 88 are parallelograms. To draw a parallelogram it is necessary to determine the lengths of its sides and of at least one of its two diagonals. In other words, the parallelogram must be divided into triangles. The length of the second diagonal of each parallelogram should be determined and used as a check. The development will not be symmetrical.

Since the bases are parallel, the lengths of the inclined sides and diagonals of the parallelogramic faces may be readily found by using a construction diagram as explained in Art. 171.

The development is probably best started with one of the three parallel sides of the prism. Let this be A D. Find the true length of A D: Set the compass on the distance A D in the top view. Retain the setting. Place the lead of the compass at X, and set the needle of the compass at L on the line X Y. Rotate the compass about the needle and open it until the lead is at T.

The compass is now set to the true length of A D. Start the development with the true length of A D. The top view is a normal view of the lines A C and D F. Set the compass to the true length of these lines. Center the compass at A, and again at D, and mark short arcs to the left of these points. C and F will be on these arcs. Next, on the construction diagram, determine the true length of C D. Set the compass to this length, center it at D in the development, and describe an arc that crosses the arc of which A is the center. The crossing point of these arcs determines the location of the point C in the development. Determine the true length of the diagonal A F, and locate the point F by a similar method. Then draw the parallelogram A D F C.

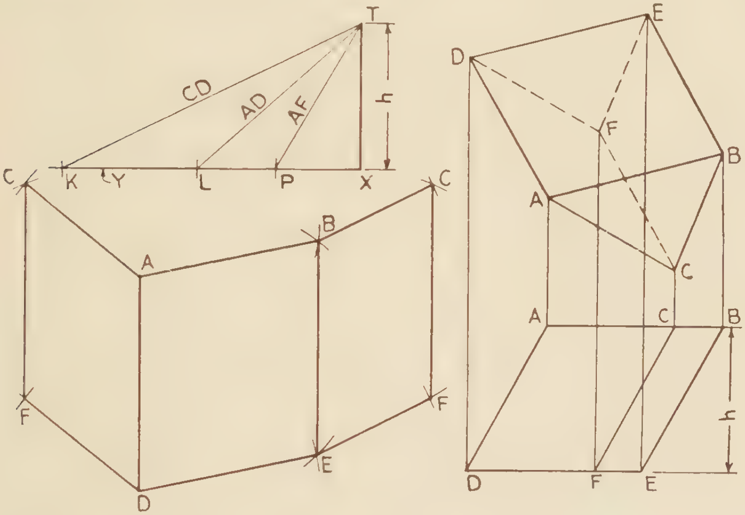


Fig. 88.—Development of oblique prism.

The parallelograms A D E B and B E F C are next determined, and the development is completed. If, however, the ends of the prism are to be closed, it is necessary to add to the development the triangular ends of the prism. Check to see that the near face of the development is the inside of the prism. Check also to make sure that the joining lines F C are equal.

**181. Development of Oblique Prism. Right-section Method.**—Another plan for developing an oblique prism is as follows: Given the front and top views of the oblique prism of Fig. 89, draw an auxiliary elevation that is also a normal view of the parallel edges of the prism. In the auxiliary elevation pass a plane X perpendicular to the axis A B. The plane X will cut from the prism the six-sided right section shown at C. This is not a regular hexagon. Its sides are unequal in length.

Each part of the oblique prism, on either side of the plane X, may now be developed as if it were the frustum of a right prism. In the development, the six sides of the right section will be unfolded into a straight line, as shown from $X_1$ to $X_1$. The folding lines of the development will be perpendicular to the sides of the right section. The distance that each folding line extends on either side of the right section is determined from the auxiliary elevation. The upper and lower edges of the pattern are then drawn. The development should be checked to make sure that the twelve edges that form the two regular hexagonal ends of the prism are all equal in length. This development could, to advantage, be drawn directly in line with the auxiliary elevation.

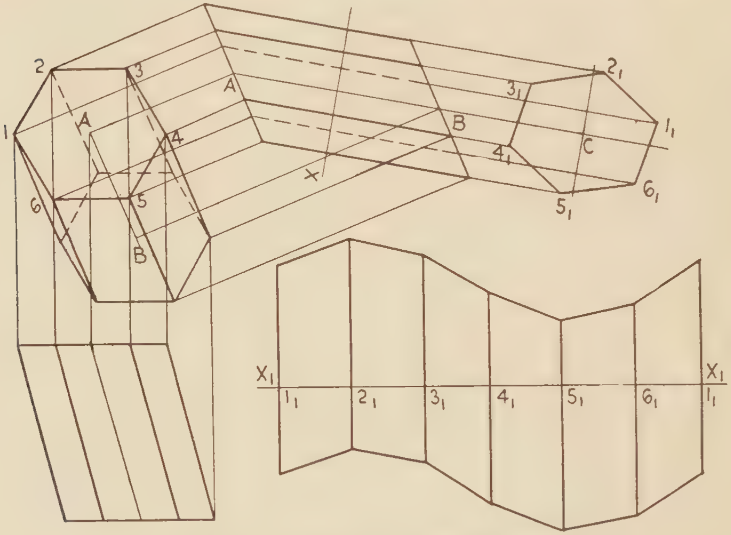


FIG. 89.—Development of oblique prism.

The method just explained should be compared with the method described in the preceding article. It is desirable to understand several methods for solving a problem, so that there may be a choice for different conditions.

**182. Problems.**

Group 62. Oblique Prisms.
Group 63. Hoppers.
Group 64. Transition Pieces.
Group 65. Elbows and Offsets.
Group 66. Prisms and Pyramids.

## CHAPTER X

# SINGLE-CURVED SURFACES

### CONE—CYLINDER—CONVOLUTE

**183.** Ruled surfaces are generated by a moving straight line. There are three types of ruled surfaces: planes, single-curved surfaces, and warped surfaces.

*A single-curved surface* is a ruled surface of which two or more consecutive, straight-line elements intersect. Cones, cylinders, and convolutes are single-curved ruled surfaces.

### THE CONE

**184. Definition.**—The cone is a single-curved surface in which the straight-line generatrix traverses a fixed point and intersects a curved directrix. The fixed point is the vertex, and the curved directrix is the base of the cone. All elements of the cone intersect at the vertex. Since the generatrix is a straight line, the cone is a ruled surface. Since all elements intersect, it is a single-curved surface. The generatrix may extend through the vertex, in which case two nappes of the cone are generated.

**185. Types of Cones.**—In Fig. 90 is shown a general-type cone, of which the directrix is an open curve. The straight-line elements of the cone indicate successive positions of the generatrix. The directrix may be either a single- or a double-curved line.

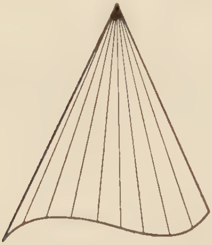


Fig. 90.—Cone.

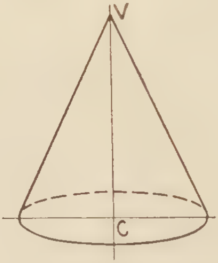


Fig. 91.—Right circular cone.

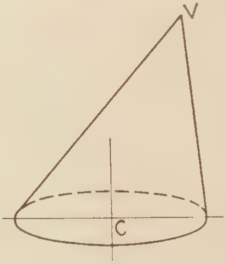


Fig. 92.—Oblique cone.

Figure 91 shows a right circular cone. The axis V C is perpendicular to the plane of the circular base. The right circular cone is also called the cone of revolution, since it may be generated by revolving the generatrix around the axis. The base of a right cone may be an ellipse or other plane curve instead of a circle.

An oblique cone is shown in Fig. 92. Here, the axis V C is not perpendicular to the plane of the base.

**186. Development of a Right Circular Cone.**—Figure 93. Visualize the development as if it were unrolled or unwrapped from the cone. The elements of the right circular cone are equal in length and intersect at the vertex. The development will be the sector of a circle having a radius equal to the length of the elements of the cone. The arc of the development sector will be equal in length to the circumference of the base of the cone.

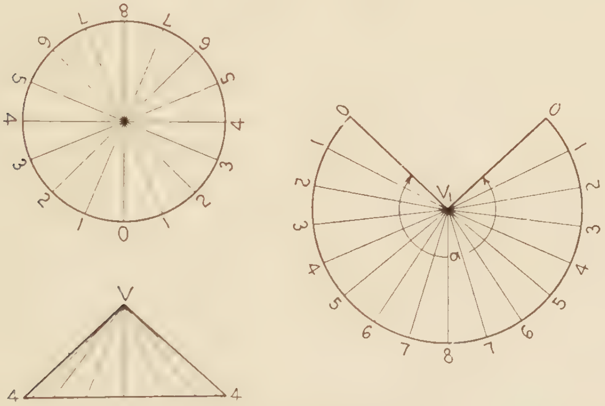


Fig. 93.—Development of right cone.

Divide the circular base of the cone into any number of equal parts; not less than twelve or sixteen. The true length of the elements, or slant height of the cone, is seen at V 4 in the front view. Using this length as a radius, draw a circle with center $V_1$ in any convenient location. On this circle, step off parts equal in length and number to the parts into which the base of the cone was divided. Care must be taken to obtain these distances accurately, since any error is multiplied in the development by the number of divisions taken. Complete the development by drawing the two radii $V_1$ 0. The elements of the cone may be shown in the development. Care must be taken to see that there are as many equal parts in the development as there are in the base of the cone.

A fairly accurate development may be obtained by the method just described. If a very accurate development is required, calculate the angle $a$ of the development sector, and measure it on the development with an accurate protractor; or, if an accurate protractor is not available, calculate the length of the chord. The angle $a$, in degrees, may be determined from the proportion:

$$\frac{a}{360} = \frac{2\pi r}{2\pi s} = \frac{r}{s},$$

in which $r$ is the radius of the base of the cone, and $s$ is the slant height of the cone.

**187. Intersection of a Cone and a Plane.**—A plane may intersect a right circular cone in a point, two straight lines, a circle, an ellipse, a parabola, or an hyperbola. The parabola, ellipse, and hyperbola are called the conic sections. The intersection of a cone and a plane may be found by passing cutting planes, or by obtaining an edge view of the plane, as is explained in Arts. 154 and 155.

**188. Development of a Truncated Cone.**—The right circular cone of Fig. 94 is intersected by the vertical plane R shown in the top view. The elliptical intersection is seen obliquely in the front view, and its true shape is shown in the cross-hatched normal view. The truncated cone is the part of the cone that is included between the circular base and the elliptical section. Before starting the development, it should be noted that the truncated cone is divided symmetrically by a horizontal plane through the vertex. The shortest line of the development is the element at the right of the cone. The development should be cut on this line. The middle line of the development is the element at the left of the cone. Since the development will be symmetrical, it should be started at the middle and developed both ways.

Divide the circular base into any number of equal parts, sixteen, for example, and number the parts in both directions from 0 to 8. Begin the development with the middle element V 8, and develop the entire cone as shown at the right.

To develop the truncated cone, it is necessary to find the true distance from the vertex of the cone to the point of intersection of each element with the plane R. This is best done in the top view by revolving the cone on its axis, as is explained in

Art. 168. In the top view, find the true distance from V to the intersection of the element V 8 with the plane R, and mark this distance on $V_1$ 8 in the development. Next, find in the top view the true length for V 7, and transfer this length to each of the elements $V_1$ 7 in the development. Take each pair of elements in turn. Complete the solution by drawing the boundary of the development as shown in Fig. 94.

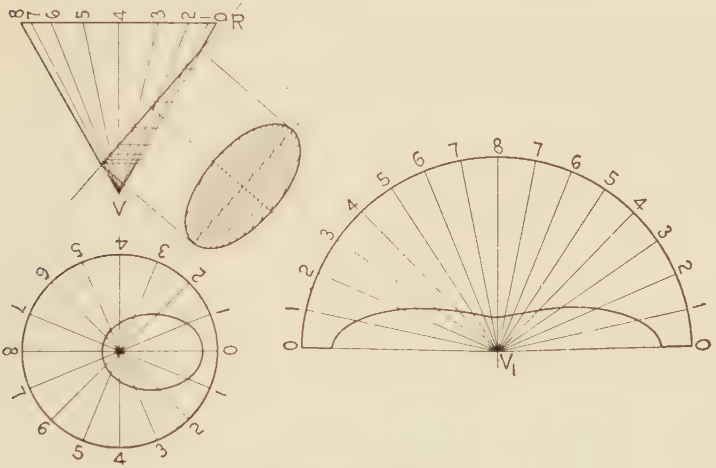


Fig. 94.—Development of truncated cone.

**189. Problems.**

Group 67. Developments of Right Circular Cones.
Group 68. Cone and Plane Intersections.

**190. Development of an Oblique Cone.**—The oblique cone is frequently used in sheet-metal work. The elements of an oblique cone vary in length, so that the development will not be a sector of a circle. In Fig. 95, the oblique cone should first be divided symmetrically by a vertical plane passed through the vertex and the center of the base. V 0 is the shortest element of the cone, and the development should be cut on this line. The center line of the development should be the longest element V 6. Divide the base of the cone into twelve or more equal parts. These are numbered consecutively from 0 on one-half of the cone. Since the cone is symmetrical, the points on the other half need not be numbered. The lengths of the elements may be found as explained in Art. 169. In the top view, draw the arc A centered at V and having its radius equal to the altitude of the cone. To find the length of any element V 1, draw the right-angled triangle V 1 1. The distance from 1 to 1 is the true length of the element V 1. It is necessary to locate only the third corner of each triangle on the arc A. The sides of the triangle need not be drawn.

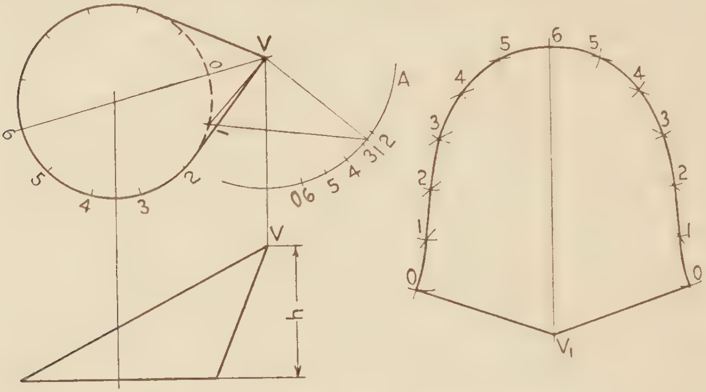


FIG. 95.—Development of oblique cone.

To start the development, determine the true length of the longest element V 6, and draw this at $V_1$ 6 in the development. Next, set the compass to the true length of the element V 5; then center the compass at $V_1$ and draw the two short arcs at 5 and 5. Set another compass to the constant distance 6 5, 5 4, etc., and retain this setting throughout the solution of the problem. Center this compass at 6 in the development and draw arcs that cross the previously drawn arcs at 5 and 5. In a similar manner locate each pair of points in turn. Draw the two elements $V_1$ 0, and complete the development by drawing a smooth symmetrical curve through the numbered points. This curve should be perpendicular to the shortest element at 0.

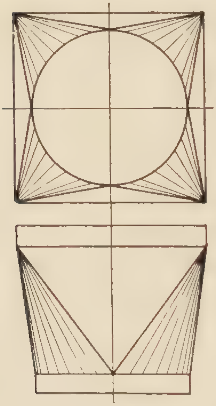


FIG. 96.—Sheet metal transition.

**191. Sheet-metal Transitions.**—A transition piece is a sheet-metal pipe that is designed to connect two pipes or openings that are different in shape or size. Figure 96 is an example of a transition designed for connecting a square opening with a round opening. Since the transition piece is to be made from sheet metal, it must be designed so that it is composed of surfaces that may be developed, that is, plane and single-curved surfaces. In the transition shown there are four triangles and four oblique cones. The vertex of each triangle is a point on the circle, and its base is one side of the square. The vertex of each oblique cone is at one corner of the square, and its base is one-fourth of the circle. In making the development for a transition piece, care should be taken not to reverse the developments of the triangle and the adjacent cone.

**192. Problems.**

Group 69. Developments of Oblique and Non-circular Cones.
Group 70. Oblique-cone Transitions.

**193. Intersection of Cones.**—The intersection of two cones is generally found by passing cutting planes. The cutting plane should be passed so that it cuts either a circle or two straight lines from each of the two cones. Since the bases of cones are commonly circles, it is possible to cut a circle from each of the two intersecting cones by passing a cutting plane parallel to the planes of both bases. The two circles thus found will normally intersect at two points. If the circular bases do not lie in parallel planes, this method cannot be used.

To obtain cutting planes that cut two lines from each cone: Draw the line connecting the two vertices, and pass the cutting plane through this line. This cutting plane will cut two lines from each cone, and these lines will cross at four points. One or more of these points may, however, be outside the limits of the drawing.

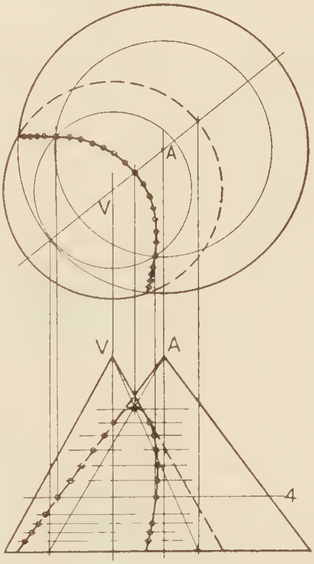


Fig. 97.—Intersecting cones.

In Fig. 97 the intersection of the two cones may be found by passing horizontal cutting planes. Plane 4, for example, cuts a circle from each of the cones. In the top view these circles intersect at two points. By means of rays, the front views of these two points are next located on the front views of the circles. This process is repeated until a sufficient number of points is found. The curve is then drawn, and the hidden parts of the cones are either dotted or erased.

In Fig. 98 the bases of the two intersecting cones do not lie in parallel planes. To find the intersection, first draw the line V A connecting the vertices. Any plane passed through this line,

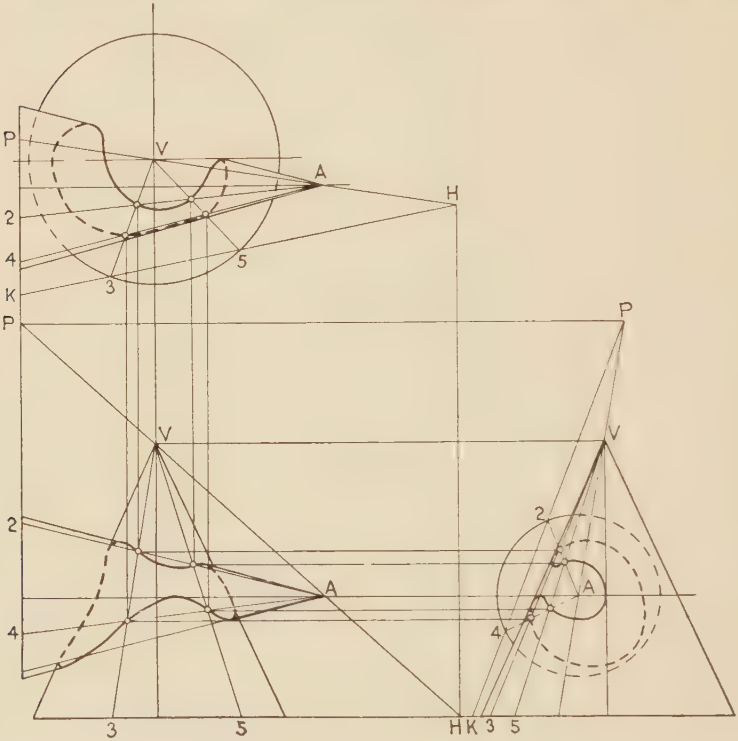


Fig. 98.—Intersection of two cones.

within certain limits, will cut two lines from each cone, and these lines will determine four points on the intersection. Extend the line V A until it meets the plane of the base of the horizontal cone at P, and the plane of the base of the vertical cone at H. The line P H coincides with the line V A. All cutting planes pass through this line. In the top view assume a horizontal line H K. This determines a cutting plane H K P. The line H K intersects the horizontal circular base of the vertical cone at 3 and 5. The elements V 3 and V 5 are cut from the vertical cone and may now be located in all views. Next locate the points K and P in the side view, and draw the line K P. This line is a line of the same cutting plane H K P, and it intersects the circular base of the horizontal cone at 2 and 4. The elements A 2 and A 4 are cut from the horizontal cone and may now be drawn in all views. The two pairs of elements thus found intersect at four points that may be independently located in each view. The accuracy of the locations of these points should be checked by means of rays connecting adjacent views. The above process is repeated until enough points are found, and then the curve of intersection is drawn.

Theoretically, the intersection of two cones may be found more quickly by passing cutting planes through the vertices, but practically it is often desirable to cut circles from the cones. Often both methods are useful in a single problem.

**194. Problems.**

Group 71. Intersecting Right Cones.
Group 72. Intersecting Oblique Cones.

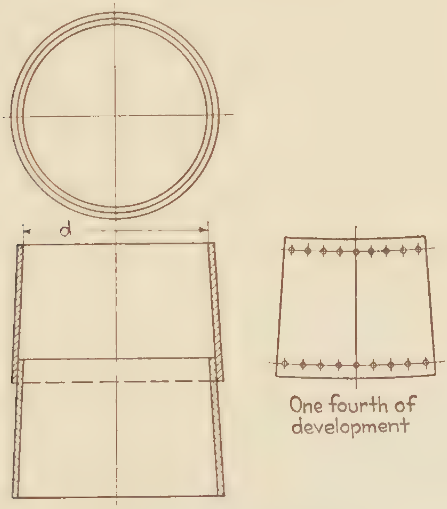


Fig. 99.—Thick metal pipe.

**195. Thick Metal Pipe.**—When a cylindrical pipe is to be made from thick metal sections riveted together, it is necessary to make the sections conical so that the end of one section will slip inside or outside the end of the adjoining section. Cylindrical sections of pipe of uniform diameter cannot be forced to telescope in this manner. Conical sections, illustrated in Fig. 99, should be used when the sheet metal is more than $\frac{1}{32}$ inch thick. One-fourth of the development of a conical section is shown. The cone from which the development is made should be taken halfway between the inner and outer surfaces of the thick metal of which the pipe is made. In developments of this kind it is generally best to calculate the slant height of the cone. This may be done by means of a simple proportion.

**196. Tapering Funnels and Spouts.**—Curved tapering pipes may be designed so that they can be made from a single right

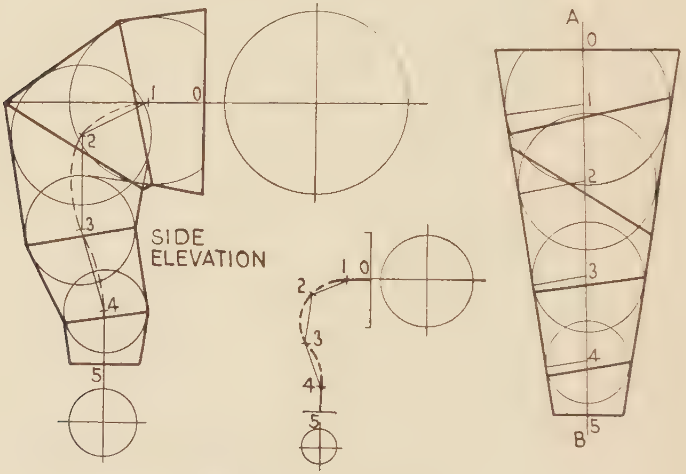


FIG. 100.—Conical funnel.

circular cone. A design of this kind is relatively easy to develop and requires a minimum amount of material. The conical funnel shown in Fig. 100 is taken as an example. The designer first determines the sizes and relative positions of the circular openings, and then sketches a curved center line to indicate the general shape of the funnel, as is shown to a reduced scale in the figure. The center line may then be divided into a number of equal or unequal parts, and these parts may be numbered as shown. Next, draw the cone from which the funnel is to be made. Take the line A B as the axis of the cone. Centered at 0 on this line, show the edge view of the large circular opening of the funnel. On the axis lay off the true lengths of the straight-line distances 0 1, 1 2, 2 3, etc. These are taken from the side elevation of the funnel. At 5 show the edge view of the small circular end of the funnel. The circles centered at 0 and 5 are bases of the frustum of a right circular cone. Draw the slanting sides of the cone. Using the numbered points on the axis as centers of spheres, draw spheres tangent to the cone. At the same time draw similar spheres centered on the corresponding numbered points in the side elevation, and draw a cone tangent to each pair of consecutive spheres. Complete the view by drawing the seam lines where the cones intersect. This completes the design of the funnel.

To proceed with the development, transfer the slant heights and the seam lines from the side elevation to the cone A B. In doing this it is necessary to reverse adjoining seam lines end for end. The built-up right circular cone may now be developed. If allowances for seams are necessary, the parts of the development must be separated enough to provide the extra metal. The appearance of the conical funnel may be improved by increasing the number of parts. A different method of designing and developing funnels is described in Art. 211.

**197. Problems.**

Group 73. Funnels and Spouts.

## THE CYLINDER

**198. Definition.**—The cylinder is a single-curved surface in which the straight line generatrix moves parallel to itself and touches a curved directrix. Since the generatrix is a straight line, the cylinder is a ruled surface. Since all elements of a cylinder intersect, the cylinder is a single-curved surface; and hence it may be developed. All elements of a cylinder are parallel. Mathematically speaking, they are said to intersect at infinity. If the vertex of a cone moves away from the base, the elements of the cone become more nearly parallel, until, in the limit, the vertex is at infinity, the elements are parallel, and the cone becomes a cylinder.

**199. Types of Cylinders.**—Figure 101 is a general-type cylinder in which the directrix is an open curve.

A right circular cylinder is shown in Fig. 102. The axis of a right cylinder is perpendicular to the planes of the bases. The right circular cylinder is also called a cylinder of revolution, since it may be generated by revolving the generatrix about the axis.

An oblique cylinder is shown in Fig. 103. The axis of an oblique cylinder is not perpendicular to the planes of the bases.

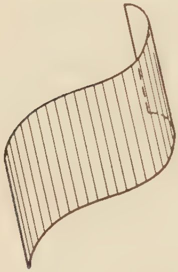


FIG. 101.—Cylinder.

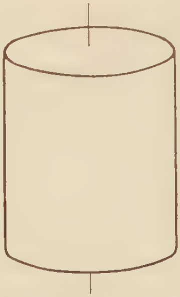


FIG. 102.—Right cylinder.

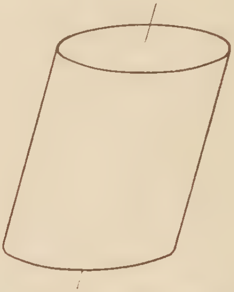


FIG. 103.—Oblique cylinder.

**200. Development of a Right Cylinder.**—Figure 104. The development of a right cylinder is a rectangle, of which one side is equal in length to the circumference of the circular base of the cylinder, and the other side is equal to the altitude of the cylinder. In the figure, the development of the entire cylinder is the rectangle A B C D.

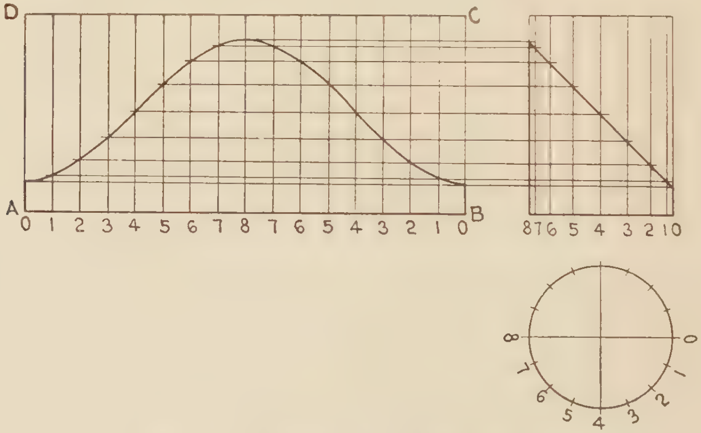


FIG. 104.—Development of right cylinder.

If, now, the rear part of the cylinder of Fig. 104 is cut off by a plane that is oblique to the axis, in order to develop the part that is left it is first necessary to divide the circular base into a number of equal parts, sixteen, for example. The elements through these points are next drawn in the top view, and here the true length of each element is seen. The shortest element may be number 0, and the longest element 8. In the development of the entire cylinder the circumference A B is also divided into sixteen equal parts. Draw the elements, and number the middle element 8, since it is longest. The other elements are then numbered consecutively as shown in the figure. The development will be symmetrical, and the cylinder will be cut along its shortest element. If the line A B of the development has been drawn in line with the front base of the cylinder, the true length of each pair of elements may be transferred directly from the top view to the development by means of parallel lines. Finally, the curved side of the development is drawn.

**201. Intersection of a Cylinder and a Plane.**—In Figs. 105, 106, and 107 are illustrated three different methods of finding the intersection of a cylinder and an oblique plane. All possible methods should be understood, and used as judgment dictates.

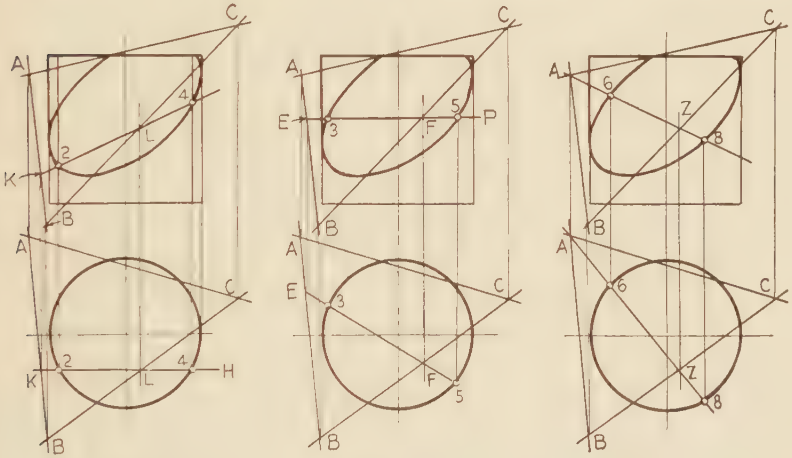


FIG. 105. FIG. 106. FIG. 107.

FIGS. 105 to 107.—Intersection of cylinder and plane.

In Fig. 105, the horizontal plane H cuts two elements from the cylinder, and a line K L from the given plane. In the top view, the line K L is seen to intersect the elements at 2 and 4. Other points on the elliptical intersection may be found by taking other horizontal cutting planes.

In Fig. 106, the frontal cutting plane P cuts a circle from the cylinder, and a straight line E F from the given plane. In the front view, the line E F is seen to intersect the circle at the points 3 and 5. These points are transferred to the top views of the line and circle. Other points on the intersection may be found by repeating this process.

In Fig. 107, any line A Z of the given plane is drawn in the front and top views. In the front view, this line is seen to pierce the surface of the cylinder at the points 6 and 8. The top views of these points are next located on the top view of the line A Z. Other points on the intersection may be found by taking other lines of the given plane. Whenever it is possible to obtain an edge view of a surface, such a view will show where lines intersect the surface.

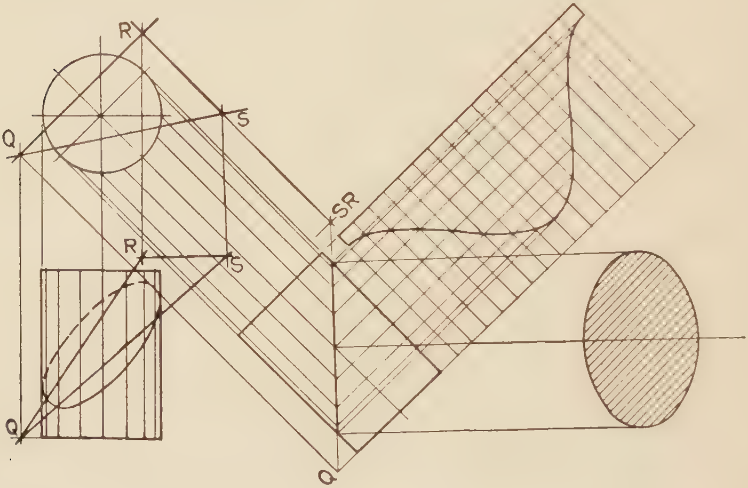


Fig. 108.—Right cylinder and oblique plane, intersection and development.

The intersection of a cylinder and a plane may also be found by obtaining an edge view of the plane, as is shown in Fig. 108. Starting with the front and top views, draw an auxiliary elevation taken in a direction parallel to any horizontal line R S of the given oblique plane Q R S. The auxiliary elevation will show the edge view of the plane. The front view of the intersection is obtained by determining the elevations of various points from the intersections in the auxiliary elevation, and then locating these points at their respective elevations in the front view. The true shape of the elliptical intersection is shown adjacent to the auxiliary elevation.

To develop the part of the cylinder that is above the given plane, the drawing should be oriented so that the auxiliary elevation becomes the front view, and then the cylinder may be developed by using the method explained in Art. 200.

**202. Problems.**

Group 74. Cylinder and Plane Intersections and Developments.

Group 75. Lines on Cylindrical Surfaces.

**203. Elbows.**—Sheet-metal elbows are designed so that one pattern will serve for all parts of the elbow. This is possible only for elbows that are to join pipes of the same diameter.

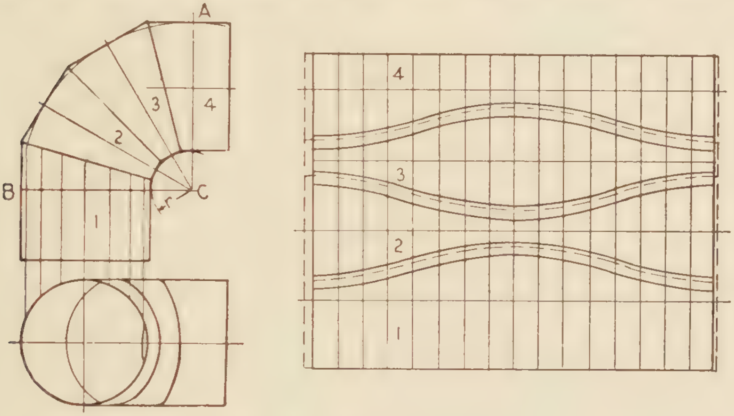


FIG. 109.—Four-piece elbow.

A 90-degree, four-piece elbow is shown in Fig. 109. The 90-degree angle is A C B. The ends of the pipe extend beyond the lines C A and C B. There are no seams on these lines. The least radius of the bend is *r*.

Note that the angle of the bend of the elbow is divided into equal angles. Each of the interior pieces of the elbow requires two of these angles, and each end piece requires one angle. The number of the equal angular divisions of the bend of the elbow is two less than twice the number of pieces in the elbow. Increasing the number of pieces in an elbow improves the appearance, decreases the internal friction, and increases the cost of the elbow. Increasing the least radius has the same effects.

The elbow consists of right cylinders cut off at an oblique angle by a plane. The development, drawn to the right of the elbow, shows how the four pieces of the elbow may be cut from a rectangular sheet of metal without waste. In addition to avoiding waste, this method of cutting the metal alternates the seams so that three seams will not come together when the parts are assembled. The allowance for seams is shown by dotted lines. It is generally necessary to furnish the shop only a single net pattern like the one shown at 3 in the development.

**204. Problems.**

Group 76. Elbows and Bends.

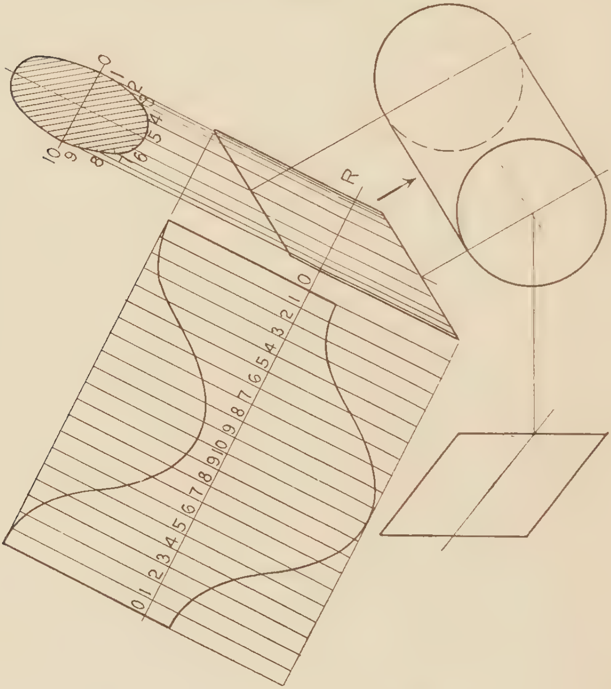


FIG. 110.—Oblique cylinder development.

**205. Development of Oblique Cylinder.**—In Fig. 110 are shown the front and top views of an oblique cylinder. To develop the cylinder, draw first an auxiliary view that shows both an edge view of the bases and a normal view of the elements. In the figure this is an auxiliary elevation taken in the direction of the arrow. Draw next an end view of the cylinder. This is shown in the figure as a cross-hatched ellipse. It is a right section of the cylinder. The front and top views of the cylinder may now be ignored, and the development may be made from the auxiliary elevation and right section.

Divide one-half of the elliptical right section into any number of parts. Preferably, these parts should be equal in length. Show the edge view of this right section at R in the auxiliary elevation. The problem now resolves itself into developing two right cylinders that have a common elliptical base at R and are cut off by two parallel oblique planes. Rectify the ellipse along the line O O. On this straight line step off the lengths of the parts into which the ellipse has been divided. Draw the elements. The middle element should be either 0, 10, or 5, in order to produce a symmetrical development. Let 10 be chosen as the middle element. Number the other elements consecutively in both directions. The length of each element may now be transferred directly from the auxiliary elevation to the development by means of parallel lines. Complete the development by drawing the outline.

**206. Problems.**

Group 77. Developments of Oblique Cylinders.

**207. Intersecting Cylinders.**—In finding the intersection of two cylinders, it is usually advisable to pass cutting planes parallel to the axes of both cylinders. Such a plane will cut two lines from each cylinder. These lines will cross at four points of the required intersection. The operation is repeated until enough points are found to indicate the intersection definitely.

The axes of the cylinders in Fig. 111 are frontal. Frontal cutting planes will be parallel to both axes. Pass the frontal cutting plane F. This will cut two elements from the vertical cylinder. The elements that the plane F cuts from the inclined cylinder are best found from the circular end view of the inclined cylinder. Here the same frontal plane is shown at $F_1$. The distance that it is drawn in front of the axis of the cylinder is determined from the top view. It cuts two elements from the inclined cylinder. The four elements thus found cross at the points 2, 3, 4, and 5 in the front view. Pass other frontal cutting planes until enough points are found, and then draw the intersection. In the top view the intersection will show as the arc of a circle. The elements that are cut from the inclined cylinder could be determined from the ellipse in the top view; but this is not generally advisable, since the ellipse is not likely to be as accurate as the circle.

**208. Cylinder and Cone Intersections.**—Two general methods are available for finding the intersection of a cylinder and a cone: Cutting planes may be passed, or the intersection may be found from a view that shows an end view of the cylinder. Cutting planes should be chosen so that they cut from the cone and the cylinder either two circles, two lines and a circle, or four lines.

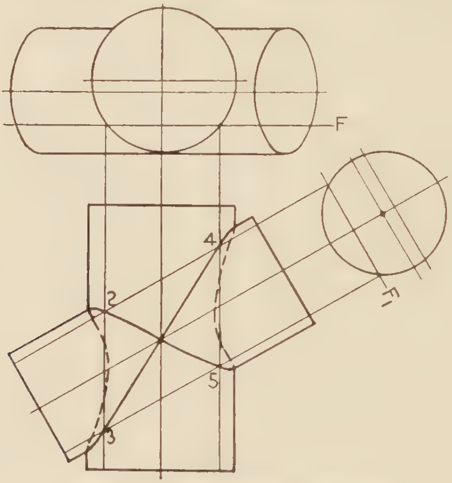


Fig. 111.—Intersecting cylinders.

The last choice is theoretically most desirable, since four points will be found for each plane that is passed. To do this, the cutting plane should be passed through the vertex of the cone, and parallel to the axis of the cylinder. However, the possibility of finding the intersection by obtaining an end view of the cylinder should not be ignored, since in such a view the exact point of intersection of each element of the cone with the surface of the cylinder is at once seen.

**209. Problems.**

Group 78. Intersecting Cylinders.
Group 79. Intersecting Cylinder and Cone.

**210. Development by Triangulation.**—Any curved surface may be approximated by dividing the surface into triangles. Since triangles are plane surfaces, they should be conceived not as lying on the curved surface, but rather as undercutting the curved surface. The greater the number of triangular divisions the nearer will the actual surface be approached.

A transition piece is shown in Fig. 112. One opening is circular and the other elliptical. The circle and ellipse do not lie in parallel planes. This surface is neither a cylinder nor a cone. Unnamed surfaces of this general type are frequently used in piping.

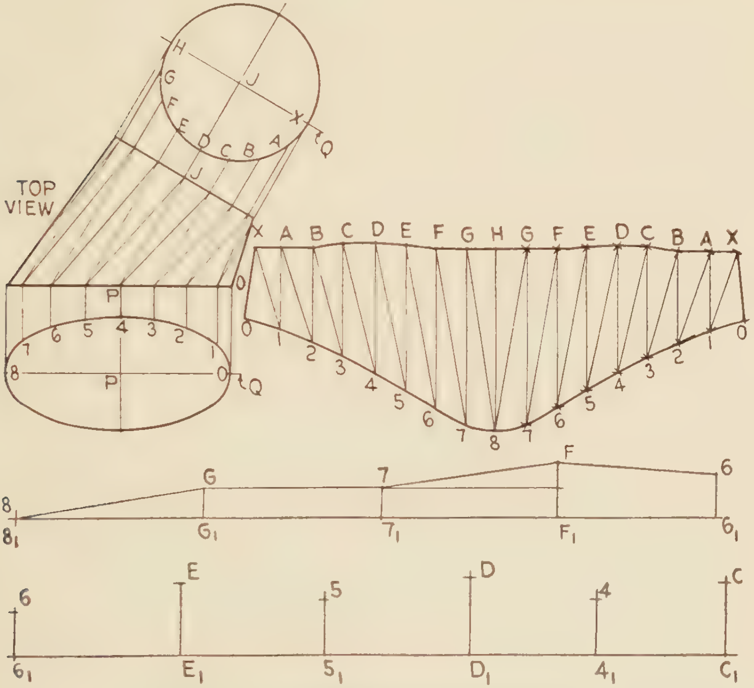


Fig. 112.—Development by triangulation.

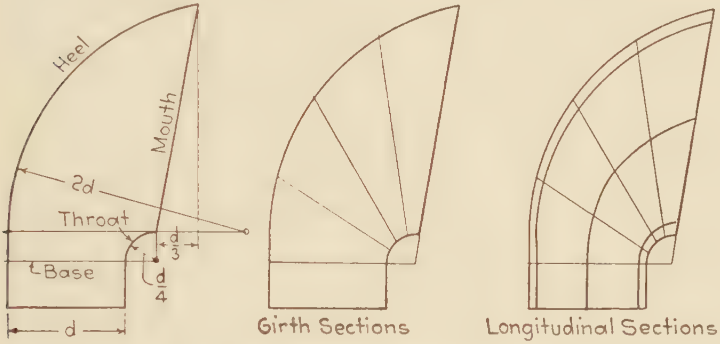
The horizontal plane Q passing through the axis J P divides the transition piece symmetrically. The development should be cut on the shortest line X O of the surface. Divide the circle and the ellipse into the same number of equal parts. These may be numbered consecutively on the ellipse, and lettered on the circle. The surface of the transition piece may now be divided into triangles, as shown in the top view. Developing the transition consists in finding the true lengths of the sides of these triangles, and building up the development line by line. The middle line of the development will be H 8. Since it is horizontal, its true length is seen in the top view. Draw H 8 in the development. Determine the true lengths of the lines H G and G 8, and draw the triangle H G 8 in the development. A similar triangle is drawn on the other side of H 8. Now, find the true lengths of the sides of the triangle G 8 7, and add one of these triangles to either side of the development. In turn, find the true lengths of the sides of the triangles G F 7, F 7 6, etc., and add these triangles to either side of the development. The completed development is shown in the figure.

The true lengths of the longer sides of the triangles may be found by means of the construction diagram shown below the development in Fig. 112. On a horizontal base line mark off in succession the lengths of the top views of the lines 8 G, G 7, 7 F, etc., as indicated at $8_1$ $G_1$, $G_1$ $7_1$, $7_1$ $F_1$, etc. At each of these lettered points, draw a perpendicular to the base line, and on this perpendicular mark the distance that the point itself is above the median plane Q. The true lengths of the lines may now be measured at 8 G, G 7, 7 F, etc. in the construction diagram.

Proof: The true length of 7 F in the diagram is the hypotenuse of a right-angle triangle, of which one leg is equal in length to the top view of 7 F, and the other leg is equal to the distance that F is above 7. In the diagram it is unnecessary to draw the true lengths. These may be obtained from the lower diagram. Time is saved in developing the transition piece by using three compasses: Set one compass to the true length of the equal parts of the circle; set another compass to the true length of the equal parts of the ellipse; and use the third compass for determining the lengths of the longer sides of the triangle in the construction diagram.

**211. Ship's Ventilator.**—The standard profile for a ship's ventilator, or funnel, is shown in Fig. 113. The diameter $d$ of the base circle is determined by the use to which the funnel is to be put. The radii of the throat and of the heel are shown in the drawing. The overhang is $\frac{d}{3}$. These proportions make the diameter of the mouth very nearly equal to $2d$. The mouth of the funnel may be either circular or elliptical.

The ventilator may be made either from girth sections or longitudinal sections, as shown in the figure. To obtain the girth sections, the heel and the throat are divided into the same number of equal parts. Four are here chosen, and these parts are connected as shown in the drawing. The seams are circles if both the base and the mouth are circular. Each girth section may be developed by the triangulation method explained in the preceding article. The developments of adjacent sections are generally cut so that three seams do not come together when the sections are assembled.



Girth Sections

Longitudinal Sections

FIG. 113.—Standard ship's ventilator.

To obtain the seams for a ventilator that is to have longitudinal sections, the circular sections are first determined as explained in the preceding paragraph, and then each circle is divided into the same number of equal parts. The longitudinal seams pass through these points on the circles. The sections are developed by triangulation.

**212. Problems.**

Group 80. Developments by Triangulation.
Group 81. Ventilators.

## THE CONVOLUTE

**213. Definition.**—The convolute is a single-curved surface that is generated by a straight line moving tangent to a helix or other double-curved line. Since the elements are straight lines, the surface is ruled. Two consecutive elements will intersect, that is, they line in a plane. No three consecutive elements of the convolute intersect. The convolute may be considered as made up of parts of planes; hence, the surface may be developed. The helical convolute is sometimes called a developable helicoid. This surface is definitely determined by the diameter and the lead of the helical generatrix.

**214. Example.**—One turn of the helix on the cylinder shown in Fig. 114 is taken as the directrix. The generatrix is here taken as a straight line of limited length. Its upper end is tangent to the helical directrix, and its lower end will trace another helix larger in diameter.

To draw the convolute, first choose and draw the helical generatrix. Next draw the element K L. It is tangent to the helix at its front point. The direction for drawing K L is determined from the triangle A B C. The side A B is made equal to one-half of the circumference of the cylinder on which the helical generatrix is wound. The side B C is made equal to one-half of the lead of the helix. The hypotenuse A C shows the slope and the half length of the helix. All lines tangent to the helix will have the same slope. In the front view, draw K L parallel to A C. In the top view, draw K L tangent to the circular top view of the helix. The length of K L may be chosen as desired. The point L will trace another helix of the diameter indicated in the top view, and the lead of this helix will be the same as the lead of the helical generatrix. In the top view, beginning at K and L, divide each of the two helices—which appear as circles—into the same number of equal parts. Trans-

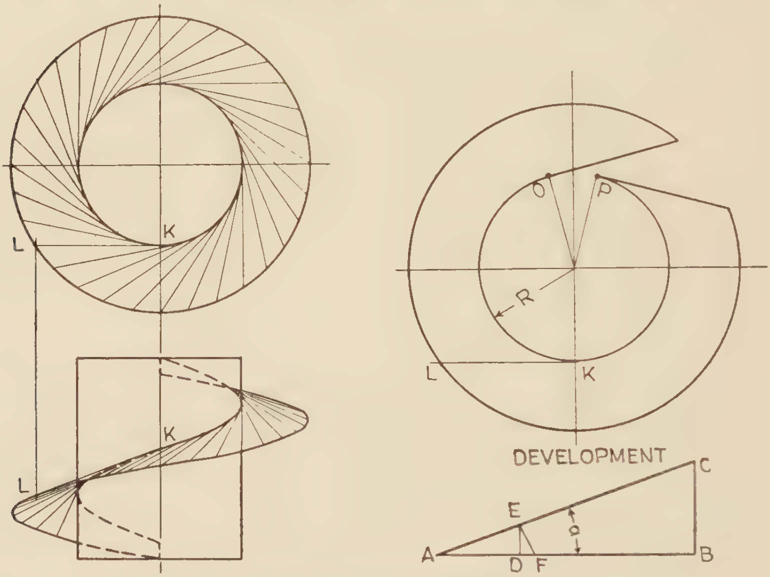


Fig. 114.—The convolute.

fer these division points to the respective helices in the front view. Straight lines joining corresponding points will be elements of the convolute. The intersection of the convolute with a plane perpendicular to the axis is an involute. Why is this so?

**215. Development of the Convolute.**—The development of the convolute may be visualized by imagining that the convolute as shown in the front view is pressed downward until it lies in a flat plane. Since the curvature of the helix is constant, the helix will become a circular arc in the development. The radius of curvature R of a helix is equal to $\frac{r}{\cos^2 a}$, in which $r$ is the radius of the cylinder of the helix, and $a$ is the angle of slope of the helix. The formula is developed by the aid of calculus. The radius of curvature may be calculated from the formula just given, or the formula may be solved graphically as follows: In the triangle A B C, Fig. 114, make A D equal to $r$. Draw D E perpendicular to A B, and then draw E F perpendicular to A C. The distance A F is equal to the radius of curvature R of the helix. Prove that the distance A F $= \frac{r}{\cos^2 a}$.

In the development draw a circle of radius R. At K draw a tangent to this circle, and make K L equal in length to the front view of this line. Draw the outside circle through L. Make the arcs K P and K O each equal in length to the straight line A C. The length of the arc O K P is now the same as the length of the helical directrix. At P and O draw tangents to the inner circle. The completed development represents one turn, or flight, of the convolute. The ring of the development might be cut radially instead of tangentially at the points O and P, but the radial cuts will be slightly curved when the convolute is formed from the development.

The convolute is used as the surface of screw conveyors. It is impossible to make the convolute perpendicular to the cylinder on which it is wound; a surface meeting this condition may be developed as explained in Art. 210.

**216. Problems.**

Group 82. Convolutes.

# CHAPTER XI

## WARPED SURFACES

### HYPERBOLIC PARABOLOID—CONOID—HELICOID—HYPERBOLOID OF REVOLUTION

**217.** A warped surface is a ruled surface in which consecutive elements do not intersect. In surfaces of this type the motion of the straight-line generatrix is usually determined by some combination of two or three curved or straight-line directrices, and a plane or cone director. The generatrix remains parallel to the plane director, or, if the director is a cone, the generatrix is in turn parallel to the consecutive elements of the cone. If the surface has three directrices, no plane or cone director is necessary.

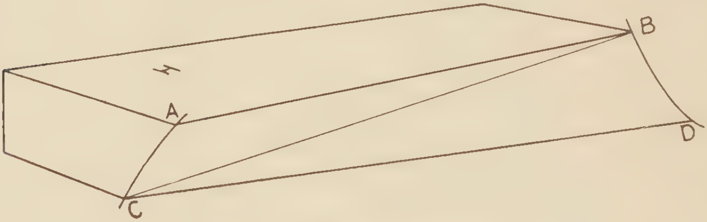


Fig. 115.—Warped surface.

Figure 115 illustrates two consecutive elements, A B and C D, of a typical warped surface. The elements are straight lines that do not intersect. In the illustration, the lines A C and B D are curved-line directrices of the surface, and a horizontal plane H is the plane director. The elements intersect the directrices and remain parallel to the plane director. Either one or both of the directrices may be straight.

Only the most commonly used warped surfaces are described in this chapter. These are: the hyperbolic paraboloid, the conoid, the helicoid, and the hyperboloid of revolution. All of these surfaces have definite uses, and are relatively easy to construct if the geometry of the surfaces is understood. They could, to advantage, be more generally used.

**218. Warped Surfaces Not Developable.**—The true warped surface cannot be developed. Such a surface, however, may be approximated by triangulation, and the approximate form developed.

The warped surface A B D C, Fig. 115, cannot be developed. It is impossible to divide it into triangles or unroll it into a plane. It is possible, however, to approximate the surface by drawing the straight line B C and substituting the triangles A B C and B C D for the warped surface. The approximate surface may now be developed by triangulation. The line B C does not lie in the warped surface even when the directrices are straight lines.

## THE HYPERBOLIC PARABOLOID

**219.** The hyperbolic paraboloid is a warped surface having two straight-line directrices and a plane director. Since the surface is ruled, the generatrix is a straight line. Since the surface is warped, it is not developable.

**220. To Draw the Hyperbolic Paraboloid.**—This surface is illustrated in Fig. 116. A C and B D are the straight-line directrices, and H is the plane director. A B and C D are elements of the surface. Other elements are shown parallel to the plane director. The hyperbolic paraboloid may be doubly ruled. The elements A B and C D of the first generation may be taken as directrices of the surface. The plane P, parallel to the directrices A C and B D of the first generation, becomes the plane director of the second generation. Other elements parallel to the plane P are shown. The elements of both generations form the same surface. Plane sections of the surface are straight lines, hyperbolas,

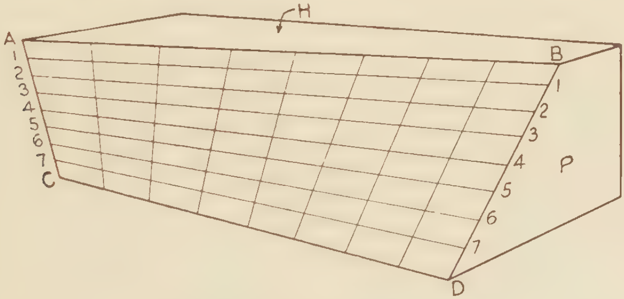


FIG. 116.—Hyperbolic paraboloid.

and parabolas. Various forms of the surface are shown in Fig. 117.

The method of drawing the different views of the surface may be explained with the aid of Fig. 116. The problem may be stated as follows: A C and B D are the directrices of an hyperbolic paraboloid. A B and C D are two elements of the surface. Draw the front, top, and left-side views.

First, locate the views of the directrices, and then divide each directrix into the same number of equal parts. Draw the element A B, and then, in turn, draw the elements connecting consecutive points on the two directrices. Time is saved by numbering the division points of each directrix consecutively from A and from B, in the different views. Each straight-line element is then drawn through the two points that have the same number.

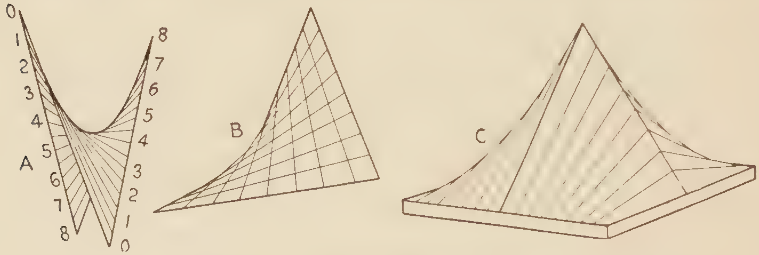


Fig. 117.—Hyperbolic paraboloids.

The surface may be doubly ruled by dividing the elements A B and C D—which are now taken as the directrices of the second generation—into the same number of equal parts, and then connecting the corresponding points by straight lines. It is not necessary to divide both pairs of directrices into the same number of equal parts, but the two directrices of a single generation must have the same number of divisions.

**221. Uses of the Hyperbolic Paraboloid.**—The fact that this surface may be doubly ruled makes it possible to construct the surface with cross-timbers, as in a roof or in a form for concrete. The timbers follow the directions of the elements of each generation. The surface is used for the wing walls of dams, abutments, and bridge portals. It has also been used for roofs and for the bows of ships. Incidentally, the pilot of the American locomotive is an hyperbolic paraboloid. It is a strongly braced structure.

**222. Problems.**

Group 83. Hyperbolic Paraboloid.

## THE CONOID

**223.** The conoid is a warped surface having a straight-line directrix, a curved directrix, and a plane director. Since it is a ruled surface, the generatrix is a straight line. The surface, being warped, cannot be developed excepting approximately by triangulation.

**224. Constructing the Surface.**—A right conoid is illustrated in Fig. 118. The elements of the surface are parallel to the plane director P, and intersect the straight-line directrix and the circular directrix. The surface cannot be doubly ruled. Plane sections parallel to the circular base are ellipses. In the right conoid the elements are perpendicular to the straight-line directrix, and the middle point of the directrix is on the axis of the circle. An oblique conoid is shown in Fig. 119. T is the plane director.

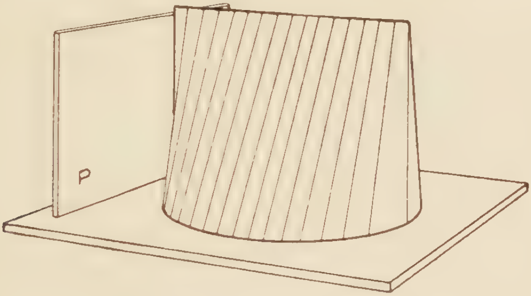


Fig. 118.—Right conoid.

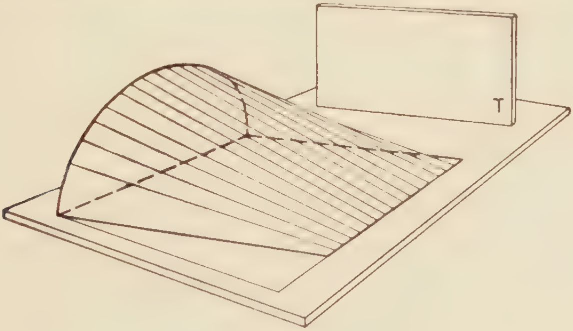


Fig. 119.—Oblique conoid.

The front, top, and left-side views of a right conoid are shown in Fig. 120. Since all of the elements are frontal, the plane director is frontal. The equally spaced elements are drawn in the top view and then in the front and side views.

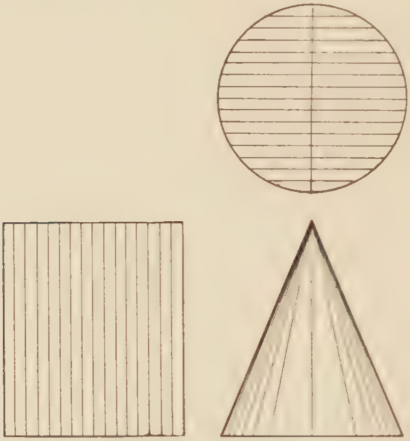


Fig. 120.—Conoid.

**225. Uses of the Conoid.**—The conoid is used in connecting a flat surface with a curved surface, for example, when connecting a flat ceiling with an arch, or when changing the cross-section of a flume or pipe from rectangular to circular. The surface is not so readily constructed as is a doubly-ruled surface. Transition pieces to be made from sheet metal should not be designed as conoidal, but rather as combinations of oblique cones and plane surfaces.

**226. Problems.**

Group 84. Conoid.

## THE HELICOID

**227.** The helicoid is a warped surface having a helical directrix, a straight-line directrix, and a plane or a cone director. It may be visualized as a surface that is generated by a straight line, which moves so as to intersect a helix and its axis, and keeps a constant angle with the axis. If the generatrix remains perpendicular to the axis, a right helicoid is generated; this is the surface of a square-threaded screw. If the generatrix makes a constant oblique angle with the axis, an oblique helicoid is generated; this is the surface of a screw having V threads. The right helicoid has a plane director; the oblique helicoid, a cone director. The generatrix may not intersect the axis of the helix, but this type of helicoid is unusual. The surface of the helicoid is warped; hence, it is not developable excepting by approximation. It cannot be doubly ruled.

The helicoid may also be visualized as generated by a straight line that intersects an axis at a fixed angle, that rotates around the axis, and at the same time moves along the axis at uniform rates.

**228. Representing the Helicoid.**—One flight, or turn, of a right helicoid is shown in Fig. 121. The elements are perpendicular to the axis, and intersect the helix and its axis. The surface is here represented as one flight of a right-hand right helicoid with the elements ruled on the upper side of the surface. In the top view the elements are spaced at equal angles around the axis, and in the front view they are spaced at equal distances measured parallel to the axis. The lead of the helix in the front view is divided into the same number of equal parts as is one turn of the helix in the top view.

An oblique helicoid is shown in Fig. 122. The elements are inclined to the axis. The surface is here represented as one flight of a left-hand oblique helicoid with the elements ruled on the upper side of the surface. In the top view the elements are shown equally spaced around the axis. As the generatrix revolves around the axis, and at the same time moves parallel to the axis, each point of the generatrix traces a helix. All of these helices have the same lead. In the front view several consecutive elements have been extended to meet the axis. The upper ends of the elements are equally spaced along the axis, and the lower ends are at similarly equally spaced levels. The upper end of the generatrix remains at a constant distance above its lower end. As the generatrix moves from one position to the next, both ends of the generatrix are lowered the same amount. This information is an aid in drawing the front view.

To draw the oblique helicoid of Fig. 122, the limiting helices should first be represented by circles in the top view, and the equally spaced radial elements drawn. The larger helix, of the required lead or pitch, should now be plotted and drawn in the front view. One of the normal elements, A B, may now be drawn. The given angle that it makes with the axis is seen at

A in the front view. As the generatrix A B moves, the point O traces the smaller helix. This helix has the same lead or pitch as has the larger helix. A B is the first element of the surface. As the lower end B is rotated and lowered to the next position C on the helix, the upper end A is simultaneously lowered an equal vertical distance to D on the axis. The vertical distance A D is equal to the difference in level between B and C. The other elements are drawn in a similar fashion. The intersection of the oblique helicoid with a plane H that is perpendicular to the axis is an Archimedean spiral. This intersection is at times useful in drawing or representing the surface.

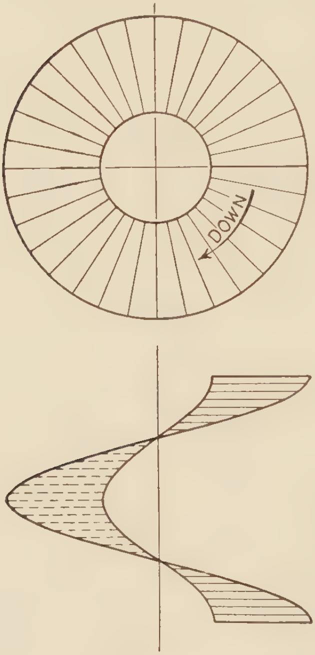


Fig. 121.—Right helicoid.

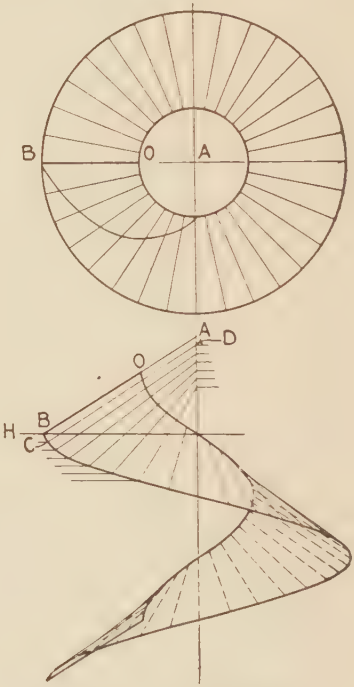


Fig. 122.—Oblique helicoid.

**229. Uses of the Helicoid.**—The helicoid is used more frequently than any other warped surface. It is the surface of screw threads, of certain "spiral" springs, of screw propellers, of spiral stairways, helical cams, etc. Since the surface of the helicoid may be approximated and developed by triangulation, it is used for screw conveyors and spiral chutes. Attention is here called to the convolute, a single-curved surface, that is also called the developable helicoid. It is described in the chapter on single-curved surfaces.

**230. Problems.**

Group 85. Helicoid.
Group 86. Helicoidal Chutes and Conveyors.

**231. Screws and Springs.**—These are such common articles that there is no need of describing their uses. We are here primarily concerned with the method of drawing them. The threads of screws are of different shapes. The shape of the thread depends upon the use to which the screw is to be put. Threads are normally made right handed, but the requirements of a design sometimes make it necessary to use a left-hand thread. Threads may be single or multiple; that is, one or more threads may be cut on a cylinder. Single threads are used when the screw is to act as a clamp or when the screw is to exert great pressure. Multiple-threaded screws are used for increasing the travel per turn, and for power transmission purposes. Within certain limits, multiple-threaded screws are more efficient than single-threaded screws; that is, they use less power in friction. Screws were once made by wrapping a round or a square wire around a cylinder and soldering it in place.

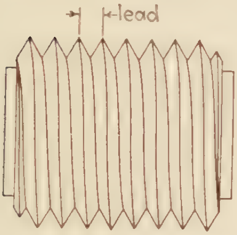


Fig. 123.—Right-hand, single, V thread.

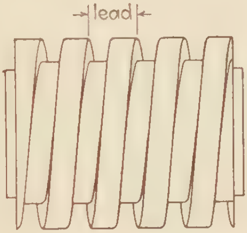


Fig. 124.—Left-hand, single, square thread.

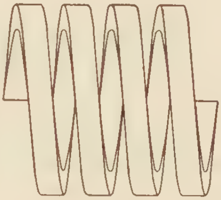


Fig. 125.—Right-hand spring.

The screw shown in Fig. 123 has a right-hand, single, sharp V thread. The angle of the V is 60 degrees. The surfaces of the thread are oblique helicoids. The curved edges that limit the surfaces are two helices. The screw is represented by drawing these helices and the profiles of the V thread. Threads are shown conventionally by drawing the helices as straight lines, eliminating the V profile, and showing the outline of the cylinder.

A left-hand, single, square-threaded screw is shown in Fig. 124. The thread is drawn by showing the visible parts of the four helices. These may be considered as being traced by the four corners of a square. The working surfaces of the thread are right helicoids. The lead, or pitch, of the thread is indicated. The size of the square is one-half the lead.

A right-hand spring made from square stock is shown in Fig. 125. It is a square thread with the cylindrical core removed. It may be made by winding a square metal wire around a cylinder. Other methods of manufacture are also used. The surfaces bounding the spring are cylinders and right helicoids. Springs made from round stock are double-curved surfaces (see Art. 254).

**232. Multiple Threads.**—The possibility of cutting more than one thread on a cylinder is illustrated in Fig. 126. The lead of the left-hand square helical groove is here taken as six times the width of the groove. Two other similar, equally spaced, left-hand, helical grooves could be cut in the cylinder between the turns of the first groove. When this is done, the left-hand, triple, square-threaded screw, illustrated in Fig. 127 is produced. Another screw, having a right-hand, double, square thread, is shown in Fig. 128.

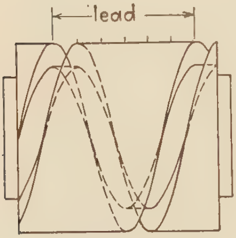


Fig. 126.—Helical groove.

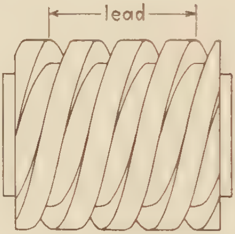


Fig. 127.—Left-hand, triple, square thread.

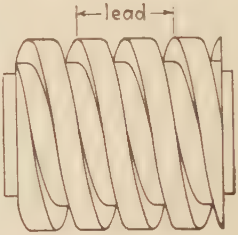


Fig. 128.—Right-hand, double, square thread.

To determine the size of a square thread, a profile similar to that shown at A in Fig. 129 may be sketched. If the thread is to be a single thread, the lead will be from 0 to 1, and the size of the square will be one-half of the lead. If the thread is to be a double thread, the lead will be measured from 0 to 2, and the size of the square will be one-fourth of the lead. If the thread is to be triple, the lead will be measured from 0 to 3, and the size of the square will be one-sixth of the lead. The size of the square of any multiple thread may be determined in this way. This could easily be reduced to a rule; but rules, unless visualized, are easily forgotten.

When it is necessary to draw the profiles of a square thread, care must be taken to see that the profiles on the opposite sides of the cylinder are properly related. At B in Fig. 129 is shown the relation between the profiles when the screw is to have an even number of threads, and at C is shown the correct relation for an odd number of threads. At B, the valleys of the threads are opposite; at C, the thread is opposite a valley. These relations may be determined by the fact that the thread travels one-half of the lead in one-half of a turn.

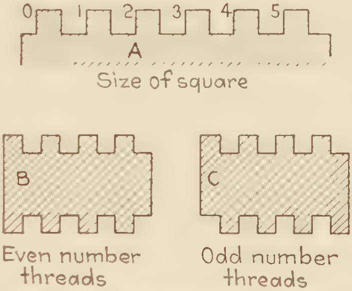


FIG. 129.—Square thread profiles.

**233. Screw Propellers.**—The driving surfaces of ship's propellers are usually helicoidal. Propellers are often called screws. The surface of the blade may be a right helicoid, an oblique helicoid, or a helicoid of radially increasing pitch. In the latter, the outer end of the generatrix moves parallel to the axis at a faster rate than does the inner end. The working surfaces of screw propellers should be very smooth, and in the case of large propellers are often machined. To make the drawings and patterns, it is necessary to know how the helicoidal surfaces are generated.

**234. Problems.**

Group 87. Screws.
Group 88. Springs.

## THE HYPERBOLOID OF REVOLUTION

**235.** The hyperboloid of revolution is a warped surface that is generated by a straight line revolving around an axis that it does not intersect. This surface may also be defined as a warped surface having as directrices three coaxial circles. Ordinarily two of the circles are the same size, and the third and smaller circle is halfway between them. The surface may be doubly ruled, but it is not developable.

**236. Drawing the Hyperboloid of Revolution.**—Figure 130 shows the front and top views of an hyperboloid of revolution.

If the upper and lower circles, and the middle, or gore, circle are given, any number of equally spaced elements may first be drawn in the top view, where they appear to be tangent to the gore circle. The elements, however, do not lie in the plane of the gore circle. Every element intersects each of the circular directrices at a single point. One end of the element K L, for example, is at K on the upper circle, and the other end is at L on the lower circle. These points are located in the front view, each on its respective circle, and then the front view of the element is drawn.

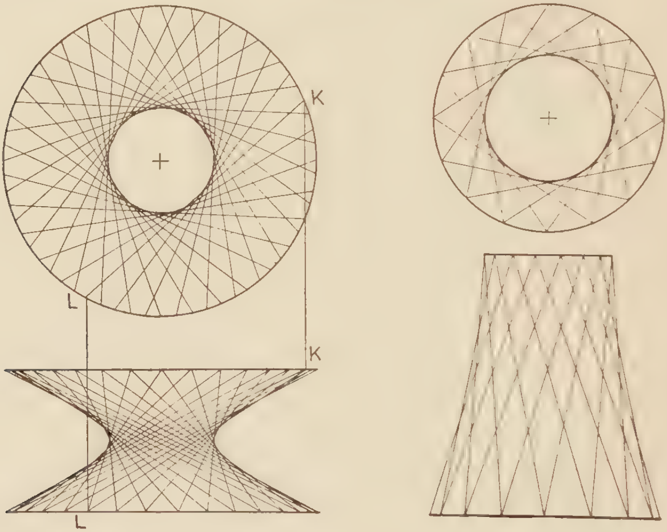


Fig. 130.

Fig. 131.

Figs. 130 and 131.—Hyperboloid of revolution.

In each view of Fig. 130 all of the elements of a single generation are shown. The elements of the second generation could be drawn by reversing the elements of the first generation, for example, by locating the point K on the lower circle, and the point L on the upper circle in the front view.

If the axis and the generatrix K L of the surface are given, the elements may be drawn as described above, after the circles generated by the points K and L and the gore circle are drawn. The gore circle is generated by the point on the generatrix that is nearest the axis. The intersection of the surface of the hyperboloid of revolution with a plane passed through, or parallel to the axis is an hyperbola.

In Fig. 131 is shown the lower part of an hyperboloid of revolution. In the top view the elements of two generations are shown. In the front view the elements of both generations that lie on the front half of the surface are shown.

**237. Uses of the Hyperboloid of Revolution.**—The doubly ruled hyperboloid of revolution, built from straight steel members, has been used for towers and for the firing masts of battleships. The surface is also used for skew gears, as explained in the next paragraph. Incidentally, the surface is used in lattice work and in reed furniture.

**238. Skew Gears.**—When two non-intersecting and non-parallel shafts are to be directly geared together, skew gears are often used. The outline of a skew gear must be an hyperboloid of revolution.

The theoretical rolling surfaces of a pair of skew gears are shown in Fig. 132. The actual blanks, into which the gear teeth are cut, must be slightly larger by a certain fraction of the height of a tooth. In the figure the center lines of the shafts are A B and C D. It is here assumed that the shafts are to rotate at the same speed.

In the top view, the angle between the axes is bisected, and the desired equal faces or widths of the two gears are indicated. The bisector E F is the common element of contact of the two gears. It is a horizontal line, and since the gear ratio is one to one, it is halfway between A B and C D. Locate E F in the front view and the auxiliary elevation. In the latter view, the circular outline of the upper gear may now be drawn. Its center is on C D; the larger circles bounding it pass through E and F; and the gore circle appears tangent to E F. The upper gear is here shown in section, together with twenty-four elements of the surface. Its top and front views may now be completed.

In the right-side view a section of the lower gear may now be drawn. Its center is on A B; the larger circles pass through E and F; and the gore circle appears to be tangent to E F. Twenty-four elements of the surface are shown. The front and top views of the lower gear may now be drawn. Only the elements that are visible in each view are shown.

In gears of this type, there is a certain amount of end slip between the teeth. If the shafts are to turn at different rates, it is necessary to take this end slip into account when determining the location of the element of contact E F. Two hyperboloids rolling together could not serve as friction gears because of the end slip along the element of contact. Skew gears should not be confused with spiral gears.

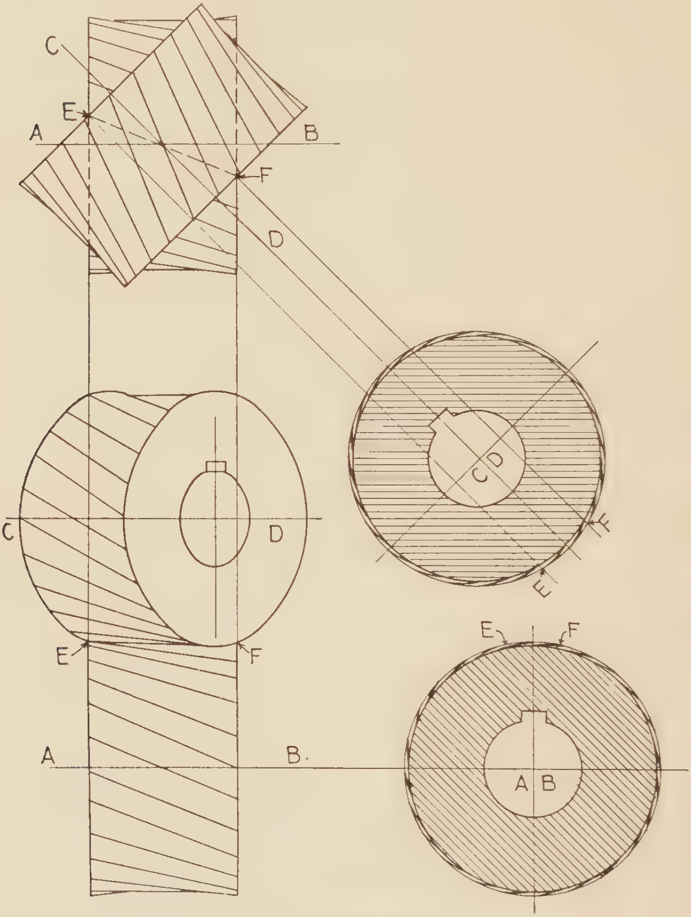


FIG. 132.—Skew gears.

**239. Problems.**

Group 89. Hyperboloid of Revolution.
Group 90. Skew Gears.

## CHAPTER XII

# DOUBLE-CURVED SURFACES

### SPHERE—ELLIPSOID—TORUS—SURFACES OF REVOLUTION—SERPENTINE—SPRINGS

**240.** Surfaces are classified either as ruled or as double-curved. Ruled surfaces are generated by a straight line. Double-curved surfaces have no straight-line elements. A double-curved surface is curved in every direction. The sphere is an example. Many double-curved surfaces are surfaces of revolution; that is, they are generated by revolving a curved line around a straight-line axis. Surfaces of revolution are easily made. They are usually turned in a lathe, formed on a potter's wheel, cast from turned patterns, or shaped with turned dies.

A double-curved surface cannot be developed, but may be approximated by enclosing it in sections of cylinders or cones. The latter, being single-curved surfaces, may be developed.

### THE SPHERE

**241.** A sphere is a double-curved surface generated by revolving a circle about one of its diameters as an axis. It is a surface of uniform curvature in every direction. Every point of the surface is equidistant from the center.

**242. Intersections.**—Every plane section of a sphere is a circle. If the plane is oblique, the circle will appear as an ellipse in the front, top, and side views.

To find the intersection of an oblique plane and a circle, pass horizontal, frontal, or profile cutting planes. Each cutting plane will cut a circle from the sphere, and a straight line from the oblique plane. The line and circle will cross at two points of the required intersection. Repeat the operation until a sufficient number of points is found (see Fig. 133).

To find the intersection of two spheres, pass horizontal, frontal, or profile cutting planes. Each cutting plane will cut a circle from each of the spheres. The circles will cross at two points that are common to both surfaces. The operation is repeated until the intersection is indicated by a succession of points as in Fig. 134.

The intersection of a sphere and a cylinder, or a sphere and a cone, may be found by passing cutting planes that cut circles from the cone or the cylinder (see Art. 157).

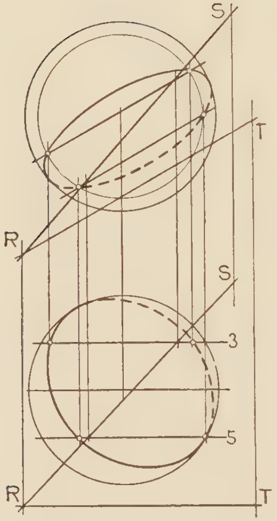


Fig. 133.—Intersecting sphere and plane.

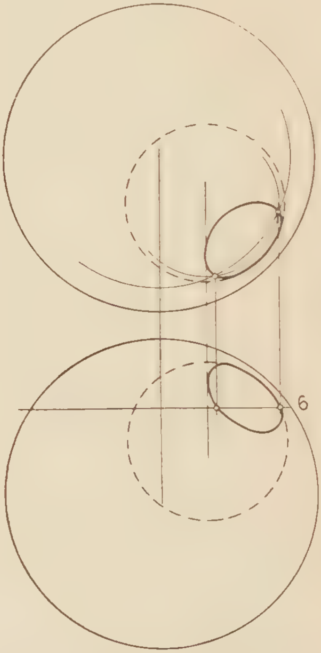


Fig. 134.—Intersecting spheres.

**243. Uses of the Sphere.**—The most common use of the sphere is in ball bearings. The ball is forged or turned approximately spherical, and, after hardening, is ground and lapped between parallel rotating disks. Specifications often require that the finished ball shall not be more than one ten-thousandth of an inch out of round. Brass and steel balls are used as valves. Hemispheres are used as the domes of buildings and for the bottoms of water tanks.

**244. Development of the Sphere.**—Two methods for making an approximate sphere from sheet metal are available; the zone method illustrated in Fig. 135, and the meridian method illustrated in Fig. 136.

When the zone method is used, the sphere is approximated by enclosing it in sections of right circular cones. The development of the conical zones is shown in Fig. 135. The operation of developing the cones is described in Art. 186. Increasing the number of zones will make the surface more nearly spherical.

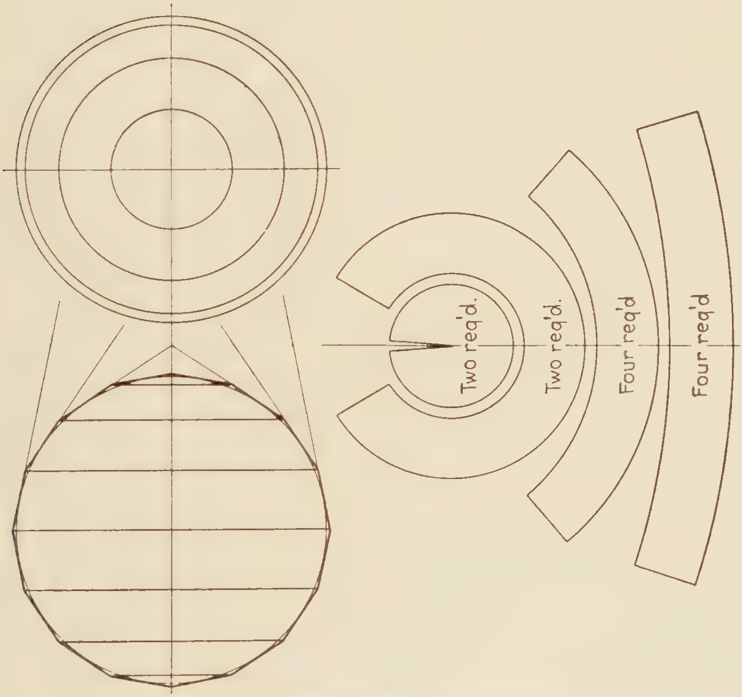


Fig. 135.—Zone development of sphere.

When the meridian method is used, the sphere is approximated by enclosing it in similar sections of a right cylinder. The design and the development are shown in Fig. 136. If the sections are to be stretched by hammering or rolling to a spherical shape so that the assembled sections will more nearly make a sphere, the sections are generally developed from a design that is drawn inside the actual sphere.

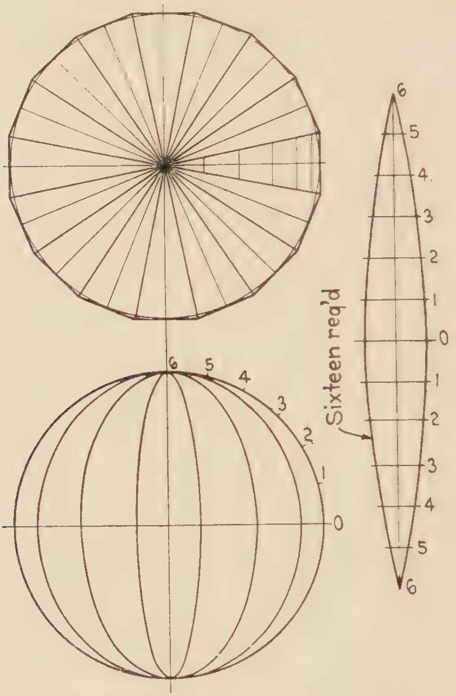


FIG. 136.—Meridian development of sphere.

### 245. Problems.

Group 91. Intersecting Sphere and Plane.
Group 92. Intersecting Sphere and Prism.
Group 93. Intersecting Spheres.
Group 94. Intersecting Sphere and Cylinder.
Group 95. Intersecting Sphere and Cone.
Group 96. Approximate Development of Sphere.

## THE ELLIPSOID

**246.** The ellipsoid is a double-curved surface that is generated by revolving an ellipse about one of its axes. A prolate ellipsoid is generated by revolving an ellipse about its major axis; an oblate ellipsoid, by revolving an ellipse about its minor axis (see Fig. 137).

**247. Intersections and Uses.**—Intersections of the ellipsoid with other surfaces are found by passing cutting planes perpendicular to the axis of revolution. The surface is used mainly for domes of buildings and in ornamental designs. The approximate surface may be developed by either the zone or the meridian method, as described in Art. 244.

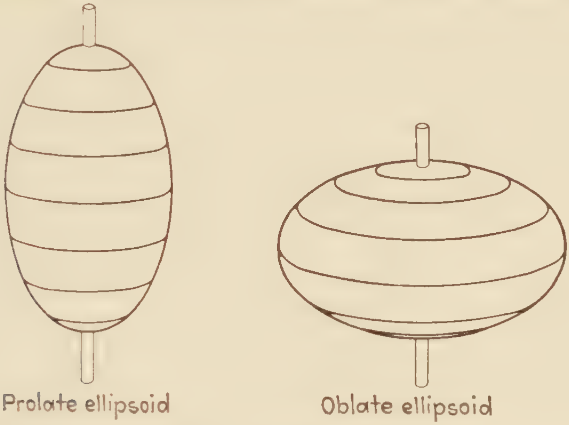


Fig. 137.—Ellipsoids.

**248. Problems.**

Group 97. Ellipsoid.

## THE ANNULAR TORUS

**249.** The annular torus, or anchor ring, is a double-curved surface that is generated by a circle which revolves about an axis that lies in the plane of the circle. It may also be visualized as the envelope of all spheres of equal size that may have their centers on a given circle (see Fig. 138).

The intersection of the torus with any other surface may be found by passing cutting planes either perpendicular to, or through the axis of revolution. In either case, two circles will be cut from the torus. The torus cannot be developed excepting when the surface is approximated by enclosing it in sections of cylinders.

**250. Uses of the Torus.**—In addition to being the surface of rings made from round stock, of pipe bends, etc., the torus is

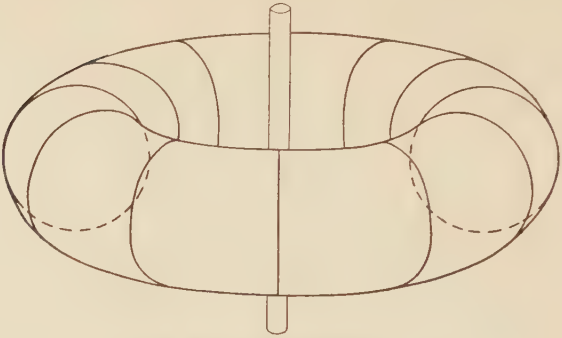


FIG. 138.—Annular torus.

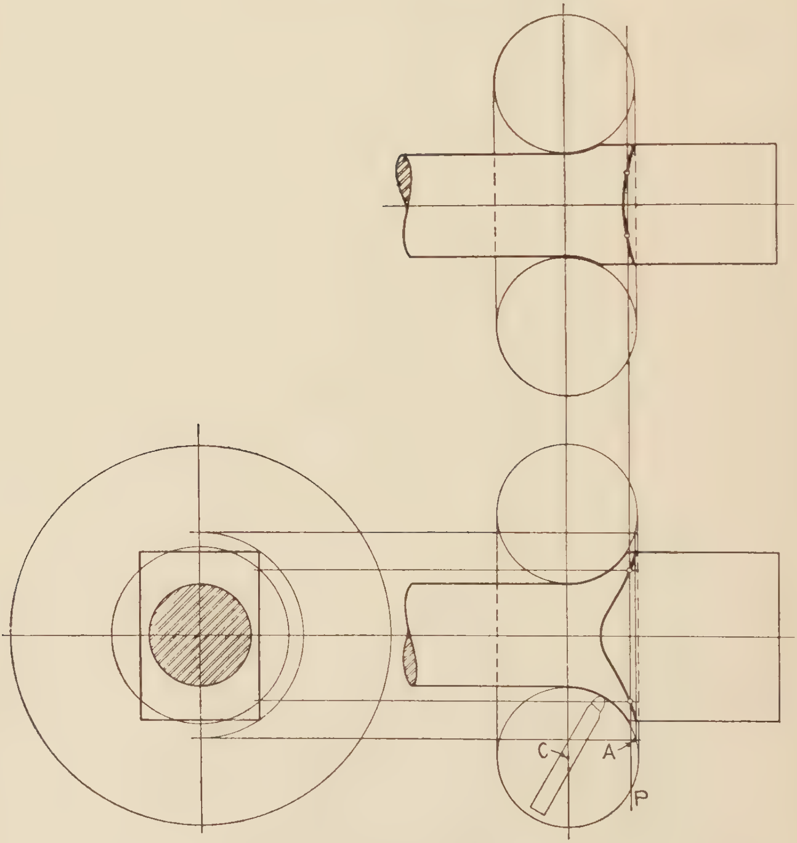


FIG. 139.—Connecting rod.

the surface that is formed when a circular fillet is turned in a lathe. As an example, the stub end of a connecting rod is shown in Fig. 139. The middle part of the rod has been turned cylindrical, while the stub end has been left rectangular in cross-section. A circular, turned fillet joins the two parts. A sharp corner would tend to start a crack in the metal. The turned fillet is a part of an annular torus. The faces of the rectangular stub end are planes. The intersections of these planes with the torus will be the curved lines that are shown on the connecting rod in the front and top views. These intersections are found by passing cutting planes—P, for example—perpendicular to the axis of revolution. Without the curved lines of intersection, the views of the connecting rod have an unfinished appearance. Note that when the point of the lathe tool is at A, it is just cutting the corners of the rectangular bar. The fillet is formed by turning the lathe tool about the center C.

**251. Problems.**

Group 98. Annular Torus.

### SURFACES OF REVOLUTION

**252.** A surface of revolution is generated by revolving a generatrix about a straight-line axis. Certain single-curved and warped surfaces—the cone, the cylinder, and the hyperboloid of revolution—are ruled surfaces of revolution. Double-curved surfaces of revolution described in this chapter are the sphere, the ellipsoid, and the torus. Others are the paraboloid, which is generated by revolving a parabola about its axis; and the hyperboloid of two nappes, which is generated by revolving the two branches of an hyperbola about the transverse axis. The parabolic reflector is a paraboloid. In addition to these, there is an unlimited number of double-curved surfaces of revolution that may be generated by revolving a curved-line generatrix about a straight-line axis. Vases and turned ornaments, the base, receiver, and transmitter of the telephone are examples.

**253. Problems.**

Group 99. Surfaces of Revolution.

### SPRING AND SERPENTINE

**254.** The serpentine is a double-curved surface that is generated by a sphere which moves so that its center follows a helix. The surface may be visualized and drawn as the envelope of all spheres of a given size that have their centers on a given helix as in Fig. 140. Surfaces of this kind are called tubular. The annular torus is also a tubular surface.

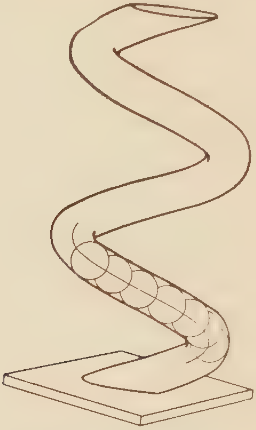


Fig. 140.—Serpentine.

The serpentine is the surface of helical springs that are made from round stock. It is also used as an architectural ornament. It cannot be developed, but it may be approximated by enclosing the surface in sections of cylinders.

**255. Problems.**

Group 100. Spring and Serpentine.

## CHAPTER XIII

## MINING PROBLEMS

**256.** The mining engineer must solve problems relating to veins of ore, strata of rock, underground workings and surveys, faults in veins, etc. Mining problems, although seemingly difficult of solution, may be solved quickly and accurately by applying the principles and the graphical methods of the geometry of engineering drawing. All engineers should have some knowledge of the possibilities of such solutions.

**257. Definitions.**—Theoretically, it is assumed that a vein of ore is bounded by two parallel planes. The distance between the bounding planes is the thickness of the vein. Veins are usually tipped to an oblique angle. They are seldom vertical or horizontal. The lower plane or boundary of the vein is called the *foot wall*, since it is the wall on which the miner stands. The upper plane is the *hanging wall*. Some veins are sheared or faulted; some are folded.

In mining problems, as in land surveying, directions are measured from some base line that is usually taken as a north and south line. Levels are measured from some base plane that is horizontal. The zero level may be taken at sea level, or otherwise, as desired.

The trend or *strike* of a vein is the direction of any horizontal line in the center of the vein. The strike is specified, for example, as N 42° E, or N 77° W.

The *dip* of a vein of ore or layer of rock is the average angle that either wall of the vein makes with a horizontal plane. It is the slope of the steepest line in the vein. After the strike is known, the dip may be specified. For example, if the strike is N 42° E, the dip may be stated as 35° to the N. W.; or, if the vein slopes the other way, 35° to the S. E.

Veins of ore are usually discovered through surface indications or *outcrops* of the ore in places where erosion has uncovered the vein. After the vein is discovered, it may be developed by removing the overburden or by digging a tunnel, incline, or shaft.

In mining, a *tunnel* is a horizontal or nearly horizontal passage having one opening at daylight and extending across the country rock to the vein. An *incline* is an entry into the mine that follows the dip of the vein or seam. A *shaft* is a vertical passage from the surface of the ground to the vein.

**258. Angle of Repose.**—Any uniform loose material that is dumped in a pile assumes the shape of a right circular cone. The base angle of the cone varies with the moisture content, the size of the particles, and the nature of the material. The base angle is called the angle of repose, the natural slope, or the angle at which the material hangs up. It may be stated in degrees, or specified as a slope, for example, of $1\frac{1}{2}$ in 1 or $\frac{1}{2}:1$. Usually, the first distance of the ratio is measured horizontally, and the second vertically; but sometimes the ratio is stated in the reverse order, so that it is necessary to know which is intended.

A material when dry has a lower angle of repose than when the same material is wet. Wet material may harden as it drys, and thus retain its slope. Wet sand stands at $1\frac{1}{2}:1$, while dry sand stands at $2:1$. These are both less than 45 degrees. Disintegrated rock, when wet, has a natural slope of $\frac{1}{2}:1$, when dry $1\frac{1}{4}:1$. The wet rock stands higher than 45 degrees.

**259. Example Solution.**—Figure 141. The upper half of this illustration represents the contour of a section of mountainous country. Each crooked contour represents a horizontal line of the surface of the ground. The contour marked 0′ is taken as the zero or base level. Contours at intervals of 100′ are drawn in heavy lines, and the level is marked on the contour. Intermediate contours at each 25 feet difference in level are shown in finer lines. The contour map is equivalent to the top view of the country. It gives a good idea of the lay of the land. The contours of a map are obtained from surveyor's topographical transit notes, or with the aid of a plane table. The elevation, or front view, shown in the lower half of the illustration is sometimes useful in solving problems. In the front view each contour appears as a horizontal line that may also be considered as the edge view of a horizontal plane.

**260. To Find the Strike and Dip.**—Outcrops in the foot wall of a vein of ore have been discovered at A, B, and C on the map of Fig. 141. The points A, B, and C determine the plane of the foot wall of the vein. The direction of the strike may be determined by drawing any horizontal line of this plane. In the front view draw the horizontal line A D. This intersects the line B C at D. In the top view locate D on B C. Draw A D. This is the top view of a horizontal line in the foot wall of the ore body, and hence it gives the direction of the strike. The strike bears N 82° E.

The dip of the vein is obtained by drawing an edge view of the foot wall or plane A B C. This edge view is an auxiliary elevation taken in the direction of the strike. It is shown at $A_X$ $B_X$ $C_X$ The angle of dip is 67 degrees to the north, or, stated more exactly, 67 degrees in a northerly direction. If a single point of outcrop in the hanging wall is known, this may be plotted in the auxiliary elevation, and the thickness *t* and the cross-section of the vein may then be shown.

**261. To Find the Line of Outcrop.**—The probable line of outcrop of the foot wall of the vein is determined by finding the intersection of the foot-wall plane, A B C, with the surface of the hillside represented by the contour lines. As usual, this intersection is found, point by point, by passing cutting planes.

For example, pass a horizontal cutting plane at the 150-foot level in the front view, Fig. 141. This plane cuts from the hillside the 150-foot contour line; and it cuts from the plane A B C a horizontal line that intersects B C at E, and that is parallel to the strike. Locate E on B C in the front view, and then on B C in the top view. Draw the top view of this horizontal line through E parallel to the strike. This straight line crosses the 150-foot contour line at two points that lie in the foot wall of the vein and on the surface of the ground. By passing a cutting plane through each 25-foot level, a series of outcrop points is found. The outcrop line, drawn through these points, represents the line along which the vein is likely to appear at the surface. If the vein is thick, the outcrop line of the hanging wall may also be drawn. The vein between the rough angle of the outcrop line has been eroded and the material washed downstream. For the purpose of finding the outcrop line, it is often more convenient to draw the levels in the auxiliary elevation, and to use this view instead of the front view. If desired, the front view of the outcrop may also be drawn.

**262. To Find the Outline of a Dump.**—At S in Fig. 141 is located the mouth of a shaft. Mine refuse dumped at S will roll downhill until the dump forms a part of a cone. Suppose that it is found by test that the natural slope of the material to be dumped is $1\frac{1}{4}$ in 1. To find the outline of the dump, draw in the front view a right circular cone with the vertex at S, and make the slope of the cone the same as the natural slope of the material that is to be dumped. The problem is to find the line of intersection of this cone with the hillside. As usual, pass horizontal cutting planes. For example, pass a horizontal cutting plane at the 325-foot level. This plane cuts the 325-foot contour line from the hillside, and a circle from the cone. The size of the circle is obtained in the front view, and the arc, centered at S, is drawn in the top view. Here, the arc crosses the 325-foot contour line at two points. These points limit the width of the dump at this level, and the arc shows the curve of the dump. A series of points is determined by passing a cutting plane for each 25-foot level. The dump line, drawn through these points, indicates the maximum possible size of the dump.

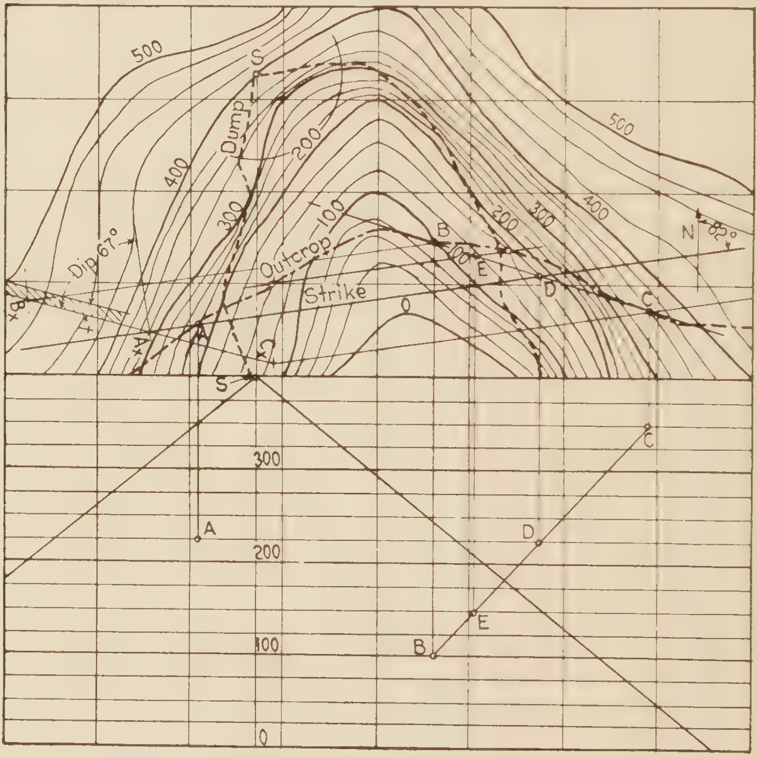


Fig. 141.—Contour map. Chinaman Gulch.

When these limits are reached, no more material can be dumped from S. The mine refuse may, however, be dumped at increasing horizontal radial distances from S, thus materially increasing the capacity of the dump by forming the frustum of a cone, of which the top circular section is centered at S. The boundary lines of the dump may thus be gradually extended. If desired, the front view of the dump may also be drawn.

**263. Bibliography.**—The solutions of problems related to faults in veins and to folded veins are not here explained. Those who desire to study this subject in greater detail are referred to the following:

Roe, J. W.—"The Application of Descriptive Geometry to Mining Problems," *Trans. Am. Inst. Mining Eng.*, Vol. 41, p. 512.

Tolman, C. F.—"Graphical Solution of Fault Problems." This book is now out of print, but it may be obtained by any library as a loan from the Congressional Library.

### 264. Problems.

Group 101. Mining Problems.

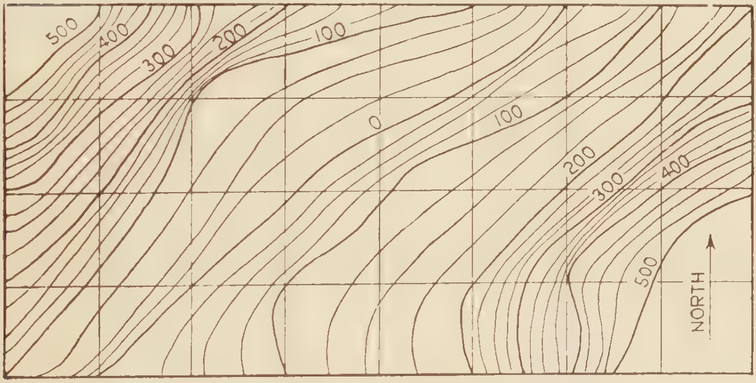


Fig. 142.—Contour map. Rainbow Canyon.

## CHAPTER XIV

## PERSPECTIVE DRAWINGS

**265.** Anyone who looks at an object sees a perspective view of the object. The observer, having two eyes, sees two different perspective views. These are merged into one view that conveys to the observer some idea of the depth of the picture. This is called binocular vision. Two photographs, taken with a stereoscopic camera, give this idea of depth in a picture when they are viewed through a stereoscope. The ordinary photograph is flat in comparison, because it is taken with a camera having one lens. The considerable difference in the pictures seen by each of the two eyes may be observed by looking first with one eye and then with the other at a group of objects at fairly close range. The apparent shifting of the objects will be readily noticed.

**266. Definitions and Principles.**—In a perspective view the rays of light converge and meet at the point where the eye of the observer is located. This point is called the station point, the point of view, or the point of sight. In a perspective view, the observer views the object from a relatively short finite distance and the rays of light converge to a point; in an orthographic view, the observer views the object from an infinite distance, and the rays are parallel.

The simplest idea of a perspective view may be had by imagining that the observer looks at an object through a window pane. The intersections, with the window pane, of the various rays of light that meet at the eye of the observer trace the perspective view of the object on the window glass. The plane on which the perspective view is drawn is called the picture plane. It is usually vertical. Perspective drawings are sometimes made on cylindrical or spherical surfaces.

**267. Uses of Perspective Drawings.**—Perspective drawings are generally used to give an idea of the proportions and appearance of structures to clients or customers who are unfamiliar with, and unable to understand, orthographic views. Architects frequently use perspective. All engineers have at least occasional use for perspective, and should know how to make drawings of this type. Freehand perspective sketches are a great convenience in construction work and in shop work. They may be used to describe to workmen and others the appearance of certain details that are not easily comprehended from orthographic views.

**268. Limitations of Perspective.**—Perspective drawings are somewhat difficult to make, and do not readily show all of the details and dimensions of a complicated structure. The fine gradations of shade and shadow necessary accurately to represent the structure in perspective can be approached only by an artist. After the structure is built, it is generally better to use photographs.

**269. Types of Perspective.** -Perspective drawings may be classified as drawn in either parallel or angular perspective. In parallel perspective, the drawing is made on a picture plane that is parallel to one of the vertical faces of the object. In angular perspective, the drawing is made on a picture plane that makes an oblique angle with all of the vertical faces of the object. Drawings in parallel perspective, particularly when circles are to be shown, are easier to make than are drawings in angular perspective. They are, however, not generally so pleasing as are drawings in angular perspective.

In linear perspective only the lines of the structure are shown. In aerial perspective shades, shadows, and colors are used to enhance the pictorial effect to a greater or lesser degree. In freehand perspective the lines are drawn freehand, and their directions are gaged by eye. In mechanical perspective the lines are drawn with the aid of straight-edges and drawing instruments, and their directions determined by geometry.

**270. Parallel Perspective.**—In Fig. 143 are shown the front, top, and right-side views of a rectangular box. The front and rear ends of the box are open and frontal. The eye of the observer is located at E. A frontal picture plane, parallel to the open ends of the box, is located at P. In each view are shown the converging rays of light that connect each corner of the box with the eye of the observer. The point of intersection of each ray with the picture plane, as seen directly in the top and side views, is transferred to the front view of the ray. The perspective view may now be completed by drawing the straight-line edges of the object, as indicated by the broad lines in Fig. 143.

The perspective view just drawn is a parallel perspective drawing of the rectangular box. In the parallel perspective view it should be noted that: All vertical lines of the object remain vertical in the picture; all horizontal-frontal lines of the object are horizontal in the picture; all receding lines of the object converge and meet at $E_F$ in the perspective view. This meeting point is called the center of vision. In angular perspective it is called the vanishing point. The perspective view may be found with the aid of either the top or the side view. Both are shown here to give a better idea of the theory of perspective. Both of these views may be used when it is desired to check the locations of points in the picture.

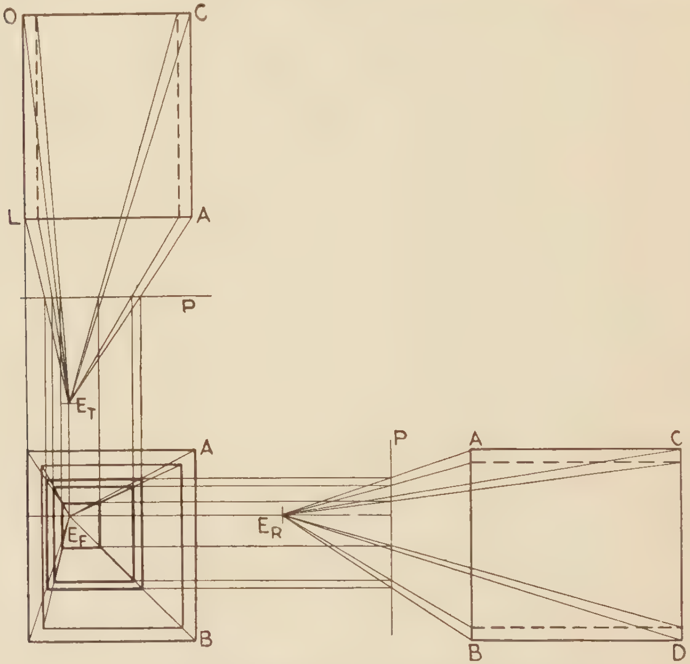


FIG. 143.—Elements of parallel perspective.

**271. Location of Picture Plane.**—The changes produced in the perspective view by moving the picture plane are illustrated in Fig. 144. Moving the picture plane closer to the observer decreases the size of the perspective picture and has the effect of moving the object away from the observer. In parallel perspective the picture plane is often taken as coinciding with the front face of the object, as shown at $P_2$ in the figure. The front face of the object then becomes a part of the perspective view, and it is only necessary to add a few lines in order to complete the picture.

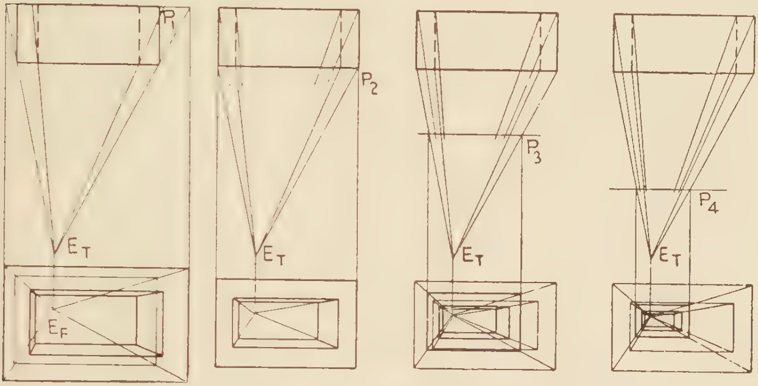


FIG. 144.—Results of moving the picture plane.

The draftsman chooses the location of the picture plane according to the size of picture desired. To prevent distortion, the picture plane should be nearly perpendicular to the central lines of sight.

**272. Distance of Station Point from Object.**—To secure a satisfactory perspective drawing, it is necessary that the position

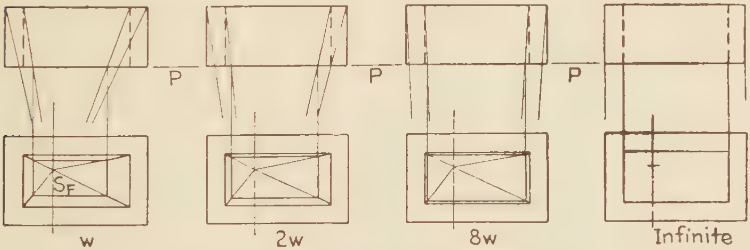


FIG. 145.—Views taken at different distances.

of the observer be chosen neither too near the object nor too far from the object. Views taken at different distances are illustrated in Fig. 145. The distances *w*, 2*w*, 8*w*, and infinite indicate, in terms of the width of the box, the distance that the respective station point is in front of the front face of the box. The top views of the station points are beyond the limits of the drawing. The picture plane in each case is taken in the front face of the box. There is quite a difference in the apparent depth of the box in the pictures taken at distances $w$ and $2w$. The picture taken at the distance $8w$ is not satisfactory. The picture taken at an infinite distance is an orthographic front view. To secure a satisfactory picture, the station point must be located neither too near nor too far from the object. The maximum visual angle of the human eye is about 60 degrees. The station point should not be so close to the object that the angle between the rays leading to the extreme points of the object is greater than 60 degrees; otherwise, the picture will be distorted.

**273. Location of Station Point.**—The effects produced in perspective views by moving the station point to the right or left, or up or down, are illustrated in Fig. 146. In the four views shown the station point is taken at a fixed distance from the front face of the box. The station point A is located directly in front of the center of the box. B is located to the right of the position A. C is located above the position B. D is chosen below and to the left of the position A. For ordinary purposes, a station point above the upper face and to the right of the right-hand face, or to the left of the left-hand face, is most desirable. The position C, for example, gives probably the best picture of the four shown. In each of these perspective views the receding lines of the box meet in the front view of the station point.

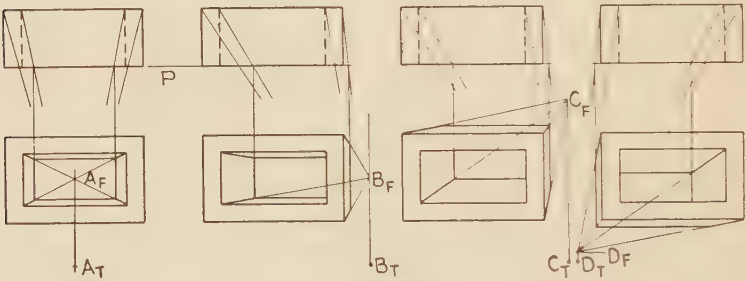


FIG. 146.—Views taken at a fixed distance.

**274. Viewing Perspective Drawings.**—A perspective drawing is best viewed from the station point that was selected by the draftsman or artist when the drawing was made. The eye of the reader should be directly in front of the front view of the station point. The picture is more satisfactory if viewed from this point than when viewed at random.

The left-hand drawing in Fig. 147 does not give a correct impression of the proportions of the object. If, however, the reader will hold the book so that $E_F$ is but 2 inches from the eye, he will note that from this point of view the picture is in better proportion. No one with normal sight cares to look at a picture from this distance. The station point was chosen too close to the object. In the right-hand perspective drawing of Fig. 147, the point of view is a normal distance from the object. This picture gives a better idea of the proportions of the object. It may be viewed from any normal reading distance.

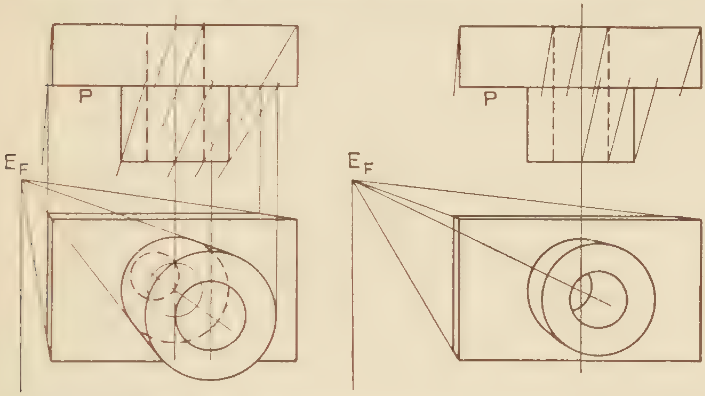


Fig. 147.—Views from two station points.

**275. Selecting the Station Point.**—To obtain a satisfactory picture, judgment must be used in selecting the location of the station point. It must be neither too near the object, nor too far away. Its elevation, in case of large structures that stand on level ground, should usually be chosen at the level of the average person's eyes. This is about 5 feet above the ground level. The location of the station point to the right or to the left should usually be chosen so that the eye is not opposite the center of the object, but it should not be too far to one side. In angular perspective the vertical faces of the object should not make equal angles with the picture plane.

**276. Solving Problems.**—Making an accurate perspective drawing is a severe test of the ability of the draftsman. The pencil must be kept needle sharp. Points must be carefully located, and lines drawn exactly through the points. Lines intersecting at a sharp angle should not be depended upon for locating points accurately. Checking points should be frequently used. Pins may be located at vanishing points and station points as an aid in drawing lines that meet at these points. In some problems vanishing points will have to be located beyond the limits of the problem sheet. In some problems the views will overlap. Subscripts should be used to differentiate between the front and top views of the station point. Hidden lines are usually not shown in the perspective view. As usual, fine, medium, and heavy lines should be used in their proper places.

### 277. Problems.

Group 102. Parallel Perspective of Straight-line Objects.

**278. Circles in Parallel Perspective.**—The front and top views of a short length of a cylindrical pipe or tube are shown in Fig. 148. The picture plane P is taken coincident with the front end of the tube. The eye of the observer is at E. In each view, rays are drawn connecting the station point with the centers of the circles of the rear face of the tube, and also with the ends of the horizontal diameters of the same circles. The points where these rays are seen to intersect the picture plane in the top view are transferred to the respective front views of the rays. These points determine the center and radii of the rear circles in the perspective view of the tube. Draw the horizontal diameters of these circles, and then draw the circles. The two circles of the front view of the tube serve also as the front circles of the perspective view, which is now completed by drawing the straight lines tangent to the front and rear circles. These lines will meet at $E_F$.

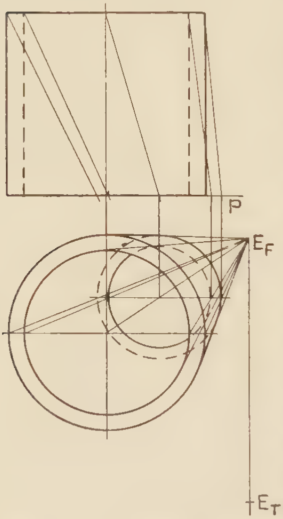


FIG. 148.—Circles in parallel perspective.

All circles parallel to the picture plane will appear as circles in the perspective drawing. The perspective view of any circle may be considered as the intersection of the picture plane with a cone of which the circle itself is the base and the station point is the vertex. If the circular base of the cone is parallel to the picture plane, the intersection will be a circle. When making parallel perspective drawings of an object having circular parts, the picture plane should be taken parallel to the circles of the object. This can be done only when all circles of the object lie in parallel planes.

**279. Problems.**

Group 103. Parallel Perspective of Circular Objects.

**280. Angular Perspective.**—The stepped block, of which the front and top views are shown in Fig. 149, is taken as an example. The details of making an angular perspective view of the steps may be explained as follows: The top view of the steps, turned so as to make a suitable angle with the frontal picture plane, is redrawn to the right of the original top view. The picture plane is here taken through the front corner of step 1. The locations of the front and top views of the station point S are chosen as desired. The top view of S has here been taken rather too close to the object, in order that all lines of the drawing may come within the limits of the page; and $S_T$ is taken below $S_F$ to keep the front and top views reasonably close together. The rays that converge at $S_T$ may now be drawn in the top view. These rays need to extend only from the corners of the object to where they meet the picture plane.

Draw a horizontal line through $S_F$. It is on a level with the eye of the observer, and is called the horizon. In the perspective view, all lines that represent the parallel horizontal lines of the object will converge and meet at vanishing points that are on the horizon. In the steps there are two sets of parallel horizontal lines. To find the vanishing point for one set of lines draw $S_T$ $K_T$ parallel to the shorter horizontal edges of the steps. The front view of this line is $S_F$ $K_F$. The line S K meets the picture plane at $K_T$ in the top view. The front view of K is at $K_F$ or $V_1$ on the horizon. $V_1$ is the vanishing point of one set of parallel lines. All of the lines of the steps that are parallel to S K will converge and meet at $V_1$ in the perspective view.

To find the other vanishing point, draw $S_T$ $L_T$ parallel to the longer edges of the steps. It meets the picture plane at $L_T$. The vanishing point is on the horizon at $V_2$. All of the edges of the steps that are parallel to S L will converge and meet at $V_2$ in the perspective view.

The front view of the steps shows the height of the steps. The different levels may be measured in the picture plane by extending to the right a horizontal line from the level of each step. The front vertical edge 1 of the steps is in the picture plane. In the perspective drawing this edge appears in its true length at 1′.

The longer vertical plane risers of the steps 2 and 3, Fig. 149, are extended in the top view until they meet the picture plane at M and N. Here these risers intersect the picture plane in the vertical lines shown respectively at 2′ and 3′. The lines 2′ and 3′ represent the actual height of the steps in the picture plane.

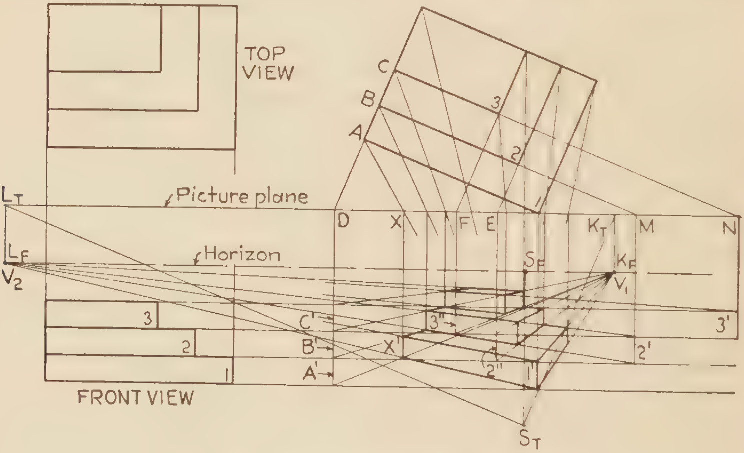


FIG. 149.—Elements of angular perspective.

Similarly, the shorter risers of steps 2 and 3 are extended until they meet the picture plane at E and F in the two vertical lines 2″ and 3″. The left-hand vertical faces A, B, and C of the steps, when extended, meet the picture plane at D and are shown respectively at A′, B′, and C′.

The edges 2′, 3′, 2″, 3″, A′, B′, and C′ are all extensions of vertical planes of the steps. From the upper and lower ends of the lines 1′, 2″, 3″, A′, B′, and C′, lines are drawn to meet at the vanishing point $V_1$. Limited parts of these lines will represent the perspective views of the edges of the steps that are parallel to the line S K. Similarly, from the upper and lower ends of the lines 1′, 2′, and 3′, lines are drawn to meet at the vanishing point $V_2$. Limited parts of these lines will represent the perspective view of edges of the steps that are parallel to the line

S L. Any horizontal line of the object may thus be extended until it meets the picture plane; and the perspective view of the line may be determined by joining the vanishing point of the line with the point where the extended line meets the picture plane.

The vertical edge A of the steps is first located in the picture plane at X, Fig. 149, and is then transferred to X′ in the perspective view. X′ is limited by the two lines that have been drawn from the upper and lower ends of A′ to $V_1$. X′ is also limited by the lines that have been drawn from the upper and lower ends of 1′ to $V_2$. This is a check on the accuracy of the work. All of the other vertical edges of the steps may be found in a similar way, and the perspective drawing may be completed by bringing out the lines of the picture.

When it is desired to locate the picture plane so that it does not pass through a vertical edge of the object, vertical measurements may still be made in the picture plane by extending vertical planes or horizontal lines to meet the picture plane as explained above in connection with steps 2 and 3.

**281. Problems.**

Group 104. Angular Perspective of Straight-line Objects.

**282. Circles in Angular Perspective.**—The square block on which is mounted a cylindrical boss, Fig. 150, is taken as an example. The top view is drawn in any desirable position with relation to the picture plane P; the front and top views of the station point are chosen, and the horizon H is drawn. In the front view the cylindrical part of the object is enclosed in an octagonal prism. The perspective drawing of the cylinder is made by first finding the perspective view of the octagonal prism, and then inscribing the curves of the circles within the perspective views of the octagons.

The perspective view of the square block and octagonal prism is found by using the methods explained in the article on angular perspective. The vanishing points $V_1$ and $V_2$ are located. The planes of the larger vertical faces of the square block and octagonal prism are extended to meet the picture plane at A, B, and C, and from these the vertical measuring lines 1, 2, and 3 are determined in the picture plane. The elevations of the corners of the object are transferred from the front view to the respective measuring lines 1, 2, and 3, and each of these points is connected with the proper vanishing point. The vertical lines of the picture are drawn, the other lines are added, and the perspective view is completed by inscribing the perspective circles within the octagons.

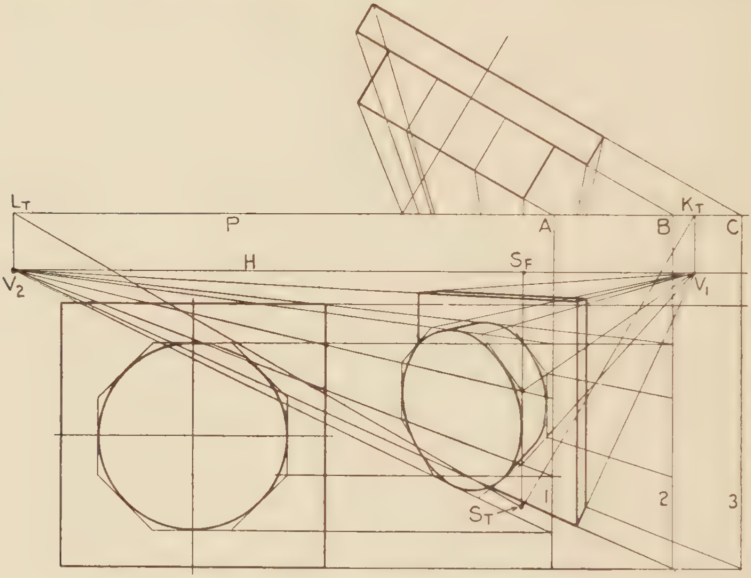


Fig. 150.—Circles in angular perspective.

### 283. Problems.

Group 105. Angular Perspective of Circular Objects.

# CHAPTER XV

## THE PROJECTION METHOD

**284.** The older method of descriptive geometry is the *projection method.* By this method objects are projected on planes of projection, as explained in the following paragraphs. The direct method, explained in this textbook, does not use planes of projection, but directs attention to the object itself.

**285. Historical.**—Gaspard Monge (1746–1818), a French mathematician, developed the principles of descriptive geometry when he was only nineteen years old. When working as a designer for the French government, he was given the job of making plans for a proposed fortress. This was a tedious process, involving long arithmetical calculations. Monge invented graphical solutions, and completed the plans in such a short time that at first the commandant refused to accept them. For a long time the graphical process was kept a state secret, and the few officers who were instructed in it were forbidden to explain the method to anyone. After some years the discovery was made public. Although Monge developed the principles of descriptive geometry, projections were used long before his time.

**286. The Projection Method.**—The theory of descriptive geometry as developed by Monge is here explained so that students of the direct method may understand something of the language of the projection method. In Fig. 151 are shown the two projection planes that are used in the projection method. One of the planes, H, is taken as horizontal, and the other plane, V, is vertical. The two planes intersect in the line G L that is called the ground line. The two projection planes divide space into four quadrants, or angles, as numbered in the illustration. Quadrants 1 and 2 are above the horizontal plane, and quadrants 3 and 4 are below the horizontal plane. Quadrants 1 and 4 are in front of the vertical plane, and quadrants 2 and 3 are behind the vertical plane. A point A is shown in the first quadrant, B in the second quadrant, C in the third quadrant, and D in the fourth quadrant. The point A is projected onto the horizontal plane at

$a_h$ by means of a projection line that passes through A and is perpendicular to the H plane. The V projection of A is located in a similar way at $a_v$ by means of a projection line that passes through A and is perpendicular to the V plane. The two projections of the point A definitely determine the location of the point with reference to the chosen projection planes. The projections of the points in the other quadrants are found in a similar manner. The V projection of a point corresponds to the front view of the point, and the H projection to the top view of the point. If a side view is desired, a profile projection plane, perpendicular to the

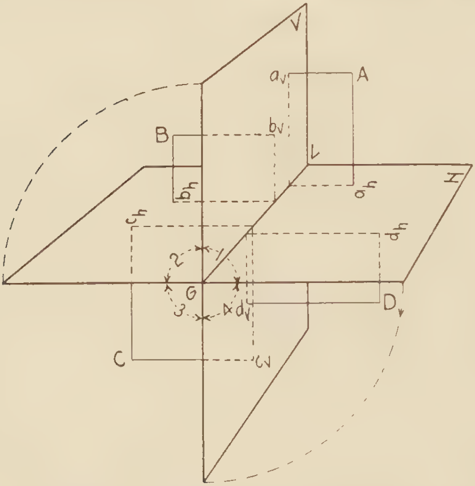


Fig. 151.—The planes of projection.

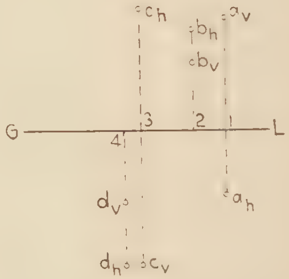


Fig. 152.—Projections of points.

horizontal and vertical planes, is introduced. Right-angle projection is called orthographic projection.

In order to represent the horizontal and vertical projection planes on a flat sheet of paper, the planes are conceived as hinged along the ground line, and the two planes are brought together by closing the second and fourth quadrants. The projections of the four points A, B, C, and D now appear as shown in Fig. 152. The point A, in the first quadrant, has its V projection above the ground line and its H projection below the ground line. The point B, in the second quadrant, has both of its projections above the ground line. The point C, in the third quadrant, has its H projection above the ground line and its V projection below the ground line. The point D, in the fourth quadrant, has both of its projections below the ground line. The two projections of a point are always in the same perpendicular. The projection of a line is drawn through the projections of points that are on the line.

**287. Relations of Views.**—When a horizontal and a vertical projection plane are used, it has seemed most natural to imagine an object in the first quadrant. This started the practice of placing the V projection, or front view, above the H projection, or top view. This custom persists in many countries and in some professions today. Civil engineers often place the top view below the front view. Architects place the plan below the front elevation. The custom in this country is to draw objects as if they were located in the third quadrant. This locates the top view above the front view, in what seems to be a natural relationship.

Workmen often have great difficulty in reading a drawing when the views are arranged in an order to which they are not accustomed. If the object represented is not symmetrical, they are likely to make a left-handed piece right-handed. One remedy is to cut the views apart and paste them together to agree with the training of the workmen.

**288. Representing Planes.**—In Fig. 153, the intersection of the oblique plane T with the V projection plane is the straight line T $t_v$. This line is called the V trace of the plane T. The H trace is the line T $t_h$. The traces of a plane always intersect on the ground line. A plane Q, parallel to the ground line, is also shown in the illustration. When the H and V planes are brought

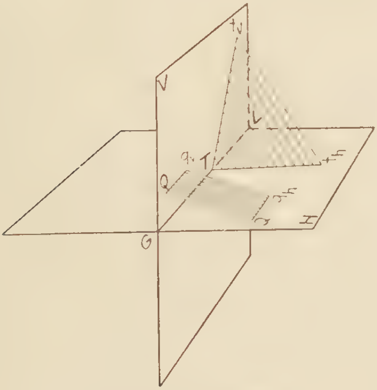


Fig. 153.—Planes.

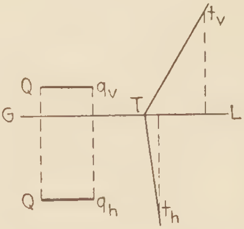


Fig. 154.—Traces of planes.

together the traces of the planes T and Q appear as shown in Fig. 154. The parts of the planes here shown are in the first quadrant. Traces of planes are seldom used in practice, and are not necessary for solving problems. The planes of a structure are determined by the points and lines of the structure, and these latter are sufficient for solving all problems relating to the surfaces of the structure.

# PROBLEMS

## GENERAL INSTRUCTIONS FOR DRAWING PROBLEMS

The following instructions are to be observed, unless specifically changed by the teacher of the class. The problems assigned are to be drawn in pencil on the standard printed drawing paper specified for this purpose. The solution of the problem should be limited to the 8-inch ruled square. Do not fold or roll the plates. Keep the paper flat and smooth. Problems not neatly and accurately drawn or not correctly lettered will be considered incorrect. A key for correcting problems is printed inside the back cover of this book.

**Appearance.**—Every drawing must be of good quality. To make a satisfactory drawing, many details must be carefully observed. Use a correctly sharpened pencil of the proper grade, neither too hard nor too soft. The pressure applied to a pencil must vary with the weight and color of line required. Each line should be of uniform color and width from end to end. Lines should be gray in color, not black. Draw all lines on the surface of the paper, not intrenching them into the paper. Do very little erasing. Lines that may not be permanent should be very fine. While making the drawing, the paper should lie on a hard, smooth surface.

**Weight of Lines.**—To make a drawing that is easily read, lines of various weights should be used.

*Very fine lines* are to be used for rays that connect similar points in the different views. These lines serve to guide the eyes of the draftsman and of the reader, and should not be omitted.

*Medium lines* are to be used for data given in the statement of the problem and for construction lines.

*Broad lines* are to be used for data that are required, so that the lines or object to be represented will stand out more clearly than will the lines used in obtaining the solution. Broad lines should be gray, not shiny black. Where required data coincide with given data, the broad lines will cover the medium lines.

*Dash lines* of medium weight are to be used to represent the hidden lines of objects. The dashes should be closely spaced and should be neither too long nor too short.

**Drawing Lines.**—The student of this subject draws a multitude of lines. The method of drawing straight lines described below, while seemingly a trifle, produces better work, saves much time, and spares the eyes.

To draw a straight line connecting two points, place the tip of the pencil at one of the points; place the straight-edge against the lead; turn the straight-edge around the lead as a center until the straight-edge is in line with the second point; then draw the line. Try this method a few times and notice its advantage over attempting to line up a straight-edge with two points at once.

Parallel and perpendicular lines should be drawn accurately parallel and perpendicular; otherwise the solution is inaccurate. To draw parallel and perpendicular lines, slide one triangle on another.

**Drawing Views.**—Keep the different views of an object far enough apart so that they will not overlap, if this is at all possible. All views are to be drawn full size unless otherwise specified. Each view is to be placed in its correct relation to its adjacent view. Letter with capital letters the corners or points of the object, using the same letter for all of the views of a single point. Letters specified in the statement of the problem should be used in the drawing. All letters and figures are to be hand printed, not written. In general, circles should be drawn first. The size of a circle or cylinder is specified by stating its diameter.

**Checking.**—After the problem has been solved, check and review the solution. Ask and answer questions like the following: Have the requirements of the problem been fully met? Does each view represent the object correctly? Does the object appear distorted in any view? What are the special points that should be checked? How could the appearance of the drawing have been improved? Is each line of the proper weight? Has the accuracy of the solution been affected by carelessness in drawing? The engineer must check the solutions of his problems because his professional standing depends upon the accuracy of his work.

## PROBLEMS—CHAPTER II

### PRINCIPAL VIEWS

**Group 1. Drawing Simple Objects from Description.**

The student should visualize the shape, the position, and the proportions of the object, from the description of the object. The position and the shape of the object determine which lines and which views should be drawn first. It is generally undesirable and often impossible to complete one view without first drawing parts of other views. Near each view letter the name of that view.

**1.** A rectangular block is 5″ wide, $\frac{3}{4}''$ deep, and $1\frac{3}{4}''$ high. Make the ends of the block, as seen in the front view, half circular. On the same centers show $\frac{7}{8}''$ holes drilled through from front to back. Centrally to the rear face add a cylindrical lug $1\frac{1}{4}''$ in diameter and $1\frac{3}{4}''$ long. Draw the front, top, and right-side views.

**2.** A cylinder $2\frac{1}{2}''$ in diameter and $1\frac{1}{2}''$ tall is capped by a block $3\frac{1}{2}''$ square and $\frac{1}{2}''$ tall. Four $\frac{1}{2}''$ holes, centered on the diagonals of the top face $1\frac{3}{4}''$ from its center, are drilled through the block. A $1\frac{1}{4}''$ vertical hole is drilled centrally through both pieces. Draw the front, top, and left-side views.

**3.** A horizontal disk, $3\frac{1}{2}''$ in diameter and $\frac{5}{8}''$ thick, has a square boss $1\frac{1}{2}''$ wide, $1\frac{1}{2}''$ deep, and 1″ high centrally located on top of the disk. A $\frac{3}{4}''$ vertical hole is drilled centrally through the boss and disk. Four $\frac{1}{2}''$ vertical holes, having centers $1\frac{1}{4}''$ respectively in front of, behind, to the right of, and to the left of the axis of the disk are drilled through the disk. Draw the front and top views.

**4.** Symmetrically placed at the middle of a cylinder $1\frac{1}{2}''$ in diameter by 2″ tall, and integral with it, is a washer 3″ square and $\frac{3}{8}''$ thick. A $\frac{1}{2}''$ hole is drilled centrally through the cylinder from end to end. Four $\frac{1}{2}''$ holes, centered $1\frac{3}{8}''$ from the axis of the cylinder and on the diagonals of the washer, are drilled through the washer. Draw the front, top, and left-side views.

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**5.** Draw the front, top, and right-side views of a rectangular block that is 3″ wide, 2″ deep, and 1″ high. Show two vertical holes, $\frac{3}{4}''$ in diameter, drilled through the block. The center of one hole is to be $\frac{3}{4}''$ to the right of, and the center of the other an equal distance to the left of the middle point of the top face. Two $\frac{3}{8}''$ holes parallel to and 1″—measured center to center—from both $\frac{3}{4}''$ holes are also drilled through the block.

**6.** A symmetrical U-shaped block is made from a rectangular block $1\frac{1}{2}''$ high by 2″ wide by 3″ deep, by cutting a slot, 1″ wide by $1\frac{1}{2}''$ high by

$2\frac{1}{2}''$ deep, into the front end of the block. A $\frac{5}{8}''$ hole is drilled through both sides of the U. The center of the hole is $1''$ from the front end of the block and $\frac{3}{4}''$ below the top. Draw the front, top, and left-side views.

**7.** Each face of a $2\frac{1}{2}''$ cube has a square recess cut into it to a depth of $\frac{1}{4}''$ and within a distance of $\frac{1}{2}''$ of the edges of the face. A $\frac{3}{4}''$ vertical hole is then drilled centrally through the block. Draw the front, top, and left-side views of the block.

**8.** A spool has a cylindrical core $\frac{7}{8}''$ in diameter and $2''$ long. The ends of the core are turned to a diameter of $\frac{5}{8}''$ to take the cylindrical disk flanges that are $2''$ in diameter and $\frac{1}{4}''$ thick. A $\frac{1}{4}''$ shaft hole is drilled through the core. Draw the front and top views and the right-end view of the spool.

**9.** Draw the front, top, and left-side views of a rectangular block that is $4''$ wide, $2\frac{1}{4}''$ high, and $\frac{1}{2}''$ deep. Add a cylinder, $1\frac{1}{2}''$ in diameter and $1\frac{1}{4}''$ long, centered on the back face of the block. Drill a $\frac{3}{4}''$ hole centrally through the cylinder and block. Parallel to this hole drill a $\frac{3}{4}''$ hole spaced $1\frac{3}{8}''$, center to center, to the right of the first hole, and a similar hole an equal distance to the left. The last two holes form the ends of slots $\frac{3}{4}''$ high that are cut through the right and left faces of the block. Show these slots. In the front view round the four corners of the rectangular block to a $\frac{3}{8}''$ radius.

**10.** The stem of a tee-shaped block is $1''$ wide, $3''$ tall, and $1\frac{1}{4}''$ deep. The top of the tee is $3''$ wide, $1''$ high, and $1\frac{1}{4}''$ deep. A tee-shaped slot is cut through from front to back, leaving the top, bottom, and side walls $\frac{1}{4}''$ thick. Draw the front, top, and right-side views.

**11.** Draw the front, top, and left-side views of a rectangular block $4''$ high, $2\frac{1}{4}''$ wide, and $2''$ deep, that is hollowed out so that the front, rear, top, and bottom faces are $\frac{3}{8}''$ thick and the side faces are open. Show a hole $1''$ square, with horizontal and vertical edges, cut centrally through the front face. Show a centrally located hole $1\frac{1}{4}''$ in diameter cut through the rear face.

**12.** A block $3''$ deep, $4''$ wide, and $1\frac{1}{2}''$ high has a symmetrical, centrally located, tee-slot cut through from end to end, leaving the top, bottom, front, and back walls $\frac{1}{2}''$ thick. The $1''$ wide opening to the slot is cut through the top face. Two $\frac{1}{2}''$ holes, countersunk at the top end for flat-headed screws, are drilled through the bottom face, $1''$ respectively to the right of and to the left of the middle of the block. Draw the front, top, and left-side views.

**13.** A disk is $3\frac{1}{2}''$ in diameter and $\frac{1}{2}''$ thick from front to back. A centrally located hole $1\frac{1}{2}''$ square and having vertical and horizontal edges is cut through the disk from front to back. Four equally spaced $\frac{1}{2}''$ holes, parallel to the axis of the disk and $1\frac{1}{4}''$ from its center, are drilled through the disk. One of the holes is directly above the axis of the disk. Through the front end of each circular hole, insert, as far as it will go, a plug $\frac{1}{2}''$ in diameter by $1''$ long and having a head $\frac{3}{4}''$ in diameter and $\frac{1}{4}''$ long. Draw the front, top, and left-side views.

**14.** Draw the front, top, and right-side views of a rectangular block that is $4''$ wide, $\frac{3}{4}''$ deep, and $3\frac{1}{2}''$ high. Cut a rectangular recess $\frac{1}{4}''$ deep in the rear face of the block, leaving the edges of the face $\frac{1}{2}''$ wide. Locate a point 1″ below and 1″ to the right of the upper, left, front corner of the block. This point is the center of a $\frac{5}{8}$″ hole to be drilled through the block parallel to the shortest edges of the block. Locate similar drilled holes at the same distance from the three other corners of the front face. Cut a round recess 1″ in diameter and $\frac{1}{4}$″ deep in the center of the front face. In the front view round the corners of the block to a $\frac{1}{2}$″ radius.

## Group 2. Drawing Objects by Discarding Specified Parts.

**1.** Show the front, top, and right-side views of all that is left of a rectangular block 2″ high by 3″ deep by 4″ wide after the following parts have been cut away: the left half of the right half of the lower half; the right half of the left half of the lower half; and the middle third lying between the front and rear thirds of the upper half.

**2.** A block is 2″ wide, 4″ deep, and 3″ high. Beginning at the lower, front, right-hand corner cut away a block that is 1$\frac{1}{2}$″ wide, 2$\frac{1}{2}$″ high, and 3$\frac{1}{2}$″ deep. Beginning at the top, rear, left-hand corner cut away all that remains of a block that is the same size as the block last mentioned. Show the front, top, and right-side views of all that now remains of the original block.

**3.** Show the front, top, and right-side views of a rectangular block, 4″ wide, 4″ high, and 3″ deep. Cut away the upper three-fourths of the front third, the lower three-fourths of the rear third, and a piece 2″ high by 2″ wide by 1″ deep from the middle of the middle third that lies between the front and rear thirds.

**4.** Draw the front, top, and right-side views of a 3″ cube. Discard all of this cube with the exception of: the upper third of the front third, the lower third of the rear third, and the middle, vertical third of the middle third lying between the front and rear thirds.

**5.** Draw the front, top, and right-side views of a rectangular block that is 4″ wide, 3″ high, and 2″ deep. Discard the front half of the lower two-thirds of the right half of the block. Discard the front half of the upper half of the lower two-thirds of the left half of the block. Discard the front half of the left half of the upper one-third of the left half of the block. Show what is left of the block.

**6.** Draw the front, top, and right-side views of a rectangular block that is 4″ wide, 3″ deep, and 1$\frac{1}{2}$″ high. Cut away the block so that only the following parts remain: the front one-third and the rear one-third of the upper one-third; the front one-sixth and the rear one-sixth of the upper half of the lower two-thirds; the lower one-third.

**7.** Draw the front, top, and right-side views of a rectangular block that is 4″ high, 3″ wide, and 2″ deep. Take from the front, upper, right-hand corner, and also from the front, left, upper corner a 1″ cube. Take a block 2″ high, 1″ wide, and 1″ deep from the middle of the lower, rear quarter of the original block.

**8.** A rectangular block is 2″ high, 1$\frac{1}{2}$″ deep, and 4″ wide. Discard the lower left quarter of the block. Discard the right half of the lower right quarter of the block. Discard the upper half of the left half of the upper right quarter of the block. Discard the upper half of the right half of the upper left quarter of the block. Draw the front, top, and left-side views of what is left of the block.

**9.** Draw the front, top, and left-side views of a 3″ cube. Discard all of the upper one-fourth of the cube excepting a $\frac{3}{4}$″ cube in the rear left corner. Discard the right half and also the front half of the lower half of the upper half of the original cube. Discard the right one-fourth and also the front one-fourth of the upper half of the lower half of the original cube.

**10.** Draw the front, top, and right-side views of a 3″ cube. Discard the upper and lower thirds of the right third, and the left third of the front third of the cube. In the middle third, between the front and rear thirds of the cube, discard the bottom and top, and the right and left thirds. In the rear third of the cube discard the upper third, and the right and left thirds of the lower third. Show three views of what remains of the cube.

**11.** A rectangular block is 5″ wide, 2″ high, and 1″ deep. Cut away the lower half of the left two-fifths of the block. Cut away the lower half of the right one-fifth of the block. Cut away the upper half of the left two-fifths of the right half of the block. Show the front, top, and right-side views of what is left of the block.

**12.** A block is 3″ square and $1\frac{1}{4}$″ high. Cut away the left half of the left half of the front half of the right half of the upper half of the block. Double the width of this groove by cutting away the block to the left of the groove. Cut away also the upper half of the front half of the rear half of the upper half of the block. Draw the front, top, and right-side views of the block.

**13.** Start with the front, top, and right-side views of a 3″ cube. Remove a $1\frac{1}{4}$″cube from the upper, front, left-hand corner, and also from the lower, rear, right-hand corner of the larger cube. Remove a triangular base pyramid, having the sides of the base 2″ long, from the upper, front, right-hand corner of the 3″ cube, and also remove a similar pyramid from the lower, rear, left-hand corner.

**14.** Draw the front, top, and right-side views of a $2\frac{1}{2}$″ cube. Take from the upper, front, right corner of this cube a $\frac{3}{4}$″ cube. Take from the front, lower, left-hand corner of the larger cube a 1″ cube. Take from the large cube one-fourth of a $2\frac{1}{2}$″ cylinder having its axis coinciding with the upper, left-hand edge of the large cube.

**15.** In the top face of a 3″ cube, cut a circular, centrally located recess 2″ in diameter and $\frac{1}{2}$″ deep. Cut from the front, lower, left corner of the cube and also from the lower, rear, left corner a $1\frac{1}{2}$″ cube. Cut from the rear, right, lower corner and also from the lower, front, right corner a $\frac{3}{4}$″ cube. Draw the front, top, and left-side views.

**16.** A rectangular block is 1″ high, $4\frac{1}{2}$″ wide, and $2\frac{1}{2}$″ deep. Draw its top, front, and right-side views. From each right-hand, front and rear corner cut a block that is $\frac{3}{4}$″ deep, $1\frac{1}{4}$″ wide, and 1″ high. From the left end, upper side, cut a block that is $\frac{1}{2}$″ high, 2″ wide, and $2\frac{1}{2}$″ deep. Show a $\frac{3}{4}$″ hole drilled vertically through the middle of the thinner part of the block.

**17.** Draw the front, top, and right-side views of a rectangular block that is 4″ wide, 3″ deep, and 1″ high. Discard the rear and front thirds of the right half of the block. Discard the upper two-thirds of the front third of the lower three-fourths of the front half of the left half of the block. Discard the upper two-thirds of the rear third of the lower three-fourths of the rear half of the rear half of the block. Show the shank at the right turned to a diameter of 1″.

**18.** A rectangular block is 2½″ deep, 1″ high, and 4″ wide. Cut a 1″ cube from the rear left, and also from the rear right corner. Discard the upper half of the front half of the block. Spot centers in the middle of the top face of each quarter of the thinner section. Show a ⅜″ vertical hole drilled through the block at each of these centers. Draw the front, top, and right-side views.

**19.** A block is 4″ wide, 2″ deep, and 4″ high. Discard all of the lower half of the lower half excepting the rear half of the right half. Discard all of the upper half of the lower half excepting the left half of the right half and the front half of the left half. Discard all of the lower half of the upper half excepting the left half of the left half. Discard all of the upper half of the upper half excepting the rear half of the left half. Show the front, top, and right-side views of all that is left of the original block.

### Group 3. Drawing Objects in Specified Directions.

Before drawing the views of the objects in this group, or the views of objects that are similar in type to those in this group, it is a great help first to make a freehand pictorial sketch like those shown in Fig. A. The ability readily to make freehand pictorial sketches is often a great aid to the engineer in clearly explaining the shapes of structures both to himself and to others.

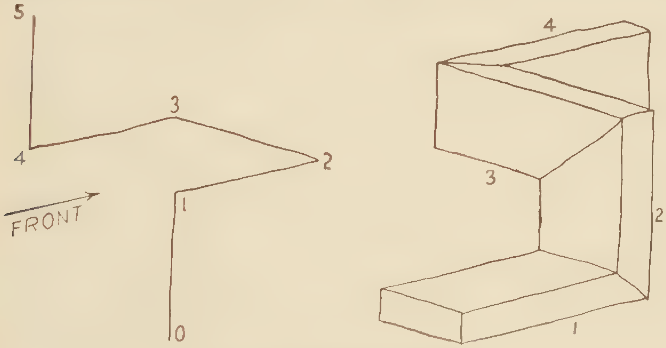

Fig. A.—Pictoral sketches.

The sketch for each problem should be drawn freehand in the blank problem space above the required side view. In the sketches the lines that represent horizontal lines should make a small angle, less than 30 degrees, with the horizontal. Lines that are vertical should be sketched strictly vertical.

**1.** A straight wire is bent at right angles at several points. The wire extends from a point 0 downward 2″ to a point 1, then 2″ to the left to a point 2, then 2″ forward to a point 3, and then 2″ to the right to a point 4. Make a pictorial sketch and show the front, top, and left-side of the bent wire.

**2.** A straight wire is bent at right angles at several points. The wire extends from a point A downward 1″ to a point B, then 1″ to the rear to C, then 1″ to the right to D, then 1″ downward to E, then 1″ to the left to F, then 1″ forward to G, and finally 1″ downward to the point H. Make a pictorial sketch and show the front, top, and right-side views of the bent wire.

**3.** A straight wire is bent at right angles at several points. The wire extends from a point 0 to the right 2″ to a point 1, then 2″ to the rear to 2, then 2″ downward to 3, then 2″ to the left to 4, then 2″ forward to 5, and then 2″ upward to the point of starting. Make a pictorial sketch and draw the front, top and left-side views of the bent wire.

**4.** A straight wire is bent at right angles at several points. The wire starts at a point A and extends 2″ to the right to B, then 3″ to the rear to C, then 1″ down to D, then 2″ to the left to E, then 1″ down to F, then 2″ to the right to G, then 3″ forward to H, and then 2″ to the left to 1. Make a pictorial sketch and draw the front, top, and left-side views of the bent wire.

**5.** A straight wire is bent at right angles at several points. Beginning at a point 0 the wire extends 2″ downward to a point 1, then 1″ forward to a point 2, then 2″ upward to a point 3, then to the rear to the point 0, then 3″ to the left to a point 4, then 1″ forward to a point 5, and then to point 3. Make a pictorial sketch and draw the front, top, and left-side views of the bent wire.

**6.** Starting at a point A, move 1″ upward, then 2″ to the rear, then 3″ to the right, then 1″ downward, then 2″ forward, then 1″ downward, then 1″ forward, then 1″ to the left, then 1″ to the rear, then 1″ upward, and then 2″ to the left. Make a pictorial sketch and show the front, top, and right-side views of the path followed.

**7.** Make a pictorial sketch and draw the front, top, and left-side views of four sticks, each ½″ square by 2″ long, that are joined together at right angles as follows: The second stick is vertical; the first stick is mitered to the top end of the second and extends forward; the third stick is mitered to the bottom end of the second and extends to the rear; the fourth stick is mitered to the rear end of the third and extends to the right.

**8.** Make a pictorial sketch and show the front, top, and right-side views of five sticks, each ½″ square and 2″ long, joined together, at right angles, as follows: The first stick is vertical; the second is mitered to the top end of the first and extends to the rear; the third is mitered to the rear end of the second and extends to the right; the fourth is mitered to the right end of the third and extends upward; the fifth is mitered to the top end of the fourth and extends to the rear.

**9.** A rectangular block is 1″ wide, ½″ deep, and 3″ tall. Mitered to the top end of this block is a similar block that extends to the rear. Mitered to the rear end of the second block is a third similar block extending to the right. Mitered to the right end of the third block is a fourth similar block that extends downward. Make a pictorial sketch and draw the front, top, and right-side views of the four joined blocks.

**10.** A rectangular block is $\frac{1}{4}''$ wide, $\frac{3}{4}''$ high, and $2''$ long from front to back. Mitered to the rear end of the first block is a similar block that extends to the right. Mitered to the right end of the second block is a similar block extending upward. Mitered to the upper end of the third block is a similar block extending to the rear. Make a pictorial sketch and draw the front, top, and left views of the four joined blocks.

## COORDINATE SYSTEM FOR LOCATING POINTS

The definite assignment of a problem in the geometry of engineering drawing requires the adoption of some method of stating the exact locations of the data of the proble.m The use of a simple coordinate system for locating points, together with coordinate problem paper and a coordinately ruled blackboard, provide means for quickly and exactly locating the given data of a problem.

The problem sheet, as shown to a reduced scale in Fig. B, is printed with a ruled space 8 inches square, reserved for the solution of the problem. The cross-section lines are spaced in inches and quarters of an inch. The origin, or zero point, is taken at the lower left-hand corner of the square. From the origin, both to the right and upward, each inch is numbered consecutively. A title is printed above the problem space. The title states the name of the course and reserves spaces for the problem number and group number, the name of the student, and the date. At the bottom of the sheet a ruled space is provided, where the statement of the problem or other notes may be lettered.

The exact locations on the coordinately ruled drawing surface of the front and top views of a point A, for example, are given by three figures, as A736. The capital letter preceding the figures states the name of the point. The first figure, 7, specifies that the front and top views of the point A are to be located in a vertical line 7 inches to the right of the origin or zero point. The second figure, 3, locates the front view, $A_F$, 3 inches above the horizontal base line that passes through the origin. The second figure always serves to locate the front view of the point. The third figure, 6, locates the top view, $A_T$, 6 inches above the horizontal base line. The third figure always serves to locate the top view of the point, and is measured from the horizontal

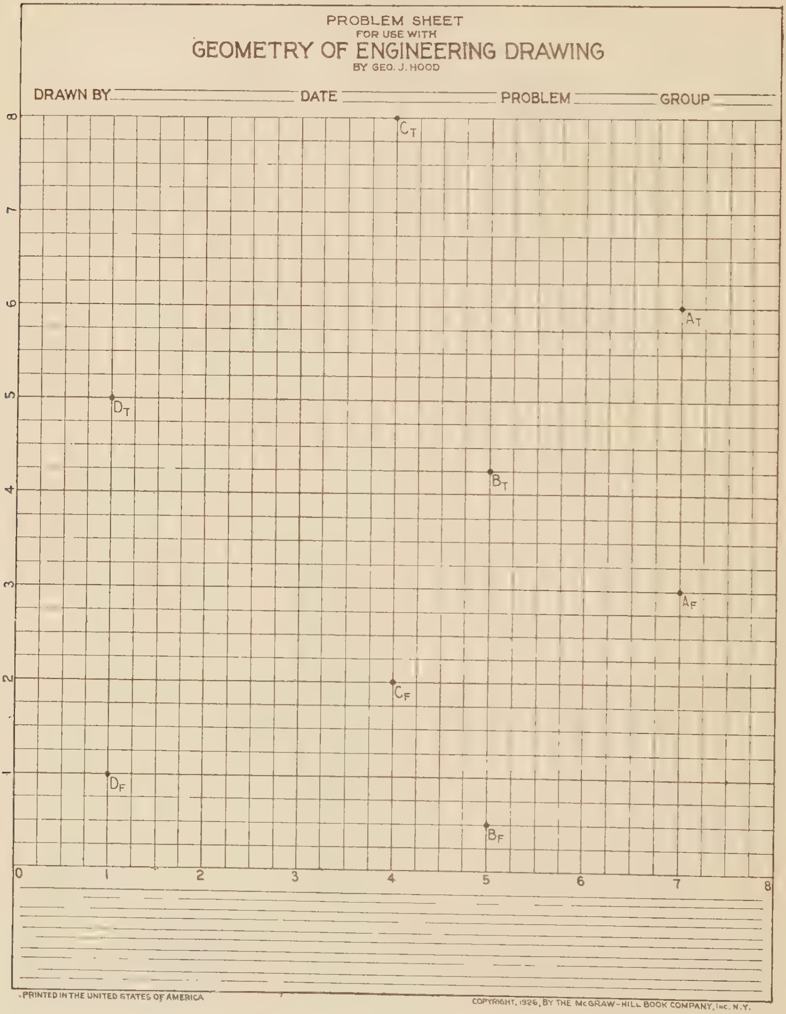
PROBLEM SHEET
FOR USE WITH
GEOMETRY OF ENGINEERING DRAWING
BY GEO. J. HOOD
DRAWN BY
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Fig. B.

base line that passes through the origin. In stating the coordinates of some points, for example, B5,$\frac{1}{2}$,4$\frac{1}{4}$, it becomes necessary to separate the three coordinates by means of commas. The letter X when used as a coordinate indicates that the coordinate X is unknown and that the location of the missing view is to be determined in the solution of the problem. E24X and F7X8 are examples of unknown coordinates. When the name of a point is repeated in the statement of a problem, the coordinates of the point usually are not repeated. In Fig. B, the top and front views of the points A736, B5,$\frac{1}{2}$,4$\frac{1}{4}$, C428, and D115 are shown. When locating the views of points on the problem sheet, subscripts should be used to distinguish the different views of a point.

## Group 4. Locating Data by Coordinates.

**1.** Draw the front, top, and left-side views of the triangle J345 K707 L824.

**2.** Draw three principal views of the triangle D527 E708 F634.

**3.** Draw the front, top, and right-side views of the triangle Q016 R307 S435.

**4.** Draw the front, top, and right-side views of the triangle A137 B428 C304.

**5.** Draw the front, top, and left-side views of the warped quadrilateral A447 B505 C736 D818.

**6.** Draw three principal views of the warped quadrilateral C2,3$\frac{1}{2}$,8 P0,1,5$\frac{1}{2}$ Q3$\frac{1}{2}$,$\frac{1}{2}$,7 R3,2$\frac{1}{2}$,6.

**7.** Draw the front, top, and right-side views of the warped quadrilateral C017 D336 E124 F408.

**8.** Draw three principal views of the warped quadrilateral B518 C647 E706 F625.

**9.** Draw three principal views of the warped quadrilateral A446 B235 D207 C128.

**10.** Draw the top, front, and right-side views of the warped quadrilateral C026 D147 F235 E308.

**11.** Draw three principal views of the triangular base pyramid B0,$\frac{1}{2}$,6$\frac{1}{4}$ C4,1$\frac{3}{4}$,7$\frac{1}{2}$ D3,3$\frac{1}{2}$,4$\frac{1}{2}$, E2$\frac{1}{2}$1$\frac{1}{2}$,6.

**12.** Draw the front, top, and left-side views of the triangular base pyramid V625 B437 C826 D508.

**13.** Draw three principal views of the tetrahedron C124 D218 E335 F206.

**14.** Draw three principal views of the tetrahedron A415 B538 C847 D627.

## Group 5. Locating Points by Measurements.

**1.** Locate A545. Locate B, 2″ to the right of, 2″ behind, and 1″ below A. Locate C, 3″ in front of, 1″ to the right of, and 3″ below B. Draw the front, top, and left-side views of the triangle A B C.

**2.** Draw the front, top, and right-side views of the triangle R017 S T. The point S is 1″ below, 1″ in front of, and 5″ to the right of R, and the point T is 2″ to the left of, 4″ above, and 1″ in front of S.

**3.** Draw the front, top, right-side, and left-side views of the triangle C D E317. The point C is 1″ to the right of, 2″ above, and 2″ behind the point D, and D is 1″ above, 1″ to the right of, and 1″ in front of E.

**4.** Draw the front, top, right, and left views of the triangle A B427 C. A is 4″ above C, 1″ to the right of B, and 1″ behind C. C is 2″ to the left of A, 2″ below B, and 2″ in front of B.

**5.** Locate the point A415. Locate the point B, 3″ behind, 1″ to the right of, and 1″ below A. Locate the point C, 2″ to the right of, 3″ above, and 1″ in front of B. Draw the front, top, and left-side views of the triangle.

**6.** Locate the point D035. The point E is 1″ to the left of F, and F is 4″ to the right of D. The point F is 4″ above E and 3″ behind D. The point E is 2″ in front of F and 3″ below D. Draw the front, top, and right-side views of the triangle D E F.

**7.** Locate the point X005. Locate the point O, 4″ to the right of, 2″ above, and 1″ behind X. Locate the point P, 4″ above, 1″ to the left of, and 2″ in front of the point Q, which is 4″ to the right of, 1″ behind, and 2″ below O. Draw three principal views of the triangle O P Q.

**8.** Locate the point K646. The point L is 1″ in front of K, and an equal distance below the point J, which is 3″ behind L and an equal distance below K. Draw the front, top, and left-side views of the triangle J K L.

**9.** Locate the point C316. The point D is 1″ to the left of E, and E is an equal distance above C. The point E is 5″ to the right of C and 2″ below D. Draw the front, top, and left-side views of the triangle C D E.

**10.** Locate the point A325. B is 3″ to the right of A and 2″ to the left of C. C is 1″ in front of A and 2″ in front of B. Draw the front, top, and left-side views of the triangle A B C.

**11.** Locate the point D, 1″ to the left of, 1″ in front of, and 1″ above E, and locate the point E, 2″ in front of, 3″ above, and 1″ to the left of the point F208. Draw the front, top, and right-side views of the triangle D E F.

**12.** Draw the front, top, and left-side views of the triangle K L M. The point K is 4″ to the left of, 4″ below, and 1″ behind the point L, and the point L is 2″ behind, 1″ to the right of, and 1″ above the point M735.

**13.** Locate the point C416. The point D is 1″ to the right of C, E is an equal distance below D, and F is an equal distance below C and to the left of E. The point D is 2″ behind C, E is an equal distance in front of D, and F is an equal distance in front of E. D is 3″ above C, and E is an equal distance to the right of D. Draw the front, top, and left-side views of the parallelogram C D E F.

**14.** Locate the point A026. Locate the point B, 1″ below, 1″ to the right of, and 2″ in front of A. Locate the point C, 3″ to the right of, and 2″ behind B, and at the same level as A. Locate the point D, 2″ behind A, 1″ above C, and 2″ to the right of B. Draw the front, top, and right-side views of the parallelogram A B C D.

**15.** Locate the point A238. Locate the point B, 2″ to the left of, 2¼″ in front of, and 2½″ below A. Locate the point C, 1¾″ below, 1¼″ in front of, and 2″ to the right of A. Locate the point D, 1″ to the right of, 1¼″ below, and 2¾″ in front of A. Draw the front, top, and right-side views of the tetrahedron having its corners at A, B, C, and D.

**16.** The point E is 2″ to the left of, 2″ behind, and 1½″ below the point D645. The point K is ½″ behind, 3″ to the right of, and 2″ below E. The point L is 2″ in front of, ½″ to the left of, and 1″ above K. Draw the front, top, and side views of the tetrahedron E D K L.

**17.** V536 is the vertex and D E F is the base of a pyramid. The corner D is 1″ in front of, 1″ to the left of, and 2½″ below V. The corner E is 3″ behind, 2″ to the right of, and ½″ below D. The corner F is 4″ in front of, 1″ above, and 2″ to the right of E. Draw the front, top, and left-side views of the pyramid.

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**18.** Locate the point A104. Locate B, 2½″ to the right of, ½″ above, and 1″ behind A. Locate C, 2″ behind, 1½″ to the left of, and ½″ above B. Locate V, 3″ above, 2″ to the right of, and 2″ behind A. V is the vertex and A B C is the base of a pyramid. Show the front, top, and right-side views of the pyramid.

**19.** Locate the point Q007. Locate the point R, 2″ to the right of, 3″ above, and 2″ in front of Q. Locate the point S, 3″ behind, 1″ to the right of, and 2″ below R. Locate the point T, 1″ above, 2″ in front of, and 1″ to the right of S. Show three principal views of the pyramid Q R S T.

**20.** Locate the point C005. Locate D, 4″ to the right of, 1″ behind, and 1″ above C. Locate E, 2″ behind, ½″ above, and 3″ to the left of D. Locate V, 2″ in front of, 1″ to the right of, and 2½″ above E. C D E is the base and V is the vertex of a pyramid. Show the front, top, and right-side views of this pyramid.

**21.** Locate the point A004. The point B is 1″ behind, 3″ to the right of, and 1″ above A. The point C is 2″ above, 2″ behind, and 2″ to the left of B. The point D is 1″ to the right of, 1″ behind, and 1″ above C. Draw the front and top views of the pyramid A B C D, and then draw the right-side view.

**22.** Locate the point V705. Locate the point A, 3″ above, 1½″ behind, and 3″ to the left of V. Locate the point B, 4″ to the right of, ½″ behind, and 1″ above A. Locate the point C, 1″ in front of, 2″ below, and 2″ to the left of the point B. First draw the front and top views of the pyramid V A B C and then draw the side view.

# PROBLEMS—CHAPTER III

## AUXILIARY VIEWS

### Group 6. Auxiliary Views.

**1.** Make a drawing of a right hexagonal prism, similar to the drawing of the pyramid shown in Fig. 15. Show all views with the exception of the oblique view. A226 D326 is a diagonal of the horizontal base of the prism. The height of the prism is $1\frac{1}{2}''$. Make angle $a = 45°$, $b = 60°$, $c = 30°$, and $e = 15°$. Letter each corner of the prism and the name of each view, and indicate all dimensions of width, depth, and height that are seen in each view.

**2.** Make a drawing of a right hexagonal prism, similar to the drawing of the pyramid shown in Fig. 15. Show all views with the exception of the oblique view. A226 D$3\frac{1}{2}$,2,6 is a diagonal of the horizontal base of the prism. The height of the prism is $1''$. Make angle $a = 45°$, $b = 60°$, $c = 45°$, and $e = 15°$. Letter each corner of the prism and the name of each view, and indicate all dimensions of width, depth, and height that may be measured in each view.

**3.** Make a drawing of an inverted right hexagonal pyramid, similar to the drawing of the pyramid in Fig. 15. Show all views with the exception of the oblique view. A2,$3\frac{1}{2}$,6 D$3\frac{1}{2}$,$3\frac{1}{2}$,6 is a diagonal of the horizontal base. The vertex is $1\frac{1}{2}''$ below the center of the base. Make the angle $a = 30°$, $b = 45°$, $c = 60°$, and $e = 45°$. Letter each corner of the pyramid and the name of each view, and indicate all dimensions of width, height, or depth that are seen in each view.

**4.** A right pyramid has a $1\frac{1}{2}''$ square base. The lower left-hand corner of the base is at A$2\frac{1}{2}$,2,$6\frac{1}{2}$, and two edges of the base are horizontal. The vertex is $\frac{3}{4}''$ directly in front of the center of the base. Draw the front, top bottom, right-side, left-side, and rear views, and also the right-auxiliary, view at an angle of 60° above the horizontal, the left-auxiliary view at an angle of 75° above the horizontal, the rear-auxiliary at an angle of 60° above the horizontal, the auxiliary elevation at an angle of 30° to the rear of the right, and an auxiliary view taken in a direction perpendicular to the top triangular face as seen in the left-side view. Letter each corner of the pyramid and the name of each view and indicate all dimensions of width, depth, and height that are seen in each view.

**5.** C425 D426 is the axis of a right prism that appears as a hexagon in the front view. The diagonals of the hexagon are $1\frac{1}{2}''$ long, and one diagonal is horizontal. Draw the following views: front, top, right, left, right-auxiliary at 45° above the horizontal, left-auxiliary at 15° above the horizontal, and the auxiliary elevation at 30° to the rear of the left.

**6.** The prism of the preceding problem is to be tipped by raising the front end until the axis of the prism makes an angle of 30° with the horizontal. Show the prism in this position by drawing the following views in the order stated: an end view taken in a direction parallel to the axis, right, front, top, auxiliary elevation 30° forward of the right, and left-auxiliary 45° above the horizontal.

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**7.** A right pyramid has a vertical rectangular face 3″ wide and 1¼″ high. The vertex V617 is 3¼″ behind the base. Show the front, top, and left-side views, and also two auxiliary views; one taken in a direction perpendicular to the left triangular face, and the other taken in a direction perpendicular to the upper triangular face.

**8.** A right pyramid has its vertex 2″ directly behind the middle point C1½,1½,5 of its rectangular base which is 3″ high and 1¼″ wide. Draw the front, top and right-side views, and show also two auxiliary views: a rear-auxiliary view taken in a direction perpendicular to the upper triangular face, and an auxiliary elevation taken in a direction perpendicular to one of the broader triangular faces.

**9.** The hexagonal base of a right pyramid has a vertical diagonal 3″ long. The vertex of the pyramid is at V226, 1″ in front of the center of the base. Draw the front, top, and right-side views, and also an auxiliary elevation taken in a direction perpendicular to the right-hand triangular face.

**10.** Draw the front, top, and right-side views of a symmetrical wedge that is 3½″ high. The horizontal base of the wedge is a rectangle 3″ wide by 1¾″ deep, and the 1½″ knife-edge of the wedge is 3½″ above and parallel to the longer edges of the base. The middle point of the base is at C2½,1,6. Draw also an auxiliary view of the wedge taken in a direction perpendicular to the right-hand triangular face.

**11.** A symmetrical wedge has a vertical rectangular base 2″ high and 3¼″ wide. The lower left corner of the base is at A118. The horizontal knife-edge of the wedge is 1¾″ long and is 3¼″ in front of the base. Draw the front, top, and right-side views of the wedge, and also an auxiliary view taken in a direction perpendicular to the right-hand triangular face.

**12.** A vertical line 2½″ long is the diagonal of a hexagon that is the common base of two equal right pyramids. The vertex of one pyramid is 4″ to the right of the vertex V4,1½,6½ of the other pyramid. Draw the front, top and left-side views, and also an auxiliary view of the pyramids taken in a direction perpendicular to the front triangular face of the left-hand pyramid.

**13.** A horizontal line 2½″ long is the diagonal of a hexagon. This hexagon is the common base of two right pyramids of equal size. The vertex of one pyramid is 4″ in front of the vertex V628 of the other. Draw the front, top, and left views, and an auxiliary view of the pyramids taken in a direction perpendicular to the upper triangular face of the front pyramid.

**14.** An inverted right pyramid of 4″ altitude, with the vertex at V206, has a horizontal rectangular base 2¼″ deep by 3½″ wide. The upper 2″ of the pyramid is to be used as a sheet-metal funnel. Show the front, top, and right-side views of the funnel. Show also two auxiliary views of the funnel taken in directions perpendicular respectively to the narrower and wider trapezoidal faces. The views will overlap.

**15.** The vertex V606 of a right pyramid is 3″ below the center of its hexagonal base. One corner of the hexagon is 4″ to the right of the opposite corner. This pyramid is made from sheet metal and the base is open. Discard the lower half of the pyramid. Draw the front, top, and left views of the upper half, and in a properly placed auxiliary view show the true shape of one of the trapezoidal faces.

**16.** An inverted right pyramid, with vertex at V217, has a horizontal rectangular base 3″ wide and 2″ deep. The vertex is 4″ below the base. Discard the lower half of the pyramid. Make a hopper by hollowing out the upper half so that the side walls are $\frac{1}{4}$″ thick. Leave the top and bottom open. Show the front, top, and right-side views of the hopper, and also an auxiliary view looking upward in a direction parallel to the left-hand face.

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**17.** A right pyramid, having a $1\frac{1}{2}$″ by 3″ rectangular base, lies on one of its broader triangular faces, with the vertex A217 of the triangle $2\frac{1}{2}$″ behind the center of its base. Draw the front, top, and right-side views, and the auxiliary view of the pyramid in which the upper triangular face is shown in its true shape.

**18.** A right pyramid of $3\frac{1}{4}$″ altitude has a 2″ by $2\frac{3}{4}$″ rectangular base. The pyramid lies on one of its narrower triangular faces, with the vertex C416 of the triangle to the right of the center of its base. Draw the front, top, and right-side views of the pyramid, and show also the right-auxiliary view of the pyramid, taken in a direction perpendicular to the upper triangular face.

**19.** A horizontal isosceles triangle of 3″ altitude and lying with the vertex behind and $1\frac{1}{2}$″ to the left of the middle point C605 of its $2\frac{1}{2}$″ base, is the lower face of a right square-base pyramid. Draw the front, top, and left-side views and the necessary auxiliary view of the pyramid.

**20.** A right pyramid having a 2″ by $2\frac{1}{2}$″ rectangular base lies on one of its narrower triangular faces. The vertex of the triangle is $2\frac{3}{4}$″ behind and $1\frac{1}{2}$″ to the right of the middle point D205 of its base. Draw the front, top, and right-side views and the necessary auxiliary view.

**21.** The base of a horizontal isosceles triangle is 2″ long, and the vertex, at A016, is 3″ to the left of the middle point of the base. This triangle is the lower face of a right hexagonal pyramid. Draw, in the order stated, an auxiliary view taken in a direction parallel to the axis, the front view, and the top view.

**22.** The base of a horizontal isosceles triangle is 1″ long, and the vertex is 3″ to the left of the middle point D826 of the base. This triangle is the lower face of a right pyramid, the base of which is an octagon. Draw the front and top views of the pyramid, and an auxiliary view taken in a direction parallel to the axis.

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**23.** The base of a right circular cone is 2″ in diameter, and the altitude is $2\frac{1}{2}$″. The vertex is directly in front of the center C426 of the base. Draw the following views: front, top, right-side, left-auxiliary at 45° above the horizontal, and auxiliary elevation at 30° to the rear of the right. State the name of each view and letter the vertex and the center of the base in each view.

**24.** The base of a right circular cone is $1\frac{1}{2}$″ in diameter. The vertex V426 is 2″ directly to the right of the center of the base. Draw the following views in the order stated: right-side, front, top, left-auxiliary at 45° above the horizontal, auxiliary elevation at 30° to the rear of the right, and a right-auxiliary view taken in a direction perpendicular to the top line of the cone. Letter the vertex and the center of the base in each view, and state the name of each view.

**25.** A right cylinder $1\frac{1}{2}''$ in diameter and $1\frac{1}{4}''$ long has a vertical axis. The center of the lower base is at C326. Draw the following views: front, top, right, right-auxiliary at 45° above the horizontal, left-auxiliary at 30° above the horizontal, rear-auxiliary at 15° above the horizontal, and auxiliary elevation at 30° to the rear of the right. Indicate the diameter and the height of the cylinder in each view where they may be measured. Near each view letter the name of that view.

**26.** Draw the following views of a cylindrical ring: top, front, right-side, right-auxiliary at 30° below the horizontal, left-auxiliary at 45° above the horizontal, and the auxiliary elevation at 60° to the rear of the right. The axis of the ring is vertical. The outside diameter is $2''$. The inside diameter is $1\frac{1}{2}''$, and the length $\frac{1}{2}''$. C326 is the center of the lower face. Draw circles first.

**27.** A circular washer, $2''$ in diameter and $\frac{1}{4}''$ thick, has a $\frac{3}{4}''$ round hole through it. C426 is the center of the lower horizontal face. Draw the top, front, right-auxiliary 30° above the horizontal, and left-auxiliary 45° above the horizontal views.

**28.** A circular washer $2''$ in diameter and $\frac{1}{4}''$ thick has a $\frac{3}{4}''$ round hole through it. C625 is the center of the upper face. The axis of the washer is tipped forward 30° from a vertical position. Draw the front, top, and left-side views, and also an auxiliary view taken in a direction parallel to the axis. The auxiliary view should be drawn first.

**29.** A circular washer is $3''$ in diameter, has a $1\frac{1}{2}''$ circular hole, and is $\frac{1}{4}''$ thick. The edge of the washer is seen in the front view. C3,2,6$\frac{1}{2}$ is the center of the right face. Draw the following views: front, top, right, right-auxiliary at 60° above the horizontal, and left-auxiliary at 45° above the horizontal.

**30.** A cylindrical washer, $2\frac{1}{2}''$ in diameter and $\frac{3}{8}''$ thick, has a hole $1''$ in diameter. The washer is first in such a position that it is seen on edge in the front and top views, and is then turned 30° to the right about a vertical axis. The center of the rear face is at C326. Draw the front, top, and right-side views, and a circular auxiliary elevation of the washer in its new position. Draw the last-named view first. Draw also a left-auxiliary view at 45° to the horizontal.

**31.** A disk $2''$ in diameter and $\frac{1}{2}''$ thick has a $1''$ square hole cut through it. The axis of the hole is on the axis of the disk. This disk is first in such a position that the front view appears as a circle, and the edges of the hole are horizontal and vertical. It is then tipped directly forward through an angle of 30°. C325 is the center of the front face. Draw the following views in the order stated: rear-auxiliary at 30° above the horizontal, right-side, front, top, and an auxiliary elevation at 45° to the rear of the right.

**32.** A225 B425 is the lower edge of the front face of a rectangular block that is $1\frac{1}{2}''$ high and $\frac{3}{8}''$ deep. A $\frac{3}{4}''$ hole centering on the front face is drilled straight through the block. Draw the front, top, right-side, left-side, and rear views, the right-auxiliary view at 45° above the horizontal, the left-auxiliary view at 60° above the horizontal, and the auxiliary elevation at 30° to the rear of the right.

**33.** A218 B518 is the rear edge of the square base of a rectangular block that is $\frac{1}{2}''$ high. A right circular cylinder $1''$ in diameter, $\frac{3}{4}''$ long, and with the axis vertical stands on the center of the top face of the rectangular block. Show the front and top views, and the right-auxiliary view at 30° above the horizontal, of the block and cylinder.

**34.** A$5\frac{1}{2}$,0,6 C306 is the diagonal of a horizontal square. This square is the base of a cube. A $1\frac{1}{4}''$ vertical hole is drilled centrally through the cube. Draw the front and top views, a right-auxiliary view taken in a direction 45° above the horizontal, and a left-auxiliary view taken in a direction 30° above the horizontal.

**35.** The square A335 B425 C315 D225 is the front face of a cube. A $1''$ hole is drilled centrally through the cube from front to back. Draw the front and top views of the cube, an auxiliary elevation at 30° to the rear of the right, and an auxiliary elevation at 45° forward of the right.

**36.** E506 F5,$1\frac{1}{2}$,6 is the axis of a right circular cylinder $2''$ in diameter. A central hexagonal hole runs lengthwise through the cylinder. The diagonal of the hexagon at each end of the hole is $1\frac{1}{4}''$ long and one corner of each hexagon is directly to the right of its center. Draw the following views: top, front, left, right-auxiliary at 45° above the horizontal, and front-auxiliary at 60° above the horizontal.

**37.** The rectangular faces of a hexagonal washer are $1\frac{1}{2}''$ by $\frac{1}{2}''$, and the hole through the washer is $1\frac{1}{2}''$ in diameter. The axis of the washer is tipped 30° to the right of the vertical, and four edges of the hexagonal faces are horizontal. C2,$1\frac{1}{2}$,6 is the center of the upper face. Draw the following views in the order stated: a right-auxiliary view taken in a direction parallel to the axis, front, top, and right-side.

**38.** A424 D724 is the diagonal of a vertical hexagon. This hexagon is the front face of a prism that is $\frac{1}{2}''$ long. A $1\frac{1}{2}''$ hole is drilled centrally through the prism from front to back. Draw the front and top views, an auxiliary elevation taken from the left and 30° to the rear, and an auxiliary elevation taken from the left and 15° forward.

# PROBLEMS—CHAPTER IV

## OBLIQUE VIEWS

### Group 7. Oblique Views Taken in Specified Directions.

**1.** Show the oblique view, taken in the direction E434 F326, of the triangle A105 B337 C526.

**2.** Show the oblique view of the triangle D435 E727 F516 taken in the direction P308 K527.

**3.** A516 C815 is the diagonal of a horizontal square. Draw the oblique view taken in the direction K208 A.

**4.** C114 E324 is the diagonal of a frontal square. Draw the oblique view of this square and its diagonals taken in the direction J056 P135.

**5.** C214 F434 is the diagonal of a frontal hexagon. Draw the oblique view of this hexagon taken in the direction K746 P535.

**6.** B405 E707 is the diagonal of a horizontal hexagon. Show the oblique view of this hexagon taken in the direction J215 K506.

**7.** A306 C607 is the diagonal of a horizontal square. This square is the base of a cube. Draw the oblique view of the cube taken in the direction of the arrow S043 V325.

**8.** C3½,0,7½ E1,2½,5 is the diagonal of a cube. The top face of the cube is horizontal. Show the appearance of the cube when looking in the direction of the arrow K658 O437. The views will overlap.

**9.** A505 G727 is the diagonal of a cube, four faces of which are vertical. Draw the oblique view of the cube taken in the direction O4,5,7½ P626.

**10.** A306 C507 is the diagonal of a horizontal square. This square is the base of a cube. Draw the view of the cube taken in the direction of the arrow O734 P526.

**11.** E115 C337 is the diagonal of a cube. The bottom face of the cube is horizontal. Show the front and top views of the cube and also a view taken in the direction of the diagonal C E.

**12.** A527 G705 is the diagonal of a cube. Four faces of the cube are vertical. Show the oblique view of this cube taken in the direction of the diagonal A G. This oblique view is an isometric view of the cube; that is, in this view all edges of the cube appear equal in length.

**13.** A104 C306 is the diagonal of a horizontal square that is the base of a right prism of 3″ altitude. Show the view of the prism taken in the direction O458 P335.

**14.** A427 D727 is the diagonal of a frontal hexagon. This hexagon is the front face of a right prism that is 1″ long. Show the view of the prism taken in the direction S704 C616.

**15.** A202 D205 is the diagonal of a horizontal hexagon. This hexagon is the lower base of a right prism that is $\frac{3}{4}''$ tall. Show the view of the prism taken in the direction P438 V315.

**16.** A205 D235 is the diagonal of a hexagon. This hexagon is the rear face of a hexagonal washer, $\frac{1}{2}''$ thick, that has through it a hole $1\frac{1}{4}''$ in diameter. Draw the oblique view of the washer taken in direction K408 P215.

**17.** V436 is the vertex and A$2\frac{1}{2}$,0,6 D$5\frac{1}{2}$,0,6 is a diagonal of the hexagonal base of a right pyramid. Show the view of the pyramid taken in the direction O843 P625.

**18.** A$4\frac{1}{2}$,2,6 is the vertex, and B324 E624 is one diagonal of the base of a right hexagonal pyramid. Draw the oblique view taken in the direction K667 P525.

**19.** The vertex A of a right hexagonal pyramid is 2″ above the center of its base, one diagonal of which is B324 E327. Discard the upper half of the pyramid, leaving a frustum 1″ tall. Show the oblique view taken in a direction parallel to the arrow K534 V417.

**20.** V328 is the vertex, and C226 F426 is one diagonal of the hexagonal base of a right pyramid. Show the oblique view taken in a direction parallel to the arrow O104 P337.

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**21.** C326 D328 is the axis of a $2\frac{1}{2}''$ right circular cylinder. Show the oblique view of the cylinder taken in the direction O104 P216.

**22.** E416 F406 is the axis of a 3″ right circular cylinder. Draw the oblique view of this cylinder taken in the direction P823 K515.

**23.** C624 is the center of a $2\frac{1}{2}''$ frontal circle. This circle is the front face of a cylinder that is $\frac{3}{4}''$ long. Draw the oblique view of this cylinder taken in the direction J212 K524.

**24.** E126 F226 is the axis of a 2″ right circular cylinder. Show the oblique view of this cylinder taken in the direction K357 L2,3,$6\frac{1}{2}$.

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## Group 8. Objects in Oblique Positions.

**1.** E416 F627 is the axis of a right prism. The diagonals of the hexagonal bases are $1\frac{1}{2}''$ long. Two of the rectangular faces are vertical. Draw the front, top, left-side, auxiliary, and end views of the prism.

**2.** C2,$1\frac{1}{2}$,5 D426 is the axis of a right hexagonal prism. The sides of the bases are $1\frac{1}{2}''$ long, and two sides of each base are horizontal. Draw the front, top, auxiliary, and end views of the prism. The views will overlap.

**3.** C2,1,$6\frac{1}{2}$ D326 is the axis of a right prism. The diagonals of the hexagonal bases are $2\frac{1}{2}''$ long, and one diagonal of each base is horizontal. Draw the front, top, right-side, auxiliary, and end views of the prism.

**4.** A217 B525 is the axis of a right prism. The diagonals of the hexagonal bases are 2″ long, and one diagonal of each base is horizontal. Draw the front, top, auxiliary, and end views of the prism.

**5.** C5,$1\frac{1}{2}$,7 D716 is the axis of a right prism. The bases are $1\frac{1}{2}''$ squares. One diagonal of each base is horizontal. Draw the front, top, left-side, auxiliary, and end views of the prism.

**6.** E225 F417 is the axis of a right prism. The ends of the prism are 1¼″ squares, and one diagonal of each square is horizontal. Show the front, top, auxiliary, and end views of the prism.

**7.** A216 B337 is the axis of a right prism having square ends. One corner of the prism is at C1,1½,6. Draw the front, top, auxiliary, and end views of the prism.

**8.** A527 B715 is the axis of a right prism. C4½,2¾,X is one corner of, and A is the middle point of one of the square ends of the prism. Draw the front, top, auxiliary, and end views of the prism.

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**9.** A square, having diagonals 2½″ long and with one diagonal horizontal, is the common base of two equal right pyramids, the vertices of which are respectively at A025 and V317. Draw the front, top, right-side, auxiliary, and end views of the pyramids.

**10.** O814 is the vertex, and P626 is the center of the hexagonal base of a right pyramid. The sides of the hexagon are 1⅞″ long, and one corner of the hexagon is directly above the axis of the pyramid. Draw the front, top, auxiliary, and end views of the pyramid.

**11.** V338 is the vertex, and C226 is the center of the octahedral base of a right pyramid. The sides of the base are 1¼″ long, and two sides are horizontal. Draw the front, top, auxiliary, and end views of the pyramid.

**12.** V726 is the vertex, and C6,3,6½ is the center of the base of a right hexagonal pyramid. The sides of the hexagon are 1½″ long, and one diagonal is horizontal. Draw the front, top, auxiliary, and end views of the pyramid.

**13.** V116 is the vertex, and C337 is the center of the hexagonal base of a right pyramid. The sides of the hexagon are ¾″ long, and two sides are frontal. Draw the front, top, right-side, auxiliary, and end views of the pyramid.

**14.** O5,1½,4 and P737 are the respective vertices of two equal right pyramids that have as a common base a 1½″ square of which one diagonal is frontal. Draw the front, top, left-side, auxiliary, and end views of the pyramids.

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**15.** V616 is the vertex, and C514 is the middle point of the base of a horizontal isosceles triangle. This triangle is the lower face of a right pyramid. The diagonals of the hexagonal base of the pyramid are 3½″ long. Show the front, top, auxiliary, and oblique views of the pyramid. The views will overlap.

**16.** A right hexagonal pyramid lies on one of its triangular faces. The vertex of the lower face is at G105, and is 2⅛″ in front of, and an equal distance to the left of the middle point of its 1½″ base. Show the front, top, and right-side views, and any necessary auxiliary and oblique views of the pyramid.

**17.** The vertex V507 of a horizontal isosceles triangle is 2″ behind and 1″ to the left of the middle point of its 2″ base. This triangle is the lower face of a right hexagonal pyramid. Draw the front, top, and left-side views of the pyramid, and also the necessary auxiliary and oblique views. The views will overlap.

**18.** V616 is the vertex, and C537 is the middle point of the horizontal base of an isosceles triangle. This triangle is one face of a right pyramid. The diagonals of the hexagonal base of the pyramid are $2\frac{1}{2}''$ long. The axis of the pyramid is nearly vertical. Draw the front, top, auxiliary, and oblique views of the pyramid.

**19.** V617 is the vertex, and C725 is the middle point of the $2''$ horizontal base of an isosceles triangle. This triangle is the lower face of a square base pyramid. Draw the front, top, left-side, auxiliary, and oblique views of the pyramid.

**20.** V315 is the vertex, and C237 is the middle point of the $1\frac{1}{2}''$ frontal base of an isosceles triangle. This triangle is the front face of a square base right pyramid. Draw the front, top, auxiliary, and oblique views of the pyramid.

**21.** The point P046 is equidistant from all points of the $2''$ circle with center at C327. Draw the front, top, auxiliary, and oblique views of the circle.

**22.** C236 is the center of a $2\frac{1}{2}''$ circle, and D828 is a point on the axis of the circle. Draw the front, top, auxiliary, and oblique views of the circle.

**23.** C636 is the center of a $3''$ circle. The point A704 is equidistant from all points of the circle. Draw the front, top, auxiliary, and oblique views of the circle.

**24.** A614 is the vertex, and C437 is the center of the $2\frac{1}{4}''$ circular base of a right cone. Show the front, top, auxiliary, and end views of the cone.

**25.** E418 is the vertex, and C136 is the center of the $1\frac{3}{4}''$ circular base of a right cone. The point of this cone is cut off leaving a frustum of $1\frac{1}{2}''$ altitude. Show the front, top, auxiliary, and oblique views of the frustum.

**26.** C626 is the center of the $3''$ circular base and V507 is the vertex of a cone. Show the front, top, auxiliary, and oblique views of the cone.

**27.** C117 D426 is the axis of a $1\frac{3}{4}''$ right circular cylinder. Show the front, top, auxiliary, and oblique views of the cylinder.

**28.** A125 B317 is the axis of a $2''$ right circular cylinder. Draw the front, top, auxiliary, and end views of the cylinder.

**29.** C2,1,$5\frac{1}{2}$ D3,3,$6\frac{1}{2}$ is the axis of a $3''$ hollow right circular cylinder having walls $\frac{3}{8}''$ thick. Show the front, top, auxiliary, and end views of the cylinder.

**30.** E$5\frac{1}{2}$,$2\frac{1}{2}$,5 F626 is the axis of a $3\frac{1}{4}''$ right circular cylinder. A $1\frac{3}{4}''$ hole is drilled centrally through the cylinder. Draw the front, top, auxiliary, and oblique views of the cylinder.

# PROBLEMS—CHAPTER VI

## STRAIGHT LINES

### Group 9. Drawing Lines in Specified Directions.

Each problem of this group states that four different lines or pliures are to be drawn. Divide the 8-inch square problem sgace into 4-inch squares. Each of the smaller squares is to be used for the views of one of the lines or figures. The first fine specified is to be drawn in the upper left-hand square, the second in the upper right-hand square, the third in the lower left-hand square, and the fourth in the lower right-hand square. Letter each line with the letters called for, and in each view use the proper subscript. Just above the lower edge of each smaller square, letter the name of the type of line or figure drawn in that square. Each line drawn should, when possible, be a general case of the type of line specified. For example, if a horizontal line is specified, the line drawn should not be horizontal and also frontal or profile.

**1.** Draw the front, top, and right-side views of: a frontal line A B, a horizontal line C D, a horizontal-profile line E F, and a vertical line G H.

**2.** Draw the front, top, and left-side views of: a profile line J K, an oblique line L M, a frontal line N O, and a horizontal-profile line P Q.

**3.** Draw the front, top, and right-side views of: a horizontal line C D, a frontal line E F, a horizontal-frontal line J K, and a profile line O P.

**4.** Draw the front, top, and left-side views of: an oblique line A B, a vertical line C D, a horizontal-frontal line E F, and a horizontal line K L.

**5.** Draw the front, top, and right-side views of: a horizontal hexagon having a horizontal-profile diagonal A B $1\frac{1}{2}''$ long, a profile hexagon with sides $\frac{7}{8}''$ long and with two sides horizontal, a frontal hexagon having sides $\frac{1}{2}''$ long and with one diagonal C D vertical, and a horizontal hexagon having a horizontal-frontal diagonal E F $1\frac{1}{4}''$ long.

**6.** Draw the front, top, and left-side views of the following: a profile equilateral triangle A B C having the lower side horizontal and $2''$ long; a horizontal equilateral triangle D E F with the right side $1\frac{1}{2}''$ long and horizontal-profile; a frontal isosceles triangle having a $1\frac{3}{4}''$ horizontal base and with the vertex $2\frac{1}{2}''$ below the base; a frontal right-angled triangle Q R S having one leg vertical and $2''$ long, one leg horizontal and $1\frac{1}{4}''$ long, and the hypotenuse slanting upward and to the left.

**7.** Draw the front, top, and right-side views of: a 2″ frontal square having the horizontal-frontal line A B for its lower side, and a 1½″ horizontal square having the horizontal-frontal line C D for its front side. Draw the front, top, and left-side views of a 2″ square having the horizontal-profile line E F for its lower side, and a 1½″ square having the vertical line O P for one side and another vertical line to the left of, and ½″ in front of O P for another side.

**8.** Draw the front, top, and left-side views of: a frontal triangle A257 B36X C3½,4½,X, a horizontal triangle D657 E7,X,7½ F8X6, a profile triangle L324 M3,1,2½ N303, and a triangle P6,0,2½ Q714 R8X3 of which the front view is an edge view.

**9.** Draw the front and top views of each of the lines G157 H367, E656 F658, C114 D312, and A704 B7,2,2½, and state the direction of each line.

**10.** Draw the front, top, and left-side views of each of the lines O378 P358, Q667 R8,4½,8, S213 T3½,1,3, and X712 Y714, and state the direction of each line.

**11.** Draw the front, top, and right-side views of each of the lines C1,5½,6 D1,4½,8, E4½,5,7 F667, J004 K123, and L514 M512, and state the direction of each line.

**12.** Draw the front, top, and left-side views of each of the lines A356 B358, C767 D7,4½,7, E213 F313, and O613 K823, and state the direction of each line.

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## Group 10. Finding the True Lengths of Lines.

In the problems of groups 10, 11, and 12, the true length of each line is to be found by means of a normal view of the line. Do not use any other method. As usual, required data should be heavy. When a distance in inches is called for, the length in inches should be marked on the true length of the line.

**1.** Find the true length of each of the lines E036 F317 and O514 P628.

**2.** Show the true length of each of the lines C123 D208 and E414 F825.

**3.** Show the true length of each of the lines E125 F408 and O505 P537.

**4.** Find the true length of the line C135 D318 and check this length by means of another normal view.

**5.** Find the true length of the line E618 F735 and check this length by means of a second normal view.

**6.** Find the true length of the line O136 P417 and check this length by means of another normal view.

**7.** Is the line A125 B438 longer than the line B C847?

**8.** Is the line C126 D507 longer than the line D E834?

**9.** What is the difference in length of the lines D145 E417 and E F836?

**10.** What is the difference in length of the lines K104 L238 and L M647?

**11.** What is the total length in inches of the straight lines A125 B317, B C434, and C D628?

**12.** What is the total length in inches of the lines C117 D335, D E436, and E F827?

**13.** What is the total length in inches of the lines A225 B227, B C428, C D408, D E638, and E F538?

**14.** How much farther is it from A024 to E538 by way of B116, C325, and D404, than from A to E in a direct line?

**15.** How much farther is it from E726 to F047 by way of C445 and D328 than from E to F in a direct line?

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**16.** Show the true shape of the triangle J106 K246 L327.

**17.** Show the true shape of the triangle Q515 R637 S815.

**18.** Show the true shape of the triangle A107 B235 C426.

**19.** Show the true shape of the triangle D734 E525 F618.

**20.** Is the triangle A834 B517 C434 isosceles?

**21.** State, in inches, the perimeter of the triangle C525 D617 E726.

**22.** State, in inches, the perimeter of the triangle D117 E247 F325.

**23.** State, in inches, the perimeter of the triangle P538 Q735 R6,0,6½.

**24.** State, in inches, the perimeter of the triangle A316 B447 C535.

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## Group 11. Measuring Given Distances on Oblique Lines.

**1.** Locate the point O on the line A046 B417 and 3″ from A. Locate the point P on the line B C704 and 2″ from C.

**2.** Locate the points A and B on the lines D147 E324 and E F715 respectively and 2″ from the point E.

**3.** Draw the front, top, and right-side views of the line J134 K318. Locate the points B and C on the line J K, and 1½″ from its middle point.

**4.** Draw the front, top, and left-side views of the line K607 L745. Locate the points E and F on the line, and 2″ from the two ends of the line.

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**5.** A417 B845 is the base of an isosceles triangle. The legs of the triangle are 3″ long, and the vertex V is directly below the base. Draw the front, top, and left-side views of the triangle.

**6.** B024 C108 is the base of an isosceles triangle. The vertex A is directly behind the base, and the legs are 2½″ long. Draw the front, top, and right-side views of the triangle.

**7.** The vertex V of an isosceles triangle is directly to the right of the base A037 B345, and 2½″ from the ends of the base. Draw the front, top, and right-side views of the triangle.

**8.** Each side of an equilateral triangle is 3″ long. One end of the base is at A835. The other end B is on the line A X428, and the vertex C is directly below the base. Draw the front, top, and side views of the triangle.

**9.** The two equal sides of an isosceles triangle are each 3½″ long. The vertex is at V218, and the ends of the base are at A and B on the lines V C135 and V D536. Draw the front, top, and right-side views of the triangle.

**10.** The two equal sides of an isosceles triangle are 3½″ long. A734 is the vertex, and the two ends of the base are respectively on the lines A E617 and A F408. Draw the front, top, and left-side views of the triangle.

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**11.** C346 D306 represents a vertical rod of fixed length. The end C remains fixed while the end D is moved 3″ to the right and 2″ to the rear. Show the rod in its new position.

**12.** The end C of the straight rod C338 D115 remains fixed while the end D is swung to a new position below C, 3″ to the right of, and 1″ in front of C. Show the rod in its new position.

**13.** The location of the end F of the straight rod E318 F626 is fixed. The end E is moved to a new position above and 3″ in front of, but not to the right or left of its original position. Show the rod in its new position.

**14.** The rod A427 B745 is swiveled about its middle point as a center, and the end B is moved until it is behind C, ½″ to the right of C, and 1″ below C. Show the rod in its new position. Check to make sure that the length of the rod has not changed.

## Group 12. True Lengths of Lines—Miscellaneous Problems.

**1.** V208 is the vertex of an isosceles triangle. F516 is one end of the base of the triangle. The other end is at E on the line V X135. Draw the front, top, and right-side views of the triangle.

**2.** V834 is the vertex and V B605 is one of the equal legs of an isosceles triangle. The other leg is on the line V X547. Draw the front, top, and left-side views of the triangle, and show the true shape of the triangle.

**3.** Draw the front, top, and left-side views of the isosceles triangle A618 B725 C. The vertex is at A, and the point C is on the line A D336.

**4.** V537 is the vertex and A716 is one end of the base of an isosceles triangle. The other end of the base is on the line V X414. Draw the front, top, and side views of the triangle.

**5.** In the triangle A234 B116 C427, the corner B is to be moved along the line C B until the triangle becomes isosceles with the vertex at C. Draw the front, top, and right-side views of the isosceles triangle.

**6.** The vertex G of an equilateral triangle is directly behind the base E205 F134. Draw the front, top, and side views of the triangle.

**7.** The vertex D of an equilateral triangle lies directly below the side E6,3½,4 F7,2½,7 of the triangle. Draw the front, top, and side views of the triangle.

**8.** The vertex A of an equilateral triangle is directly in front of the base B047 C328. Draw the front, top, and side views of the triangle.

**9.** A135 C427 is the diagonal of a vertical square. Show the front, top and right-side views of the square.

**10.** The right-side view of a square is the edge view of the square. Its diagonal is A138 C315. Draw the front, top, and right-side views.

**11.** The edge view of a square is seen in the front view. A745 C517 is a diagonal of the square. Draw the front, top, and side views of the square.

**12.** The left-side view of a square is the edge view of the square, and C516 E747 is one of its diagonals. Draw the front, top, and side views of the square.

**13.** A514 D637 is the diagonal of a hexagon. The left-side view is an edge view of the hexagon. Draw its front, top, and left-side views.

**14.** A725 D538 is the diagonal of a vertical hexagon. Draw the front, top, and left-side views of the hexagon.

**15.** The front view is an edge view of a hexagon, and A145 D3,1,6½ is one of its diagonals. Draw the front, top, and side views.

**16.** The right-side view is the edge view of a hexagon, and C216 F3½,4,7½ is one of its diagonals. Draw the front, top, and side views of the hexagon.

**17.** P5,3,5½ is a point on the surface of a sphere, and C626 is its center. Show the front, top, and left-side views of the sphere.

**18.** C226 is the center of a sphere, and P3,1½,7 is a point on the surface. Show the front, top, and side views of the sphere.

**19.** P7,3¼,5 is a point on the surface of a sphere, and O626 is its center. Draw the front, top, and left-side views of the sphere.

**20.** O626 is the center of a 3½″ sphere. Is the point P537 inside or outside the sphere?

**21.** C226 is the center of a 3″ sphere. Is the point P2½,3¼,7 inside or outside the sphere?

**22.** O426 is the center of a 4″ sphere. Is the point P3½,3½,7¼ inside or outside the sphere?

**23.** C217 is the center of a 2″ sphere, and O436 is the center of a 3¾″ sphere. Will the spheres intersect?

**24.** C225 is the center of a 2½″ sphere, and O4,3½,6 is the center of a 3″ sphere. Will the spheres intersect?

**25.** O108 is the center of an 8″ sphere, and C327 is the center of a 2″ sphere. Is the smaller sphere entirely inside the larger?

**26.** O6,3,6½ is the center of a 5″ sphere, and C625 is the center of a 1½″ sphere. Is the small sphere entirely inside the large sphere?

**27.** E117 F417 is one edge of the horizontal base of an equilateral tetrahedron. Draw the front, top, and side views of the tetrahedron.

**28.** A415 B715 is the front edge of the horizontal base of an equilateral tetrahedron. Draw the front, top, and side views of the tetrahedron.

**29.** C108 D408 is one edge of the horizontal square base of an equilateral right pyramid. Draw three views of the pyramid.

**30.** C407 D604 is the left edge of the horizontal base of an equilateral tetrahedron. Draw the front, top, side, and necessary auxiliary views of the tetrahedron.

**31.** C117 is the left-hand corner, and D315 is the front corner of a horizontal triangle. This triangle is the base of an equilateral tetrahedron. Draw the front, top, and right-side views of the tetrahedron.

**32.** E425 G726 is the diagonal of a horizontal square. This square is the common base of two equilateral pyramids. Show the front, top, and auxiliary views of the two pyramids.

**33.** A517 and B717 are the centers of two 2″ spheres. A third 2″ sphere has its center on the same level as A and B and touches the first two spheres. A fourth 2″ sphere rests in the hollow formed by the first three spheres. Draw the front, top, and side views of the four spheres.

**34.** A517, B717, C715, and D515 are the centers of 2″ spheres. Show the front, top, and auxiliary views of these spheres with a fifth 2″ sphere resting in the hollow formed by the four spheres.

**35.** C616 is the center of a 1″ sphere that rests on a horizontal surface. Around this sphere in a square formation are grouped eight similar spheres. Four 1″ spheres are placed in the cups formed by the lower spheres, and a single 1″ sphere is placed on top. Show the front and top views and an auxiliary elevation at 45°. Do not show hidden lines.

**36.** C516 is the center of a 1″ sphere that rests on a horizontal surface. Around this sphere, and touching it, are grouped six similar spheres. In three of the cups formed by these spheres are placed three 1″ spheres, and a single 1″ sphere is placed on top. Draw the front, top, and side views. Do not show hidden lines.

## Group 13. Calculating the Length of Lines.

The calculated answers to the problems in group 13 should not vary more than 0.01 inch from the correct answer. Solve each problem in the usual problem space. Show all calculations.

**1.** Calculate the length of the line C248 D516.

**2.** Calculate the length of the line E835 F147.

**3.** Calculate the length of the line J416 K735.

**4.** Calculate the length of the diagonal of a 14″ cube.

**5.** Calculate the length of the diagonal of a 5″ cube.

**6.** Calculate the length of the diagonal of a rectangular box that is 5′ long, 2′ 10″ wide, and 18″ deep.

**7.** In a bridge, the center of an upper pin-bearing is 10′–11$\frac{1}{4}$″ to the right of, and 25′–6″ above, the center of a lower pin-bearing. Calculate, to a hundredth of an inch, the distance between centers of the holes in the tie rod that is to connect the pins in these bearings.

**8.** In a watch, the distance between two pivot bearings is 0.4368″. One of two rectangular coordinates that measures the distance between these bearings is 0.1835″. Calculate the other coordinate within one ten-thousandth of an inch.

## Group 14. Drawing a Line to Intersect a Given Line.

In all intersecting line problems, the lines are regarded as unlimited in length. When necessary, the lines should be extended beyond the stated points.

**1.** Draw the line E02X F416 intersecting the line C114 D338 at the point P. Draw the line M634 N7X6 intersecting the line K616 L737 at O.

**2.** Draw the line C218 D1X5 intersecting the line A008 B334 at the point P. Also draw the line E845 F61X intersecting the line G747 H705 at a point O.

**3.** Draw the line C834 D5X5 intersecting the line A235 B717 at K. Draw the line E0X6 F8X7 intersecting the line C D at O and A B at P.

**4.** Draw the line E707 F41X intersecting the line C108 D234 at L. Draw the line J52X K21X intersecting the line E F at A and C D at B.

**5.** The line E4X7 F836 intersects the line C505 D748 at J. The line A42X B70X intersects the line E F at K and the line C D at L. Draw the front, top, and left-side views.

**6.** Draw the front, top, and right-side views of the line L118 M3X4 that intersects the line J035 K406 at O, and the line K N21X that intersects the line L M at P.

**7.** Draw the line C825 D51X intersecting the line A608 B634 at O, and the line E807 F5X5 intersecting the line A B at P.

**8.** Draw the line C047 D32X intersecting the line A238 B214 at K. Draw the line E015 F3X7 intersecting the line A B at L.

**9.** In the front, top, and left-side views of the triangle A435 B608 C724 show the horizontal line E C, the frontal line A D, and the profile line B F.

**10.** In the front, top, and right-side views of the triangle D146 E338 F204 draw the profile line F C, the frontal line D B, and the horizontal line E A.

**11.** In the front, top, and left-side views of the triangle J538 L604 N846 draw the frontal line N O, the profile line L M, and the horizontal line J K.

**12.** In the front, top, and left-side views of the triangle C406 E715 G837 draw the frontal line C D, the horizontal line E F, and the profile line E A.

## Group 15. Do the Given Lines Intersect?

**1.** Below the views of each of the following pairs of lines state whether they intersect or not: A134 B318 and C026 D434, E545 F507 and K535 L516.

**2.** Do any two of the lines A105 B137, C017 D335, and E034 F217 lie in the same plane?

**3.** Locate the points where the lines A125 B647, C327 D408, and E136 F547 intersect.

**4.** Do any two of the lines A035 B327, C007 D345, E527 F845, and J418 K536 intersect?

**5.** Will the line E0X8 F4X4 intersect the three lines A145 B228, A C527, and A D604?

**6.** Does the line K234 L826 intersect the three lines A834 B418, A C426, and A D3,2½,5?

## Group 16. Drawing a Line to Intersect Two Given Lines.

**1.** Through O4X6 draw a frontal line that intersects the lines Q147 R304 and S534 T708 at A and B. Through P41X draw a horizontal line that intersects the lines Q R and S T at C and D.

**2.** Through E31X draw a horizontal line that intersects the lines A123 B208 and C445 D808 at O and P. Through F5X7 draw a frontal line that intersects the lines A B and C D at Q and R.

**3.** Draw the front, top, and right-side views of a horizontal line A32X B that intersects the lines J034 K308 and L106 M447, and a vertical line that intersects the lines J K and L M at C and D.

**4.** Draw the front, top, and left-side views of a vertical line A B, a horizontal-profile line C D, and a horizontal-frontal line E F, intersecting the lines K408 L834 and M807 N645.

**5.** Draw the front, top, and right-side views of: a vertical line A B, a horizontal-profile line C D, and a horizontal-frontal line E F, intersecting the lines O428 P045 and J148 K304.

**6.** Draw the front, top, and left-side views of the line A6X6 B5X4 that intersects the lines E414 F736 and J608 K846 at C and D.

**7.** Locate the line E5X5 F2X6 that intersects the sides of the triangle A037 B115 C446 at O and P. Draw the front, top, and right-side views.

**8.** Locate the line A04X B81X so that it intersects the three lines D308 E145, E F717, and F D at the points J, K, and L.

## Group 17. Locating Points at Intersections.

**1.** Draw the front, top, and right-side views of the triangle D014 E336 F208. Locate the point O where the medians meet.

**2.** Draw the front, top, and left-side views of the triangle D415 E648 F807. Locate the point A where the medians meet.

**3.** Locate the point A where the medians of the triangle J435 K828 L617 meet. Draw the front, top, and left-side views.

**4.** Draw the front, top, and right-side views of the pyramid A246 B104 C018 D427. Take any corner as the vertex of the pyramid and find the point where the medians of the base meet. Draw a straight line connecting this point with the vertex. This straight line is a median of the pyramid. Take, in turn, two other corners as a vertex, and draw two other medians of the pyramid. Locate the point O where the medians of the pyramid meet. The point O is the center of gravity of the solid pyramid. Taking each median of the pyramid as a unit length, how far is the point O from the vertex?

**5.** Draw the front, top, and left-side views of the tetrahedron A438 B746 C836 D604. Locate the center of gravity O of this tetrahedron by using the method described in the preceding problem.

**6.** Draw the front, top, and right-side views of the pyramid D047 E438 F104 G316. Locate C, the center of gravity of this pyramid, by using the method described in the second problem preceding.

## Group 18. Are the Given Lines Parallel?

**1.** Draw, as required data, all groups of parallel lines among the following: A145 B417, C445 D807, E317 F634, and K017 L334.

**2.** Draw, as required data, all groups of parallel lines among the following: A025 B046, C116 D137, E438 F467, and O216 P258.

**3.** Draw, as required data, all groups of parallel lines among the following: A115 B435, C517 D827, E536 F836, and J108 K408.

**4.** Draw, as required data, all groups of parallel lines among the following: A146 B327, C338 D546, E225 F706, and O246 P827.

**5.** Draw, as required data, all groups of parallel lines among the following: A146 B106, C237 D437, E525 F528, and J626 K826.

**6.** Draw, as required data, any groups of parallel lines among the following: J014 K315, L137 M436, and N227 O528.

**7.** Draw the front, top, and left-side views of any parallel lines that are determined by the points A728, B515, C706, D447, and E537.

**8.** Draw the front, top, and side views of all groups of parallel lines that are determined by the points A335, B405, C515, D647, E827, and F706.

## Group 19. Drawing One Line Parallel to Another.

**1.** Draw the lines A646 O and A P parallel, respectively, to the lines C045 D208 and E028 F336.

**2.** Through the point O636 draw the lines O A and O B parallel respectively to the lines C146 D518 and E027 F434.

**3.** Draw the lines A226 B and A C parallel respectively to the lines K435 L608 and M525 N837.

**4.** Through the point P326 draw the lines P X and P Y parallel respectively to the lines A418 B534 and C646 D808.

**5.** A145 B016 and B C327 are two sides of a parallelogram. Draw the front, top, and side views of the parallelogram. Do all four corners of this parallelogram lie in a plane?

**6.** A545 B734 and A D628 are two edges of a parallelogram. Draw the front, top, and left-side views of the parallelogram.

**7.** Draw the front, top, and left-side views of the parallelogram A545 B618 C707 D.

**8.** Draw the front, top, and left-side views of the parallelogram C827 D706 E325 F.

## Group 20. Drawing Parallel Line Objects.

**1.** By means of straight lines, join the middle points of the adjacent sides of the warped quadrilateral A408 B534 C728 D815. Draw three views. What is the resulting figure?

**2.** Show the front, top, and side views of the warped quadrilateral A015 B148 C217 D436. By means of straight lines, join the middle points of the adjacent sides of the quadrilateral. What is the resulting figure?

**3.** Draw the front, top, and right-side views of the warped quadrilateral A115 B145 C148 D307. Using straight lines, connect the middle points of the adjacent sides of the quadrilateral. What is the resulting figure?

**4.** Draw the front, top, and side views of the warped quadrilateral A508 B534 C805 D645. By means of straight lines, join the middle points of the adjacent sides. What is the resulting figure?

**5.** Draw the front, top, and left-side views of the warped quadrilateral C606 D636 E638 F425. Using straight lines, connect the middle points of the adjacent sides. What is the resulting figure?

**6.** By means of straight lines, join the middle points of the adjacent sides of the warped quadrilateral A014 B708 C834 D127. What is the resulting figure?

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**7.** A506 B624 C815 is the triangular base of a prism. The sides of the prism are $2\frac{1}{4}''$ long and are parallel to the line O104 P045. Draw the front, top, and side views of the prism.

**8.** The triangle A524 B715 C8,3,$4\frac{1}{2}$ is the front end of a prism. The three parallel edges of the prism are $2\frac{3}{4}''$ long and are parallel to the line K424 L237. Draw the front, top, and left-side views of the prism.

**9.** The point A is $1''$ in front of, $2''$ to the left of, and $\frac{1}{2}''$ above B835. The point D is $\frac{1}{2}''$ behind, $1''$ to the left of, and $2\frac{1}{2}''$ below A. The point H is $1''$ to the left of, $2\frac{1}{2}''$ behind, and $\frac{3}{4}''$ above D. The lines A B, A D, and D H are three edges of a parallelopiped. Draw its front, top, and side views.

**10.** The point B is $1\frac{1}{2}''$ to the right of, $\frac{1}{2}''$ behind, and $1''$ above the point A104. The point D is $\frac{1}{2}''$ behind, $\frac{3}{4}''$ to the left of, and $1\frac{1}{2}''$ above A. The point E is $2\frac{1}{2}''$ behind, $\frac{3}{4}''$ to the right of, and $2''$ above A. The lines A B, A D, and A E are three edges of a parallelopiped. Draw its front, top, and side views.

**11.** A014 B205 C3,$1\frac{1}{2}$,$3\frac{3}{4}$ is the base of a triangular prism. Its parallel sides are $3''$ long, and parallel to the line O515 P638. Show the front, top, and side views of the prism.

**12.** Three intersecting edges of an equilateral parallelopiped lie on the lines A108 E025, A F425, and A G347, and are $2\frac{1}{4}''$ long. Draw the front, top, and right-side views of the parallelopiped.

**13.** The lines A634 B8,$2\frac{1}{2}$,5, A D5,$\frac{1}{2}$,$4\frac{1}{2}$, and D H4,$1\frac{1}{4}$,7 are the edges of a parallelopiped. Complete the front, top, and left-side views. Locate the middle point of each face, and join each of these points with the middle points of the adjacent faces. Draw the principal views of the resulting figure.

**14.** A116 D337 is the diagonal of a vertical hexagon. This hexagon is one base of an oblique prism. The parallel edges of the prism are $1\frac{1}{2}''$ long, and P535 is a point on its axis. Draw the front, top, and right-side views of the prism.

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## Group 21. Are the Given Lines Perpendicular?

**1.** Draw as required data all of the following groups in which the lines are perpendicular: A167 B358 and C248 D336, E557 F757 and G668 H6,$4\frac{1}{2}$,$6\frac{1}{2}$, K2,$\frac{1}{2}$,2 L404 and M114 N312, O623 P603 and Q712 R714.

**2.** Draw as required data all of the following groups in which the lines are perpendicular: A038 B256 and C156 D338, E638 F856 and K545 L848, O424 P415 and R305 S327.

**3.** Draw as required data all of the following groups in which the lines are perpendicular: O135 P436 and C318 D445, K527 L845 and A645 B818.

**4.** Draw as required data all of the following groups in which the lines are perpendicular: A126 B435 and C235 D308, E557 F857 and M646 N668, K513 L712 and P612 O714.

**5.** Draw as required data all pairs of the following lines that are perpendicular to each other: A118 B335, C636 D835, E346 F618, G505 H807, K016 P046.

**6.** Draw as required data all pairs of the following lines that are perpendicular to each other: C136 D618, E408 F653, and K224 S726.

## Group 22. Horizontal and Frontal Lines Perpendicular to Oblique Lines.

**1.** Draw O P the frontal perpendicular bisector of the line A145 B408, and K L the horizontal perpendicular bisector of the line B C724.

**2.** Draw the frontal line C00X D2XX that is perpendicular to and intersects the line A134 B418 at a point O. Draw also the horizontal line K71X L5XX that is perpendicular to and intersects the line E507 F845 at a point P.

**3.** Draw the frontal line A11X B3XX that is perpendicular to and intersects the line C038 D315 at O. Draw also the horizontal line E41X F6XX that is perpendicular to and intersects the line J505 K747 at P.

**4.** Draw a horizontal line A C perpendicular to the line A145 B317, and a frontal line B D perpendicular to the line A B. Also, through the point O72X draw a frontal line O P perpendicular to the line E518 F634, and a horizontal line O Q perpendicular to the line E F.

**5.** Draw the horizontal line C0X7 D3XX and the frontal line E0XX F3X6 perpendicular to and intersecting the line A148 B205. Draw the horizontal lines L5XX M6X5 and O4X6 P5XX perpendicular to and intersecting the line J435 K827.

**6.** Draw the lines C4X7 B718 and D6X5 A835 perpendicular to the line A B. Also draw the horizontal line O Q and the frontal line O P perpendicular to the line E127 F434 at its middle point.

## Group 23. Profile Lines Perpendicular to Oblique Lines.

**1.** Draw the line A034 B that intersects and is perpendicular to the profile line O207 P245 at B.

**2.** Draw the line K418 L2XX perpendicular to and intersecting the profile line E205 F247.

**3.** Through P412 draw the line that is perpendicular to the profile line E708 F745 and intersects it at Q.

**4.** Draw the line A446 B perpendicular to and intersecting the profile line C638 D614 at B.

**5.** Draw the profile line AX24 B perpendicular to and intersecting the line D145 E308 at B.

**6.** Perpendicular to the line A145 B437 and intersecting it at P, draw the profile line OX15 P.

**7.** Draw the profile line AX36 B perpendicular to and intersecting the line C828 D415 at B.

**8.** Draw the profile line CX46 D perpendicular to and intersecting the line E304 F028 at D.

## Group 24. Drawing Oblique Lines Perpendicular.

**1.** Through C135 draw the line that is perpendicular to the line K436 L708 and intersects it at D.

**2.** Through the point O445 draw the line that intersects and is perpendicular to the line J735 K318 at P.

**3.** Perpendicular to and intersecting the line A448 B806, draw the line O137 K.

**4.** Through C117 draw the line that is perpendicular to and intersects the line E134 F408 at a point O.

**5.** Through a point P on the line C116 D424 draw the line P O747 perpendicular to the line C D.

**6.** Through the point O6,4½,7 draw the line that is perpendicular to the line S5,2½,8 T704 and intersects it at a point P.

**7.** Draw the line P8X3 K714 perpendicular to the line K L538.

**8.** Draw the line G20X F426 perpendicular to the line E138 F.

**9.** Draw a line through O44X perpendicular to the line C735 D417 and intersecting it at C.

**10.** Draw the line E1X8 B436 perpendicular to the line B C718.

## Group 25. Figures Having Right Angles.

**1.** A524 B118 is the hypotenuse of a right-angle triangle. The third corner of the triangle is on the line B D706. Draw the front and top views of the triangle.

**2.** E745 F528 is the hypotenuse of a right-angle triangle. One leg is in the direction E745 K228. Show the top, front, and side views of the triangle.

**3.** A734 B508 is one leg of a right-angle triangle. The hypotenuse is along the line A734 D127. Draw the front and top views of the triangle.

**4.** A136 B528 is one leg of a right triangle. The hypotenuse is along the line A D705. Draw the right triangle A B C.

**5.** Draw the front, top, and left-side views of the rectangle C607 D715 E64X F.

**6.** Draw the front and top views of the rectangle A726 B318 C2X7 D.

**7.** A618 C735 is the diagonal of a square. E54X is a point on the second diagonal. Draw the front, top, and left-side views of the square.

**8.** A116 C437 is the diagonal of a square. E3X6 is a point on the second diagonal. Draw the front, top, and right-side views of the square.

## Group 26. Drawing One Line Perpendicular to Another and Intersecting a Third Line.

**1.** Through the point D624 draw a line that is perpendicular to the line C217 D, and intersects the line A857 B335 at a point E.

**2.** Through the point C draw a line that is perpendicular to the line C518 D847, and intersects the line A125 B348 at a point E.

**3.** A certain perpendicular bisector of the line C324 D705 passes through a point P on the line E747 F136. Draw the front and top views of the bisector.

**4.** A118 is one end of the hypotenuse of a right-angle triangle, and the other end is on the line E537 F206. One vertex is at B635. Draw the front and top views of the triangle.

**5.** A825 B517 is the base of an isosceles triangle having its vertex on the line C654 D226. Draw the top and front views of the triangle.

**6.** E308 F024 is the base of an isosceles triangle. Its vertex is on the line A317 B145. Show two views of the triangle. Draw its altitude.

## Group 27. Finding the Distance from a Point to a Line.

When a specified distance is required in a problem, the line representing the distance is to be drawn as required data in each view, and the true length of this line is to be found and shown as required data.

**1.** Find the distance from the point C707 to the line K524 L447.

**2.** Find the distance from the point P228 to the line E407 F114.

**3.** Find the distance from the point P047 to the line J218 K235.

**4.** Find the distance from the point T225 to the line R148 S406.

**5.** Which is farther from O308, the line A027 B335 or the line C446 D718, and how much farther?

**6.** To which side of the triangle D227 E608 F434 is the point P426 nearest?

**7.** Find the actual distance between the parallel lines A136 B508 and C434 D806.

**8.** Find the distance between the lines E134 F318 and K525 L617.

**9.** C325 D134 is the base and V408 is the vertex of a triangle. Find the true length of the altitude of the triangle.

**10.** V047 is the vertex and B205 C416 is the base of a triangle. Find the true length of its altitude.

**11.** V646 is the vertex and A114 B238 is the base of a triangle. Draw the triangle, and find the distance from the vertex to the base.

**12.** A548 is the vertex and B415 C704 is the base of a triangle. Draw the altitude of the triangle and find its true length.

**13.** Find the true length of the altitude of the triangle V348 E034 F225, considering V as the vertex.

**14.** A6,3$\frac{1}{2}$,8 is the vertex and B4,$\frac{3}{4}$,7 C534 is the base of a triangle. Draw its altitude and find its true lengeth.

**15.** Determine which altitude of the triangle A418 B545 C226 is the longest.

**16.** Determine which altitude of the triangle D227 E518 F345 is the longest.

## Group 28. Drawing One Line Perpendicular to Two Lines.

**1.** Draw the line A X perpendicular to the lines A306 B114 and A C535.

**2.** Draw the line A D perpendicular to the lines A514 B128 and A C245.

**3.** Show the front, top, and left-side views of the line V617 O5XX that is perpendicular to the lines V C405 and V D846.

**4.** Draw the line A C perpendicular to the lines A305 B117 and A D416.

**5.** Draw the line O P perpendicular to and intersecting the lines A247 B606 and C638 D215.

**6.** Draw the lines A145 B328, B C4X6, and B D perpendicular to each other.

**7.** Draw the lines A815 B627, B C40X, and C D2XX perpendicular to each other.

**8.** Draw the lines C036 D204, D E4X5, and E F4½,X,X perpendicular to each other.

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**9.** Draw the line O436 P that is perpendicular to but does not intersect the lines G126 H518 and D334 E617.

**10.** Draw the line A435 BX0X perpendicular to but not intersecting the lines C618 D725 and E026 F307.

**11.** Draw the line O426 P perpendicular to but not intersecting the lines D717 E505 and E F246.

**12.** Draw the line P537 O perpendicular to but not intersecting the lines A125 B207 and B C618.

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## Group 29. Objects Having Edges or Axes at Right Angles.

**1.** Prove by means of a drawing whether or not the diagonals of a cube are mutually perpendicular.

**2.** The triangle A534 B7,4,4½ C4,3½,5 is the base of a right prism of 3¼″ altitude. Show the front, top, and left-side views of the prism.

**3.** Show the front, top, and right-side views of a right prism of ½″ altitude that has for its base the triangle A027 B3,1¾,8 C4,3,6½.

**4.** A016 B204 is one edge of a rectangle. The adjacent edge is horizontal and 1″ long. This rectangle is the base of a right prism. The third edge of the prism is 2½″ long. Show the front, top, and right-side views of the prism.

**5.** The rectangle A127 B418 C5X7 D is one face of a rectangular solid of ¾″ altitude. Draw the front and top views of the solid.

**6.** The rectangle A437 B618 C7X5 D is the common base of two right pyramids of 2″ altitude. Draw the front, top, and left-side views of the twin pyramids.

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**7.** The diagonals of an octahedron are equal in length and each is a perpendicular bisector of the other two. A517 B745 is the diagonal of an octahedron. S31X is a point on the second diagonal. Draw the three diagonals, locate the corners of the octahedron, and draw the front, top, and side views of the octahedron.

**8.** The three diagonals of an octahedron are equal in length and each is a perpendicular bisector of the other two. A517 B634 is the diagonal of an octahedron. K4X6 is a point on the second diagonal. Draw the front, top, and side views of the octahedron.

**9.** The diagonals of an octahedron are equal in length and each is a perpendicular bisector of the other two. A3,3,7½ B514 is the diagonal of an octahedron. K61X is a point on the second diagonal. Draw the front and top views of the octahedron.

**10.** The diagonals of an octahedron are equal in length and each is a perpendicular bisector of the other two. A735 B427 is the diagonal of an octahedron. K6,2½,X is a point on the second diagonal. Draw the front, top, and side views of the octahedron.

## Group 30. Common Perpendicular to Two Lines.

**1.** Find O P, the shortest distance from the line C527 D706 to the line E645 F738.

**2.** Locate O P, the common perpendicular to the lines A136 B417 and C117 D425.

**3.** Draw A B, the common perpendicular to the lines C447 D728 and E507 F826.

**4.** Locate the shortest line O P that intersects the lines A125 B417 and C113 D436.

**5.** Find the line C P that is perpendicular to and intersects each of the lines E115 F248 and J307 K424.

**6.** Find O P, the common perpendicular to the lines E506 F718 and G637 K714. Check the elevations of the points O and P in the front view with the elevations found in the auxiliary view.

**7.** Find the shortest distance from the line E125 F338 to the line J128 K405.

**8.** Determine the shortest distance from the line C525 D708 to the line E628 F7,3½,4.

**9.** Locate the line C P that intersects and is perpendicular to each of the lines J104 K336 and M007 N318.

**10.** Locate O P, the common perpendicular to the two lines D507 E614 and F734 G808.

## Group 31. Straight-line Questions.

**1.** In what views is it possible to tell whether or not a line is horizontal?

**2.** Can a line in the rear face of an object be called frontal?

**3.** What views, and how many, is it necessary to observe to determine whether or not a line is a profile line?

**4.** Is it possible to determine from any single view whether or not a line is vertical?

**5.** In what views are all points of a horizontal-profile line equidistant from the observer?

**6.** Is it possible to tell from any single view whether or not a line is oblique?

**7.** Does a vertical line ever lean?

**8.** Given a frontal line, in what views would all points of the line be equidistant from the observer?

**9.** Is a profile line located in the right face or the left face of an object?

**10.** In what view are all points of a horizontal line equidistant from the observer?

**11.** In what views are all points of a profile line equidistant from the observer?

**12.** In what view can it be determined whether or not a vertical line and an oblique line are perpendicular?

**13.** What views show the true length of a vertical line?

**14.** In what view may it be determined whether or not two oblique lines are perpendicular?

**15.** How is a frontal line limited in direction?

**16.** Is it possible to tell from any single view whether or not a line is horizontal-profile?

**17.** In what view is it possible to tell whether an oblique line and a profile line are perpendicular?

**18.** In what views may it be determined whether a line is frontal?

**19.** In what view may it be determined whether or not an oblique line and a horizontal line are perpendicular?

**20.** In what principal views are all points of an oblique line equidistant from the observer?

**21.** In what views will two perpendicular profile lines appear perpendicular?

**22.** In what view may the true length of a horizontal line be seen?

**23.** In what views will two non-parallel horizontal lines appear parallel?

**24.** Can two non-intersecting lines be perpendicular?

**25.** In what view is the true length of a frontal line shown?

**26.** Are all horizontal lines perpendicular to all vertical lines?

**27.** How is the true length of an oblique line determined?

**28.** What is the distance from a point to a line?

**29.** How may the location of a straight line be specified?

**30.** In what views is it possible to tell whether or not two profile lines are parallel?

**31.** If two oblique lines appear perpendicular in a principal view, are the lines perpendicular?

**32.** In what view does a straight line appear longer than it really is?

**33.** How is a straight line generated?

# PROBLEMS—CHAPTER VII

## CURVED LINES

In the problems of this chapter, the curves should be determined by plotting a large number of positions of the generating point. The method used in plotting the points should in each case be developed from the law which governs the movement of the generating point. The points on an ellipse, for example, should be plotted by means of one or two compasses, and not by means of a trammel or string, except in the few problems where some such method is specified. Plot a sufficient number of points to secure a smooth, accurate curve. Approximate methods of drawing curves should not be used. The size of a circle is specified by stating its diameter.

### Group 32. The Circle.

**1.** On the center C326, draw the front, top, and right-side views of a 2″ frontal circle, a 2″ profile circle, and a 2″ horizontal circle.

**2.** On the center O526, draw the front, top, and left-side views of a 3″ profile circle, a 3″ frontal circle, and a 3″ horizontal circle.

**3.** A $2\frac{1}{2}''$ profile circle is tipped to the right until its axis makes an angle of 30° with the horizontal. Draw the front, top, right-side, and normal views of the circle and its axis in the new position.

**4.** The front edge of a $2\frac{1}{2}''$ horizontal circle is raised until the plane of the circle makes 30° with the horizontal. Draw the front, top, left-side, and normal views of the circle in its new position.

**5.** The rear edge of a 2″ horizontal circle is raised $\frac{3}{4}''$. Show the circle in its new position by drawing its front, top, right-side, and normal views.

**6.** A $2\frac{1}{4}''$ frontal circle is tipped by moving its lower edge 1″ to the rear. Show the circle in its new position by drawing the front, top, left-side, and normal views.

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**7.** A135 B427 is the axis of a $2\frac{1}{2}''$ circle having its center at the middle point of A B. Draw the front, top, right-side, edge, and normal views of the circle and its axis.

**8.** C626 is the center of a 3″ circle. The point A745 is equidistant from all points on its circumference. Draw the front, top, edge, and normal views of the circle.

**9.** Draw the front, top, edge, and normal views of a $2\frac{1}{2}''$ circle with center at C426, and with the axis passing through the point K318.

**10.** C436 D604 is the axis of a 2″ and a 1¼″ circle with a common center at C. Draw the front, top, edge, and normal views of the circles.

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**11.** C626 is the center, A518 is a point on the axis, and P536 is a point on the circumference of a circle. Draw the front, top, edge, and normal views of the circle.

**12.** P2X7 is a point on the circumference, A438 is a point on the axis, and C326 is the center of a circle. Draw the front, top, edge, and normal views of this circle.

**13.** B105 is a point on the axis, O226 is the center, and P2½,2½,X is a point on the circumference of a circle. Draw the front, top, edge, and normal views of the circle.

**14.** Draw the front, top, edge, and normal views of the circle of which P425 is the center, K608 is a point on the axis, and D3½,X,6 is a point on the circumference.

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**15.** D745 E617 is the axis of a circle, and P7½,2,6 is a point on its circumference. Draw the front, top, edge, and normal views of the circle.

**16.** A105 X348 is the axis, and P327 is a point on the circumference of a circle. Draw the front, top, edge, and normal views of the circle.

**17.** A447 is a point on the circumference of a circle, and J235 K627 is its axis. Draw the front, top, edge, and normal views of the circle.

**18.** Draw the front, top, edge, and normal views of the circle of which C024 D418 is the axis and A227 is a point on the circumference.

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## Group 33. The Ellipse.

**1.** A136 B636 is the major axis of a frontal ellipse. The distance between the foci is 4½″. Draw the front, top, and right-side views of this ellipse.

**2.** C236 D238 is the minor axis of a profile ellipse. The major axis is 4″ long. Draw the front, top, and right-side views of the ellipse.

**3.** O125 and P725 are the foci of an horizontal ellipse. The minor axis is 2″ long. Draw the front and top views of this ellipse.

**4.** Draw the front and top views of an ellipse, of which P556 is a point on the curve, and of which one focus is at E236 and the other at F636.

**5.** One focus of an ellipse is at K617 and the other focus is at L657. The ellipse passes through the point A648. Draw its front and left-side views.

**6.** A135 B427 is the major axis of a vertical ellipse. The distance between the foci is 3″. Draw the front, top, right-side, and normal views of the ellipse.

**7.** A146 B316 is the major axis of an ellipse. The horizontal minor axis is 1½″ long. Draw the front, top, right-side, and normal views of the ellipse.

**8.** The two foci of an ellipse are at J215 and K236, respectively. The horizontal minor axis of the ellipse is 1¼″ long. Draw the front, top, right-side, and normal views of the ellipse.

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**9.** A126 B726 is the major axis of an ellipse, and P4,3½,6 is a point on the ellipse. By means of a trammel, locate points on the curve and then draw the front and top views of the ellipse.

**10.** The major axis of an ellipse is A412 B417. The horizontal minor axis is 3″ long. By means of a trammel, locate points on the curve. Draw the front and top views of the ellipse.

## Group 34. The Parabola.

**1.** F548 is the focus, and A678 B608 is the directrix of a parabola. Draw the front and top views of the parabola, and draw a tangent at the point P3XX on the upper part of the curve.

**2.** C012 D018 is the directrix, P317 is a point on the curve, and X715 is a point on the axis of a parabola. Draw the front and top views of the parabola, and draw a tangent to the curve at the point P.

**3.** B018 C818 is the directrix, and D428, E438, and F448 are the respective foci of three parabolas. Draw the front view of these parabolas.

**4.** J702 K708 is the directrix, and its middle point is on the axis of six horizontal parabolas. The respective foci are 1″, 2″, 3″, 4″, 5″, and 6″, from the directrix. Draw the top views of the parabolas.

**5.** F338 is the focus and V538 is the vertex of a vertical parabola. Draw its front and top views, and draw a tangent to the curve at the point P3XX on the upper part of the curve.

**6.** P205 and O605 are points on a parabola, and B008 C808 is its directrix. Draw the top view of this parabola and draw a tangent to the curve at the point O.

**7.** F2½,1,6 is the focus, and A114 B438 is the directrix of a parabola. Draw the front, top, and normal views of the parabola.

**8.** F625 is the focus, and C408 D846 is the directrix of a parabola. Draw the front, top, and normal views of the parabola.

**9.** F6½,2,6½ is the focus, and A628 B745 is the directrix of a parabola. Draw the front, top, and normal views.

**10.** C145 D207 is the directrix, and F1½,2,6½ is the focus of a parabola. Draw the front, top, and normal views.

## Group 35. The Hyperbola.

**1.** E348 and F548 are the foci of a frontal hyperbola. The constant difference is 1½″. Draw the front and top views of both branches of the hyperbola, and draw a tangent at the point P6XX on the lower part of the curve.

**2.** The foci of a frontal hyperbola are at A448 and B548. The constant difference is ¾″. Draw the front and top views of both branches of the curve, and draw a tangent to the curve at the point P2XX on the upper part of the curve.

**3.** E404 and F405 are the foci of an hyperbola. P607 is a point on the hyperbola. Draw the front and top views of both branches, and draw a tangent to the curve at P.

**4.** P707 is a point on an hyperbola, and the foci are A204 and B604. Draw both branches of the hyperbola, and draw a tangent to the curve at the point P.

**5.** The foci of a vertical hyperbola are E226 and F327. The common difference is 1″. Draw the front, top, and normal views of the hyperbola.

**6.** O713 is a point on an hyperbola, and the foci are A527 and B635. Draw the front, top, and normal views of the curve.

**7.** J726 and K735 are the foci of an hyperbola, and P705 is a point on the curve. Draw the front, top, and normal views of the hyperbola, and draw a tangent to the curve at P.

**8.** P108 is a point on an hyperbola, and the foci are at B145 and C126. Draw the front, top, and right-side views of the hyperbola, and draw a tangent to the curve at P.

## Group 36. The Archimedean Spiral.

**1.** The generating point of a right-hand horizontal Archimedean spiral starts at the pole P314, and, after revolving 90° around the pole, is at A313. Draw the top view of one and one-quarter turns of the spiral.

**2.** P548 is the pole of an Archimedean spiral, and A5,$3\frac{1}{2}$,8 and B648 are points on the curve. Draw two turns of the normal view of the curve.

**3.** The pole of an Archimedean spiral is at P526, and the lead is $\frac{1}{2}''$. The spiral is in a plane perpendicular to the line P X627, and appears left-handed when viewed in the direction P X. Draw the front, top, and normal views of two and one-half turns of the spiral.

**4.** P526 is the pole of an Archimedean spiral, and the lead is 1″. The spiral is perpendicular to the line P K716, and appears as a right-hand spiral when viewed in the direction P K. Draw the front, top, and normal views of two and one-half turns of the spiral.

## Group 37. The Involute.

**1.** C448 is the center of a $1\frac{1}{2}''$ frontal circle. Draw the front view of three-fourths of a turn of a right-hand and of a left-hand involute of this circle. The top point of the circle is to be taken as a point on each involute.

**2.** C$3\frac{1}{4}$,0,5 is the center of a horizontal 1″ circle. Draw the top view of a left-hand involute of this circle. Start the involute at the front point of the circle, and draw as many turns as possible within the problem space.

**3.** C536 is the center and C A737 is the axis of a $\frac{3}{4}''$ circle. Draw one turn of the involute of this circle in the front, top, and normal views. The involute is seen as right-hand when viewed in the direction C A, and starts at the top point of the circle.

**4.** C336 is the center, and C D026 is the axis of a $\frac{7}{8}''$ circle. Draw the front, top, and normal views of three-fourths of a turn of the involute of this circle. The curve appears as a right-hand involute when viewed in the direction C D. The rear point of the circle is the starting point.

## Group 38. The Cycloid.

**1.** C158 is the center of a 2″ frontal circle. Draw the front view of the cycloid that is generated by the lowest point of this circle when the circle rolls on a horizontal line.

**2.** C458 is the center of a $1\frac{1}{2}$″ frontal circle. Show the front view of the complete cycloid that is generated by the top point of this circle when it rolls on a horizontal line.

**3.** Show the front view of a complete cycloid that is generated when a 3″ frontal circle rolls on the upper side of the line A008 B888.

**4.** The line A107 B701 is the director of a $2\frac{1}{2}$″ horizontal circle that rolls on the rear side of this line. Draw the normal view of the cycloid that is generated by a fixed point on this circle.

## Group 39. Epicycloids and Hypocycloids.

**1.** C404 is the center of a 4″ horizontal director circle. Show the epicycloids and hypocycloids that are generated by a 2″ circle that rolls entirely around the director circle. Both curves should start at the rear point of the director circle.

**2.** C408 is the center of a 12″ frontal director circle. Symmetrically disposed about a vertical line, show an epicycloid and a hypocycloid generated by a 2″ rolling circle.

**3.** C448 is the center of a 3″ frontal director circle. 'Draw the front view of the epicycloids and the hypocycloids that are generated by a 1″ circle rolling entirely around the director circle.

**4.** C448 is the center of a 3″ frontal director circle. Symmetrically disposed about a vertical line draw an epicycloid and a hypocycloid generated by a 2″ rolling circle.

## Group 40. Double-curved Lines.

**1.** Draw the front and top views of a smooth, double-curved line that terminates in the points A125 and B417, and then draw the side view.

**2.** Draw the front and top views of a double-curved line that terminates in the points A737 and C416, and passes through the point B645, and then draw its side view.

**3.** Draw the front and top views of a double-curved line that terminates in the points C018 and E347, and passes through the point D125, and then draw its side view.

**4.** Draw the front and top views of a double-curved line that terminates in the points E415 and G707, and passes through the point F546, and then draw its side view.

## Group 41. The Helix.

**1.** A406 B446 is the axis of a right-hand helix of 4″ diameter and 1″ lead. Show the front and top views of four turns of this helix on a cylinder.

**2.** C026 D526 is the axis of a left-hand helix of $2\frac{1}{4}''$ diameter and $2\frac{1}{2}''$ lead. Show the front, top, and end views of two turns of this helix on a cylinder.

**3.** E467 F407 is the axis of a $1\frac{1}{2}''$ right circular cylinder. Show the front and top views of the cylinder, and on the cylinder show three turns of a left-hand helix of 2″ lead.

**4.** C405 D425 is the axis of a 5″ right circular cylinder. On the front and top views of this cylinder show four turns of a right-hand helix of $\frac{1}{2}''$ lead.

## Group 42. The Conical Helix.

**1.** V446 is the vertex, and C406 is the center of the 4″ base of a right circular cone. Draw the front and top views of the cone, and on the cone show two turns of a right-hand conical helix of 2″ lead.

**2.** Show the front and top views of four turns of a left-hand conical helix of 1″ lead, that is on a right circular cone with its vertex at V424, and the center of its 3″ base at C428.

**3.** A right-hand conical helix of 1″ lead is wound around a right circular cone of which V425 is the vertex and C405 is the center of its 6″ base. Show the front and top views of the cone, and of two turns of the helix.

**4.** V412 is the vertex, and C418 is the center of the 2″ base of a right circular cone. Around this cone are wound four turns of a right-hand helix of $1\frac{1}{2}'$ lead. Draw the front and top views of the cone and helix.

# PROBLEMS—CHAPTER VIII

## PLANES

### Group 43. Points and Lines in Planes.

**1.** Locate the front, top, and right-side views of the points A1X7 and B21X in the plane Q016 R338 S425.

**2.** Draw the front, top, and left-side views of the points C82X and D6X6 of the plane R535 S648 T804.

**3.** Locate the front and top views of the line O23X P6X6 of the plane A307 B528 C445.

**4.** Draw the front and top views of the line A8X8 B02X of the plane D206 E445 F637. Check the solution.

**5.** Locate the line E72X F2X7 in the plane J116 K434 L708. Check.

**6.** Show the front, top, and left-side views of the line E53X F84X of the plane R518 S636 T727.

**7.** In the front, top, and right-side views of the plane K015 L208 M346 show the frontal line M C and the horizontal line K D.

**8.** In the plane Q524 R618 S835 draw the front, top, and left-side views of the frontal line S X and of the horizontal line Q Y.

**9.** Show the front and top views of the horizontal line E F, and of the frontal line E G of the plane C346 D435 B518.

**10.** Draw the front, top, and left-side views of the frontal line B O, and of the horizontal line A E of the plane A407 B656 C828.

**11.** Show the front, top, and left-side views of the horizontal line S B, and of the frontal line R A of the plane R505 S628 T746.

**12.** Draw the front, top, and right-side views of the horizontal line T C, and of the profile line S D of the plane Q028 S216 T434.

**13.** Show the front and top views of the triangle J23X K4X7 L52X in the plane D045 E408 F824. Check the result.

**14.** Draw the front, top, and left-side views of the triangle C52X D63X E70X in the plane Q427 R644 S816. Check.

**15.** Draw the front and top views of the triangle A0X5 B3X7 C8X6 in the plane R137 S524 T708. Check the solution.

**16.** In the front, top, and right-side views, show the plane R02X S23X T41X that contains the triangle A135 B207 C346. Check.

### Group 44. Plane Figures.

**1.** A517 C726 is the diagonal of a square that lies in a vertical plane. Draw the front, top, left-side, and normal views of the square.

**2.** The front view is the edge view of a square. A134 B215 is the front edge of the square. Draw the front, top, right-side, and normal views of the square.

**3.** B128 D315 is the diagonal of a square. The edge view of the square is seen in the side view. Draw the front, top, right-side, and normal views of the square.

**4.** B236 E317 is the diagonal of a hexagon that lies in a plane of which the front view is an edge view. Draw the front, top, right-side, and normal views of the hexagon.

**5.** C727 F415 is the diagonal of a hexagon. The side view is an edge view of the plane of the hexagon. Show the front, top, left-side, and normal views of the hexagon.

**6.** A527 D735 is the diagonal of a hexagon that lies in a vertical plane. Show the front, top, left-side, and normal views of the hexagon.

**7.** C516 D735 is the diameter of a circle, the edge view of which is seen in the side view. Draw the front, top, left-side, and normal views of this circle.

**8.** One diameter of a circle is E145 F216. The edge view of this circle is seen in the front view. Draw the front, top, and normal views of this circle.

**9.** In a normal view show the true shape of the triangle B534 C617 D826.

**10.** Draw the front, top, right-side, edge, and normal views of the triangle D114 E223 F306.

**11.** In a normal view show the true shape of the triangle J137 K216 L425.

**12.** A234 C408 is the diagonal of a square that lies in the plane A C P615. Draw the front and top views of the square.

**13.** A square having A C for its diagonal lies in the plane A317 C234 K545. Draw the front and top views of the square.

**14.** The line D627 F435 of the plane D F A806 is the diagonal of a square that is in the plane. Draw the front, top, edge, and normal views of the square.

**15.** Draw the front, top, left-side, edge, and normal views of the hexagon of which A626 D734 is the diagonal, and K808 is a point in the plane of the hexagon.

**16.** A735 D427 is the diagonal of a hexagon. K126 is a point in the plane of the hexagon. Draw the front and top views of the hexagon.

**17.** P237 is a point in the plane of a hexagon, and A125 D314 is one of its diagonals. Draw the front, top, right-side, edge, and normal views of the hexagon.

**18.** Show the front and top views of the circle that passes through the points D415 E327 and F1,1½,6.

**19.** Draw the front, top, right-side, edge, and normal views of the circle that is inscribed in the triangle P0,1,4½ Q2,0,6½ R3,3,3½.

**20.** Draw the front, top, edge, and normal views of the circle that is inscribed in, and of the circle that is circumscribed about the triangle C535 D617 E736.

**21.** The point C225 is the center of a 3″ circle that lies in the plane C D337 E415. Draw the front, top, edge, and normal views of the circle.

**22.** In the plane A415 C526 B838 is a 2½″ circle, with its center at C. Draw the front, top, edge, and normal views of this circle.

## Group 45. Parallel Planes.

**1.** Through the point P226 pass the plane P Q R parallel to the plane J526 K708 L845.

**2.** Through the point A108 pass the plane A B C parallel to the plane Q517 R655 S726.

**3.** By means of a horizontal line and a frontal line, represent the plane C208 D E that is parallel to the plane Q527 R646 S704.

**4.** Represent by means of a frontal line and a horizontal line, the plane O526 P Q that is parallel to the plane A028 B234 C317.

**5.** Draw the plane D227 E42X F4X7 parallel to the plane R505 S638 T824.

**6.** Parallel to the plane D125 E247 F306 draw the plane A537 B6X7 C43X.

**7.** Pass the plane B137 C2X6 D3X8 parallel to the plane R426 S645 T807.

**8.** Parallel to the plane J116 K248 L305 draw the plane D516 E43X F73X.

**9.** Pass the plane J205 K33X L1X6 parallel to the plane C424 D615 E838.

**10.** Parallel to the plane E136 F205 G347 pass the plane P717 Q4X6 R5X8.

## Group 46. Line Perpendicular to a Plane.

**1.** Draw the line A418 B perpendicular to the plane Q235 R614 S727.

**2.** Draw the front, top, and right-side views of the line B448 C3XX that is perpendicular to the plane R018 S335 T507.

**3.** Show the front, top, and left-side views of the line K645 P that is perpendicular to the plane J416 K L707.

**4.** Perpendicular to the plane A007 B346 C825 draw the line O638 P5XX.

**5.** Draw the front, top, and right-side views of the line K317 L0XX that is perpendicular to the plane D006 E147 F235.

**6.** Show the front, top, and left-side views of the line C635 D4XX that is perpendicular to the plane J424 K608 L737.

**7.** Draw the line E736 F71X perpendicular to the plane A814 B638 C426.

**8.** Perpendicular to the plane A005 B247 C326 draw the line C D33X.

**9.** Through A236 draw the line that is perpendicular to the plane C117 D425 E736.

**10.** Draw the line E446 G perpendicular to the plane D224 E F608.

**11.** The triangle A414 B607 C826 is the lower base of a right prism of $1\frac{1}{2}''$ altitude. Draw the front, top, and left-side views of the prism.

**12.** J105 K336 L524 is the base of a pyramid. The altitude of the pyramid is $2''$, and is perpendicular to the base at the point K. The vertex is to the left of K. Draw the front and top views of the pyramid.

**13.** The line C F is the diagonal of a hexagon that lies in the plane C1,2,$4\frac{1}{2}$ F316 K4,3,$3\frac{1}{2}$. This hexagon is the lower base of a right prism of $1''$ altitude. Draw the front, top, right-side, auxiliary, and end views of the prism.

**14.** P807 is a point in the plane of a hexagon, and B617 E7,$3\frac{1}{2}$,5 is one of its diagonals. This hexagon is the base of a right pyramid of $1\frac{1}{2}''$ altitude. The vertex is in front of the base. Draw the front, top, left-side, auxiliary, and end views of the pyramid.

**15.** A hexagon lies in the plane A227 D335 J407, and A D is one of its diagonals. This hexagon is the lower base of a right prism of $\frac{3}{4}''$ altitude. Show the front, top, auxiliary, and end views of the prism.

**16.** C016 D228 is one edge of a square. Another corner of the square is $1''$ below the corner C. This square is the base of a right pyramid of $1\frac{1}{2}''$ altitude. The vertex is above the base. Draw the front and top views of the pyramid and also any other necessary views. Four solutions are possible, but only one can be drawn within the limits of the problem space.

**17.** A147 B505 is one edge of the square A B C D. The corner D is to the right of, and $1''$ above A. The square is the lower base of a cube. Draw the front, top, and any other necessary views of the cube.

**18.** A125 C337 is the diagonal of the square A B C D. The point B is $\frac{3}{4}''$ above C, and to the right of the center point of the square. This square is the base of a right pyramid of $1\frac{1}{2}''$ altitude. The vertex is to the left of the center point of the base. Draw the front and top views and any other necessary views.

## Group 47. Plane Perpendicular to a Line.

**1.** Pass the plane Q437 R2XX S6XX perpendicular to the line A245 B507.

**2.** Through the point P225 draw the plane P Q R perpendicular to the line A318 B735.

**3.** Draw the plane A725 B C perpendicular to the line E225 D537.

**4.** Pass the plane K536 L3XX M4XX perpendicular to the line E134 F408.

**5.** Pass the plane A237 B11X C40X perpendicular to the line J125 K327, and pass the plane D845 E7X8 F5X6 perpendicular to the line O646 P716.

**6.** Pass the plane Q226 R426 S3X7 perpendicular to the line A317 B345, and pass the plane J816 K7X8 L5X5 perpendicular to the line C647 D607.

## Group 48. Plane Perpendicular to a Plane.

**1.** Draw the plane E126 F417 G perpendicular to the plane Q534 R608 S845.

**2.** Pass the plane R517 S846 T6XX perpendicular to the plane D046 E307 F425.

**3.** Through the line R038 S314 pass the plane that is perpendicular to the given plane C436 D718 E825.

**4.** Through the line C628 D545 pass the plane C D E perpendicular to the plane J106 K247 L325.

**5.** Pass the plane Q7XX R507 S435 perpendicular to the plane R S T214.

**6.** Show the plane A104 B635 C7XX that is perpendicular to the plane A B D228.

**7.** Locate the plane J525 K L perpendicular to the vertical plane through E417 F725, and to the horizontal plane through A236.

**8.** Pass the plane E F G perpendicular to the plane A125 B317 C504, and to the vertical plane through D736 E417.

**9.** Pass the plane Q535 R S perpendicular to the planes A135 B318 C424 and D215 E507 F646.

**10.** Find the plane G K L that is perpendicular to the planes B225 C438 D614 and E346 F505 G737.

**11.** Perpendicular to the planes J225 K148 L207 and Q624 R735 S817 pass the plane B426 C6XX D2XX.

**12.** Pass the plane O426 P Q perpendicular to the planes A124 B338 C816 and D018 E546 F727.

## Group 49. Intersection of a Line and a Plane.

**1.** Find O, the intersection of the line A245 B708 and the plane D318 E627 F445.

**2.** Locate the point O on the line C714 D438 and in the plane Q106 R634 S828.

**3.** Locate the point P where the line D138 E605 intersects the plane A024 B318 C857.

**4.** Locate the point P where the line C434 D208 intersects the plane Q137 S416 T848.

**5** Locate the point O on the line A324 B647 and in the plane Q145 R418 S736.

**6.** Locate the point K in the plane R545 S618 T824 and on the line E004 F316.

**7.** Locate P, the intersection of the line A236 B218 with the plane Q336 R527 S405.

**8.** Locate the point P in the plane D145 E327 F404 and on the line A234 B207.

**9.** Locate O and P, the respective intersections of the lines A346 B306 and C107 D707 with the plane E245 F408 G614.

**10.** Locate E and F, the respective intersections of the lines C146 D106 and A626 B825 with the plane J217 K617 L445.

**11.** Locate the points O and P where the line E246 F206 pierces the planes A038 B104 D636 and A B C327.

**12.** Locate the points K, L, and M, where the line O428 P134 pierces the planes A038 B504 C528, A B D527, and A B E526.

**13.** Locate two points where the lines Q527 R638 and Q S846 intersect the plane J045 K308 L534.

**14.** Locate all points where the line A207 B434 intersects the faces of the tetrahedron Q527 R004 S238 T715.

**15.** Find the points of intersection of the line A035 B808 with the faces of the tetrahedron C124 D308 E446 F715.

**16.** Find the points E and D, where the line A128 B635 intersects the faces of the tetrahedron G238 J325 K407 L546.

## Group 50. Intersections of Several Lines and a Plane.

**1.** Show three views of the intersection of the triangular base pyramid V536 A415 B618 C806 with the plane D306 E708 F635.

**2.** Show four views of that part of the tetrahedron A125 B246 C324 D417 that is above the plane E037 F114 G426.

**3.** A right prism of 3″ altitude has a horizontal hexagonal base of diagonal E207 H405. Show the part of the prism lying below the plane A005 B327 C315.

**4.** V228 is the vertex, and A134 D314 is one diagonal of the hexagonal base of a right pyramid. Show the part of the pyramid between the vertex and the plane Q003 R136 S424.

**5.** V026 is the vertex, and C326 is the center of the hexagonal base of a right pyramid. The diagonals of the base are 2″ long and one diagonal is vertical. Show four views of the part of the pyramid that is between the vertex and the plane Q137 R434 S407.

**6.** A026 B626 is the axis of a $1\frac{1}{4}''$ right circular cylinder. Show its intersection with the plane D034 E328 F615.

**7.** C506 D546 is the axis of a $2\frac{1}{2}''$ right circular cylinder. Show its intersection with the plane K705 L302 M638.

**8.** Show three views of the intersection of the plane R317 S716 T623 with the right cone of which V523 is the vertex, and C526 is the center of the $1\frac{1}{2}''$ circular base.

## Group 51. Distance from a Point to a Line.

**1.** Find the shortest distance from the point C624 to the line E547 F205.

**2.** What is the shortest distance from the point P615 to the line C324 D548?

**3.** Determine the shortest distance from the point A614 to the line O345 P408.

**4.** Find the distance from the point B417 to the line E124 F838.

**5.** Find the shortest distance from the point P637 to the line K138 L314.

**6.** Determine the shortest distance from the point O308 to the line E235 F627.

## Group 52. Distance from a Point to a Plane.

**1.** Find A B, the shortest distance from the point A448 to the plane D205 E617 F734.

**2.** How far is it from the point O234 to the plane C125 D337 E504?

**3.** What is the shortest distance from the point E447 to the plane A145 B308 C425?

**4.** What is the shortest distance from the plane P336 Q417 R645 to the point A328?

**5.** What is the shortest distance from the point C608 to the plane K158 L405 M747?

**6.** Find the shortest distance from the plane R007 S434 T838 to the point A145.

## Group 53. Intersection of Planes.

**1.** Find E F, the intersection of the planes A124 B208 C446 and Q537 R828 S605.

**2.** Find O P, the intersection of the planes A137 B405 C648 and D226 E448 F735.

**3.** Find E F, the intersection of the plane A034 B846 C508 and Q108 R314 S547.

**4.** Find O P, the intersection of the planes A145 B307 C425 and D547 E215 F634.

**5.** Locate E F, the intersection of the planes A708 B434 C027 and Q114 R327 S805.

**6.** Locate J K, the intersection of the planes A708 B534 C427 and D037 E326 F115.

**7.** Locate K L, the intersection of the planes A718 B424 C136 and D634 E308 F025.

**8.** Locate the intersection X Y of the planes A018 B135 C327 and D527 E718 F846.

**9.** Find E F, the intersection of the plane A308 B615 C145 with the horizontal plane through D026.

**10.** Find D E, the intersection of the plane A238 B324 C547 with the vertical plane through F115 G617.

**11.** Find A B, the intersection of the plane D138 E416 F837 with the frontal plane through O335.

**12.** Find the intersection E F of the oblique plane Q326 R545 S738 with the horizontal plane 2″ below Q.

**13.** Show the front and top views of the quadrilateral that is cut from the tetrahedron A246 B406 C524 D638 by the plane Q005 R348 S824.

**14.** Draw the front and top views of the triangle that is the intersection of the tetrahedron C216 D336 E447 F615 with the plane Q248 R304 S817.

**15.** Locate the point P where the planes A035 B218 C324, D236 E405 F527, and G538 J745 K625 meet.

**16.** Find the point X that is in the planes A016 B408 C325, D816 E704 F626, and J508 K614 L446.

**17.** Locate the point P where the planes B214 C408 D635, E027 F235 G816, and J018 K808 L346 meet.

**18.** Locate K, the intersection of the planes A037 B308 C416, D526 E608 F837, and R005 S814 T645.

## Group 54. Angle between Lines.

**1.** Find the true size of the angle A135 B217 C446.
**2.** Find the true size of the angle C407 D535 E718.
**3.** Find the angle between the lines A115 B337 and B C414.
**4.** Find the true size of the angle D516 C105 E228.

**5.** Draw the bisector of the angle D117 E334 F506.
**6.** Locate the bisector of the angle C405 D624 E816.
**7.** Determine the bisector of the angle J017 K208 L316.
**8.** Find the bisector of the angle A547 B517 C735.

**9.** Find the line A405 D that intersects the line B548 C827, and makes an angle of 30° with the line A B.

**10.** B105 C325 is the base, and A037 is a point in the plane of a right-angle isosceles triangle. The vertex V is to the rear of the base. Draw the front, top, right-side, edge, and normal views of the triangle.

**11.** A745 B617 is the base of an isosceles triangle. D436 is a point in the plane of the triangle. The base angles are 30°. The vertex C is to the left of the base. Show the front, top, edge, and normal views of the triangle.

**12.** V217 is the vertex, and A325 is the middle point of the base of an equilateral triangle. P036 is a point in the plane of the triangle. Draw the front, top, right-side, edge, and normal views of the triangle.

**13.** Show the true size of the angle H that the line A515 B647 makes with a horizontal plane, and of the angle F that the line A B makes with a frontal plane.

**14.** Given the line D117 E336, show the true size of the angle that this line makes with all horizontal planes. Also, show the true size of the angle that this line makes with all profile planes.

**15.** Show the front and top views of the lines A237 B1X4 and C535 D62X that make 30° with all horizontal planes. B is below A, and D is behind C.

**16.** Draw the front and top views of the lines E436 F5X4 and E G31X that make an angle of 45° with all horizontal planes. F is below E, and G is in front of E.

**17.** One end of a line is at A436. The line is 2″ long and makes an angle of 30° with all horizontal planes, and an angle of 45° with all frontal planes. Show the front and top views of the eight possible locations of this line.

**18.** Find the front and top views of the line C236 D that is 2½″ long, makes an angle of 22½° with all horizontal planes, and 15° with all frontal planes. D is above, to the right, and to the rear of the point C.

**19.** Locate the front and top views of the line E536 F that makes an angle of 22½° with all frontal planes, and 30° with all horizontal planes. The line is 3″ long, and F is to the left of, in front of, and below E.

**20.** Draw the front and top views of the line O P417 that makes an angle of 60° with all frontal planes, and 15° with all horizontal planes. The line is 3″ long. The end O is to the right of, in front of, and above the point P.

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**21.** Show the front, top, edge, and normal views of the angle B028 A237 C equal to the angle that the line B A makes with the plane Q435 R518 S724.

**22.** Draw the front, top, edge, and normal views of the angle A327 B137 C that the line A B makes with the plane R024 S236 T414.

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## Group 55. Angle between Planes.

**1.** Show the true size of the angle between the planes A115 B545 C317 and D726 E428 F206.

**2.** Find the true size of the angle between the planes D115 E247 F336 and J025 K416 L538.

**3.** Find the true size of the angle between the planes P237 Q516 R105 and A648 B324 C806.

**4.** Find the true size of the angle between the planes A534 B648 C715 and D425 E517 F836.

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**5.** Find the true size of the angle between the plane Q218 R516 S145 and a horizontal plane.

**6.** Find the true size of the angle between the plane R447 S505 T814 and a horizontal plane.

**7.** Find the true size of the angle F that the plane Q146 R215 S357 makes with all frontal planes, and the angle H that it makes with all horizontal planes.

**8.** Find the true size of the angle F that the plane R317 S445 T526 makes with all frontal planes, and also find the true size of the angle H that this plane makes with all horizontal planes.

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## Group 56. Timber Framing.

**1.** Show the face, edge, and end views of the ends of a 2″ × 4″ common rafter for a roof having a rise of 8″ in a run of 12″. Scale 3″ = 1′.

**2.** Show the face, edge, and end views of the ends of a 2″ × 4″ common rafter for a roof having a rise of 15″ in a run of 12″. Scale 3″ = 1′.

**3.** To a quarter-size scale, show the face, edge, and end views of the ends of a 2″ × 6″ hip rafter for a roof having a rise of 8″ in a run of 12″.

**4.** To a quarter-size scale, show the face, edge, and end views of the ends of a 2″ × 6″ hip rafter for a roof having a rise of 15″ in a run of 12″.

**5.** Show the face, edge, and end views of the ends of a 2″ × 6″ jack rafter for a hip roof having a rise of 8″ in a run of 12″. Scale one-quarter size.

**6.** Show the face, edge, and end views of the ends of a 2″ × 6″ jack rafter for a hip roof having a rise of 15″ in a run of 12″. Scale one-quarter size.

**7.** The rectangles A025 B125 C123 D023 and E458 F558 G556 H456 are the upper ends of two 4″ × 8″ posts. K114 L447 is the center line of a 2″ × 4″ brace that is cut to fit the posts. The 4″ faces of the brace are vertical. Show the front, top, face, edge, and end views of the brace.

**8.** An equilateral tetrahedral frame, having edges 16″ long, is made from six similar pieces that are 2″ × 4″ in cross-section. Draw the face, edge, and end views of one of these pieces to a scale of 3″ = 1′.

**9.** A004 C408 is the diagonal of a horizontal square. This square is the base of a right pyramidal roof of 8″ altitude that is drawn to a one-quarter size scale. Show the face, edge, and end views of one of four similar 2″ × 3″ hip rafters for this roof. The upper edge of each rafter is to be beveled.

**10.** V646 is the vertex, and A604 D608 is the diagonal of the hexagonal base of a right pyramid. This pyramid is to be made from six similar pieces of wood that are $\frac{1}{2}$″ thick. The bottom is to be left open. Show the front and top views of the pyramid and the true shape of one of the six pieces. Show also the exact angle to which the piece is beveled. The views will overlap.

**11.** The line A115 B415 is the front edge of a horizontal equilateral triangle. The triangle is the base of an equilateral pyramid. The pyramid is made from four boards, $\frac{1}{2}$″ thick, and cut alike. Draw the front, top, and right-side views of the pyramid including all dotted lines, and show the true size of the angle of the beveled joints.

**12.** The horizontal rectangle A048 B448 C445 D045 is the base of a right pyramid. The vertex is 3″ below the base. Each face of the pyramid is made from a single board $\frac{3}{8}$″ thick. Discard the lower one-half of the pyramid. The frustum that is left is open at the top and bottom, and is used as a hopper. Show the face and edge views of one of the narrower end boards, and show the true size of the angle to which the beveled joint is cut.

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## Group 57. Miscellaneous Problems.

**1.** Find the locus of a point that is equidistant from the points A534 and B308.

**2.** Find the locus of a point that is equidistant from the points C647 and D205.

**3.** Find the locus of a point that is equidistant from the points E245 and F707.

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**4.** Find the locus of a point that is equidistant from the points A747, B318, and C235.

**5.** Find the locus of a point that is equidistant from the points A734, B418, and C345.

**6.** Find the locus of a point that is equidistant from the points D606, E447, and F115.

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**7.** Find the locus of a point that is equidistant from the points A047, B308, C524, and D736.

**8.** Find the locus of a point that is equidistant from the points C727, D304, E118, and F446.

**9.** Show the front and top views of the sphere that passes through the points A427, B635, C2,4,$5\frac{1}{2}$, and D3,1,$4\frac{1}{2}$.

**10.** Find the line K L that is parallel to the line E017 F835, and intersects the lines A225 B607 and C348 D524 at O and P. Check the solution.

**11.** Find the line that is parallel to the line C026 D847, and intersects the lines K214 L438 and M207 N735 at A and B, respectively. Check.

**12.** Locate the line A B that intersects the lines J105 K447 and M228 N734, and is parallel to the line O648 P017. Check.

**13.** Find the line that is perpendicular to the plane Q035 R225 S337, and intersects the lines C207 D628 and E346 F834 at A and B, respectively. Check.

**14.** Draw the line O P perpendicular to the plane Q546 R814 S708, and intersecting the lines A126 B517 and C236 D724 at K and L. Check the result.

**15.** Locate the line that intersects the lines A518 B834 and C436 D707 at E and F, and is perpendicular to the plane R036 S315 T428. Check the result.

**16.** Find the line that passes through the point O545, and intersects the lines C207 D625 and E104 F738 at A and B. Check the solution.

**17.** Locate the line that passes through the point A726, and intersects the lines J228 K304 and E408 F645 at the points B and C. Check.

**18.** Locate the line A526 D that intersects the lines J208 K334 and L135 M707 at B and C. Check.

**19.** A535 is one end of the axis of a right prism. The other end is in the plane J415 K647 L824. The diagonals of the hexagonal bases are $2\frac{1}{2}''$ long, and one diagonal is parallel to the line J L. Draw the front, top, auxiliary, and end views of the prism.

**20.** V225 is the vertex of a right pyramid having a $1\frac{1}{2}''$ square base in the plane Q007 R145 S317. Two sides of the base are parallel to the line R S. Draw the front and top, and the necessary auxiliary and oblique views of the pyramid.

**21.** C225 is the center of a $3''$ sphere. Show the front and top, and the necessary auxiliary and oblique views of that part of the sphere that is above the plane C D024 E146.

**22.** The line A D in the plane A215 D336 K137 is the diagonal of a hexagon that lies in the given plane. The hexagon is the base of a right pyramid of $2''$ altitude. The vertex is above the base. Draw the front, top, right-side, auxiliary, and end views of the pyramid.

# PROBLEMS—CHAPTER IX

## INTERSECTION AND DEVELOPMENT OF SURFACES

### INTERSECTION PROBLEMS

### Group 58. Miscellaneous Intersections.

This group of problems is divided into subgroups of three problems each. The problems of each subgroup are chosen so that all three methods of finding intersections can be used to advantage in solving the three problems of the subgroup. Each problem requires a different method from the other two. The order of a problem in the group does not indicate the method that should be used in solving it. Each problem should be visualized, and the best method for solving it should be determined.

**1.** A746 is the vertex, and X206 is the center of the horizontal square base of an oblique pyramid. One diagonal of the square is frontal and 4″ long. V346 is the vertex, and C606 is the center of the horizontal hexagonal base of an oblique pyramid. The diagonals of the hexagon are 3″ long, and one diagonal is frontal. Show the front and top views of the intersecting pyramids. To solve this problem readily, pass cutting planes through the line V A.

**2.** V624 is the vertex, and O628 is the center of the base of a right pyramid. One diagonal of its hexagonal base is 4″ long and vertical. T426 is the vertex, and C826 is the center of the base of a second right pyramid. One diagonal of its square base is 3″ long and vertical. Show the intersecting pyramids in the front, top, and left-side views.

**3.** A146 C345 is the diagonal of a horizontal square. This square is the upper base of a right prism of 4″ altitude. E$\frac{1}{2}$,3,8 H3$\frac{1}{2}$,1,8 is the diagonal of a frontal hexagon. This hexagon is the rear end of a right prism. Show the front, top, and right-side views of the intersecting prisms.

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**4.** V624 is the vertex, and C628 is the center of the 4″ vertical diagonal of the hexagonal base of a right pyramid. Show the front, top, and left-side views of that part of the pyramid lying between the hexagonal base and the plane R106 S808 T545.

**5.** V225 and P325 are the vertices, and C228 and D328 are the respective centers of the 4″ circular bases of two right cones. Find the intersection of the cones. Draw the front, top, and right-side views.

**6.** V246 is the vertex, and C606 is the center of the horizontal hexagonal base of an oblique pyramid. One corner of the base is 2″ in front of its center. A646 B206 is the axis of an oblique prism. Each of the horizontal hexagonal bases of the prism has one corner 1½″ to the left of its center. In the front and top views, show the intersection of the pyramid and prism.

**7.** C226 and O3,1½.5 are the centers of 3″ spheres. Find the intersection of the spheres. Draw the front, top, and right-side views.

**8.** V424 is the vertex, and C226 is the center of the horizontal 3″ diagonal of the hexagonal base of a right pyramid. Draw the front, top, and auxiliary views of that part of the pyramid that is between the hexagonal base and the plane R005 S244 T406.

**9.** V446 is the vertex, and C406 is the center of the horizontal hexagonal base of a right pyramid. The diagonals of the hexagon are 4″ long, and one diagonal is frontal. A424 is the vertex, and O428 is the center of the frontal hexagonal base of a second right pyramid. The diagonals of this hexagon are 4″ long, and one diagonal is horizontal. In the front and top views, show the intersection of the pyramids.

**10.** V624 is the vertex, and O628 is the center of the base of a right pyramid. One diagonal of the hexagonal base is 4″ long and vertical. T426 is the vertex, and C826 is the center of the 3″ square base of a second right pyramid. Two sides of the square are horizontal. In the front, top, and left-side views, show the intersecting pyramids.

**11.** A404 D408 is the diagonal of a horizontal hexagon. The hexagon is the base, and V436 is the vertex of a pyramid. K526 L506 is the axis of a right prism. The sides of the hexagonal bases of the prism are 1½″ long. Two faces of the prism are frontal. In the front and top views, show the intersection of the pyramid and prism. The points of intersection of the lines V A and V D, with the vertical faces of the prism, may be found by working the first method backward.

**12.** V424 is the vertex, and C428 is the center of the 4″ circular base of a right cone. Find the intersection of this cone with the 3″ sphere having its center at P3,1½,6.

**13.** B3,0,5½ D7,0,6½ is the diagonal of a horizontal square. This square is the base, and V546 is the vertex of a right pyramid. E638 K825 is the diagonal of a vertical square. This square is the base of a right pyramid with the vertex at X4,2½,4½. Show the front and top views of the two intersecting pyramids.

**14.** A316 is the center of a 4″ sphere, and B408 is the center of a 6″ sphere. In the front and top views, show as much of the intersection of these two spheres as appears within the limits of the plate.

**15.** The axis of a right prism is E446 F716. The sides of the hexagonal ends are 1¼″ long, and four sides are horizontal. In the front, top, and left-side views, show all of the prism that is below the plane Q318 S818 T636.

## INTERSECTION AND DEVELOPMENT PROBLEMS

Developments or views should not overlap or extend beyond the problem space. To accomplish this, it is a good plan to draw the development on other paper, and then to locate the development on the problem sheet. Large developments may be placed diagonally on the sheet. Some developments are specified as a separate problem. Developments that are not carefully and neatly drawn will be considered incorrect. When a development is called for, the complete development should be shown, unless otherwise specified in the statement of the problem. Bases and cut sections are not required unless so specified. The near side of the pattern should be the inside. The net development should be shown. Allowances for seams and wiring are not to be shown. Show the dotted parts of intersections.

Clear visualization is necessary if problems are to be readily solved. Much ingenuity is needed in choosing the best method of solving a problem, and in carrying out the details of the solution. Much time and work are saved by using more than one compass when making developments.

### Group 59. Right Pyramids.

**1.** The vertex V236 of a pyramid is 2″ above the horizontal equilateral triangular base. The sides of the base are 3″ long. One corner of the base is directly in front of the center of the base. Draw the front, top, and right-side views of the pyramid, and show the development of the surface, including the base.

**2.** B218 D214 is the diagonal of a horizontal square. This square is the base of a pyramid of 1½″ altitude. Show the development, and the front and top views of as much of the pyramid as is above the plane Q034 S4,¾,5 T038. Also, in an auxiliary view, show the true shape of the plane section.

**3.** The vertical square base of a right pyramid is 2″ behind the vertex P625. One diagonal of the base is vertical and 3″ long. Show the front and top views, and the true shape of the section cut by the vertical plane R437 S815. Show also, in one connected development, the cut section, the base, and the surface lying between them.

**4.** The 2″ square base of a right pyramid is 3″ behind the vertex V224. Two sides of the base are vertical. Show the front, top, and auxiliary views, and the development of that part of the pyramid that is between the vertex and the plane J005 K506 L126.

**5.** V526 is the vertex, C725 is the center of the square base, and D7½,2½,6 is a corner of the base of a right pyramid. Show the front, top, and end views, and a stretchout of the part of the pyramid that is between the base and a profile plane 1″ to the right of V.

**6.** B305 is one corner and C206 is the center of a horizontal hexagon. This hexagon is the base of a right pyramid of 4″ altitude. Show the true shape of the intersection of this pyramid with the plane K145 L405 M148, and develop the surface of the pyramid that lies between the plane and the hexagon.

**7.** X324 is the vertex, and A034 is one corner of the profile hexagonal base of a right pyramid. Show the front, top, and right-side views of the pyramid and its intersection with the plane R115 S218 T245. Develop the part of the pyramid that is to the right of the plane.

**8.** A7$\frac{1}{2}$,2,7 is one corner of the frontal hexagonal base, and V625 is the vertex of a right pyramid. L415 M815 N637 is a plane that cuts the pyramid in two. Show the front, top, and left-side views, and the development of the part of the pyramid that is behind the given plane.

**9.** V056 is the vertex, and C236 is the center of the hexagonal base of a right pyramid. One corner of the base is 1$\frac{1}{2}$″ in front of C. Show the front, top, and end views, and make a pattern of the part of the pyramid that is to the right of a profile plane that is 1″ to the left of C.

## Group 60. Oblique Pyramids.

**1.** Show the front, top, and left-side views, and develop the entire surface of the tetrahedron A536 B836 C717 D714.

**2.** The lines E036 F016 and K227 L224 are two edges of a triangular base pyramid. Show the front and top views of the pyramid, and develop the entire surface.

**3.** The lines B535 C727 and D516 E715 are two edges of a tetrahedron. Draw its front, top, and left-side views. Develop the entire surface.

**4.** One diagonal of the frontal square base of an oblique pyramid is D527 F717, and the vertex is at V4,2$\frac{1}{2}$,4$\frac{1}{2}$. Show the development of the surface.

**5.** In an oblique pyramid X7$\frac{1}{2}$,3,6$\frac{1}{2}$ is the vertex, and A406 C705 is one diagonal of the horizontal square base. Develop the surface of the pyramid.

**6.** The vertex of an oblique pyramid is at A046, and B138 E318 is one diagonal of the frontal hexagonal base. Develop the surface of the pyramid.

**7.** C115 F317 is one diagonal of the horizontal hexagonal base of an oblique pyramid. The vertex is at V435. Obtain the development of the pyramid.

## Group 61. Right Prisms.

**1.** The triangle A2,0,6$\frac{1}{4}$ B406 C2$\frac{3}{4}$,0,4$\frac{1}{4}$ is the base of a right prism of 3″ altitude. Find the intersection of this prism with the plane Q005 R327 S315, and show the true shape of the intersection.

**2.** Develop that part of the prism of the preceding problem that is below the plane Q R S. Include the base and the cut section.

**3.** The triangle D014 E035 F027 is the base of a right prism. Show the front, top, and right-side views, and the development of the part of the prism that is between the base D E F and the plane J208 K144 L148.

**4.** A015 C316 is the diagonal of a horizontal square. The square is the lower base of a cube. In the front and top views, show that part of the cube that is above the plane Q308 R606 S026.

**5.** Make a pattern for the entire surface of the part of the cube that is shown in the preceding problem.

**6.** The parallelogram A006 B107 C306 D205 is the base of a right prism of 4″ altitude. Show the front, top, and right-side views of that part of the prism lying below the plane J017 K618 L135.

**7.** Show the development of the lower part of the truncated prism of the preceding problem, including the base and the cut section.

**8.** C535 D715 is the axis of a right hexagonal prism. One corner of the lower base is $1\frac{1}{4}''$ in front of D. Draw the front, top, and end views of all of the prism that is to the right of the profile plane through K608, and show its development.

**9.** A626 B628 is the axis of a right hexagonal prism. One corner of the rear base is $1\frac{1}{4}''$ to the right of B. Show the front, top, and auxiliary views, and the stretchout of the part of the prism that is behind the plane P407 Q808 R845.

**10.** C125 D526 is the axis of a right hexagonal prism. The rectangular faces of the prism are $1\frac{1}{2}''$ wide, and two are vertical. In the front and top views, show the part of the prism that lies behind the plane R308 S704 T425. Do not draw an edge view of the plane.

**11.** Develop the part of the prism that is shown in the preceding problem. Include the base and the cut section.

**12.** A416 B C716 D is a horizontal square. This square is the base of a cube. In the front, top, and left-side views, show that part of the cube that is below the plane that passes through the middle points of the front and rear vertical edges of the cube and also through the middle points of the edges A B and A D. In addition show the true shape of the cut section. Is this a regular hexagon?

**13.** Develop the entire surface of the part of the cube that is shown in the preceding problem.

## Group 62. Oblique Prisms.

**1.** A125 B335 is one edge, and B is the upper corner of a frontal equilateral triangle. This triangle is one base, and A D237 is one edge of an oblique prism. Draw the front and top views of the prism and develop the surface.

**2.** A237 C335 and E116 G214 are the respective diagonals of two horizontal squares. The squares are the bases of an oblique prism. Develop the surface.

**3.** The triangle A025 B335 C215 is one base of an oblique prism. A D136 is one of the three parallel edges of the prism. Show the front and top views, and the development of the surface and bases of the prism.

**4.** A003 B203 is the front edge of a horizontal equilateral triangle. D024 E224 is the front edge of a second horizontal equilateral triangle. These triangles are the two bases of an oblique prism. Draw the front, top, and right-side views of the prism. Obtain a right section of the prism, and develop the surface.

**5.** C636 D716 is the axis of an oblique prism. The diagonals of each horizontal hexagonal base are 2″ long, and one diagonal of each base is frontal. Draw the front and top views, obtain a right section, and develop the surface of the prism.

**6.** A737 B705 is the axis of an oblique prism. C637 is a corner of the upper horizontal hexagonal base. Pass a plane perpendicular to the axis at its middle point. Show the front, top, and left-side views of the part of the prism that is below this plane, and develop the surface of this part of the prism.

## Group 63. Hoppers.

**1.** A symmetrical hopper has an inlet 16″ square, an outlet 8″ square, and a height of 32″. Show the top and front views and the stretchout to a scale one-eighth full size.

**2.** The inlet of a symmetrical sheet-metal hopper is 8″ square, the outlet 3″ square, and the depth 10″. To a scale of 3″ = 1′ draw the front and top views and a pattern of the hopper.

**3.** The horizontal inlet of a symmetrical hopper is a 4″ by 12″ rectangle. The horizontal outlet is a 2″ by 4″ rectangle having its longer sides parallel to the longer sides of the inlet. The depth of the hopper is 4″. To a one-quarter size scale, draw the front, top, and side views and the development.

**4.** The horizontal inlet of a symmetrical hopper is a 4″ by 12″ rectangle. The horizontal outlet is a 2″ by 4″ rectangle having its longer sides parallel to the shorter sides of the inlet. The depth of the hopper is 4″. To a one-quarter size scale, draw the front, top, and right-side views and the development.

**5.** A hopper has a horizontal inlet 3″ square, a horizontal outlet 1″ square, and a height of $3\frac{1}{4}$″. Two of the sides are vertical. Show its top and front views. Draw the development one-half size.

**6.** A148 B448 C445 D145 is the inlet, and E118 F218 K216 L116 is the outlet of a hopper. Develop the hopper to a one-half size scale.

**7.** The rectangle A048 B348 C346 D046 is the inlet, and the rectangle E128 F238 G236 H126 is the outlet of a hopper. Draw the front and top views, and show the development of the hopper.

**8.** The horizontal rectangle A534 B536 C836 D834 is the inlet, and the frontal square E624 F724 G714 H614 is the outlet of a hopper. Draw its front, top, and left-side views, and make a pattern.

**9.** The horizontal rectangle A058 B358 C356 D056 is the inlet, and E138 F248 G247 H137 is the outlet of a hopper. Draw its front and top views and its development.

**10.** C$6\frac{1}{2}$,2,5 is the center of a horizontal hexagon, of which the diagonals are 3″ long, with one diagonal frontal. D$6\frac{1}{2}$,0,5 is the center of a horizontal 1″ square, with two sides frontal. The hexagon is the inlet, and the square is the outlet of a hopper. Draw its front, top, and left-side views, and its development.

**11.** The inlet of a symmetrical hopper is an 8″ horizontal square, and the depth is 6″. The outlet is a 4″ horizontal square, having its diagonals parallel to the sides of the inlet. To a one-quarter size scale, draw the front and top views and the stretchout.

**12.** The horizontal inlet of a symmetrical hopper is 3″ square. The outlet is a horizontal hexagon, of which two of the 1″ sides are parallel to two sides of the inlet. The depth is 2″. Draw the front, top, and right-side views and one-half of the development of the hopper.

## Group 64. Transition Pieces.

**1.** The rectangles A006 B306 C305 D005 and E137 F237 G234 H134 are the open ends of a transition piece. Draw its front, top, and right-side views, and show one-fourth of the development.

**2.** The rectangles A037 B137 C134 D034 and E015 F315 G314 H014 are the open ends of a transition piece. Draw its front, top, and right-side views, and draw the development to a one-half size scale.

**3.** The horizontal rectangles A037 B137 C134 D034 and E017 F217 G215 H015 are the open ends of a transition. Draw the front, top, and right-side views. Draw the development one-half of full size.

**4.** A025 C105 and E237 G317 are the respective diagonals of two frontal rectangles. Two sides of each rectangle are vertical. These rectangles are the ends of a transition piece. Draw the front and top views and the development of the piece.

**5.** A045 C115 is the diagonal of a frontal rectangle, two sides of which are horizontal. E1½,3½,7 G3½,1½,7 is the diagonal of a frontal square. The rectangle and the square are the open ends of a pipe connection. Draw the front and top views and make a one-half size development.

**6.** The open ends of a pipe connection are the rectangles A048 B248 C247 D047 and E316 F416 G414 H314. Draw its front, top, and right-side views.

**7.** Develop the pipe connection of the preceding problem.

**8.** B747 D545 is the diagonal of a horizontal square. A second 2″ horizontal square, having its sides parallel to the diagonals of the first square, has its center 4″ below the center of the first square. These squares are the open ends of a transition piece. Draw its front and top views and show one-half of the development.

**9.** A445 B745 is the upper edge of a frontal square. P5½,2½,7 is the center of a second square, of which the 3″ sides are parallel to the diagonals of the first square. These squares are the open ends of a pipe connection. Show its front and top views and one-fourth of the development.

**10.** A036 D436 and G228 J224 are the respective diagonals of two horizontal hexagons. These hexagons are the open ends of a transition piece. Draw its front and top views, and show one-half of the development.

**11.** A137 C335 is the diagonal of a horizontal square. O216 is the center, and P314 is a point on the extended diagonal of a horizontal 2″ square. These squares are the open ends of a pipe connection. Draw its front and top views, and show one-half of the development.

**12.** The horizontal rectangles A135 B337 C436 D234 and E116 F416 G415 H115 are the open ends of a pipe connection. Draw its front, top, and right-side views.

**13.** Develop the pipe connection of the preceding problem. Make a model. Stiffen the model by adding the rectangles in the open ends.

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## Group 65. Elbows and Offsets.

**1.** The lines A434 B634 and E838 F836, respectively, are the upper edges of two 1″ by 2″ vertical rectangles. These rectangles are the open ends of a three-piece pipe elbow. The end sections are to be alike, and the middle section is to have the same cross-section as the ends. The vertical planes of the seams are to pass through the point C735. Draw the front and top views of the elbow, and show the full-size developments of the middle section and one end section.

**2.** The lines A434 B634 and E838 F836, respectively, are the upper edges of two 1″ by 2″ vertical rectangles. These rectangles are the open ends of a four-piece pipe elbow. The end sections are to be alike, and the middle sections are to have the same cross-section as the ends. The vertical planes of the seams are to pass through the point C735. Draw the front and top views of the elbow, and show the full-size developments of one of the middle sections and one end section.

**3.** A045 B047 is the upper edge of a 1″ by 2″ profile rectangle. E215 F217 is the left edge of a 1″ by 2″ horizontal rectangle. These rectangles are the open ends of a three-piece pipe elbow. The end pieces are to be alike, and the middle piece is to have the same cross-section as the ends. The planes of the seams are to pass through the line K125 L127. Draw the front and top views of the elbow, and show how the complete full-size development may be cut, without waste, from a sheet of metal.

**4.** A045 B047 is the upper edge of a 1″ by 2″ profile rectangle. E215 F217 is the left edge of a 1″ by 2″ horizontal rectangle. These rectangles are the open ends of a four-piece pipe elbow. The end sections are to be alike, and the middle sections are to have the same cross-section as the ends. The planes of the seams are to pass through the line K125 L127. Draw the front and top views of the elbow, and show how the full-size elbow may be cut from a sheet of metal without waste.

**5.** The lines A068 D067 and E338 H337, respectively, are the left edges of two 1″ by 2″ horizontal rectangles. These rectangles are the open ends of a three-piece offset pipe connection. The end pieces are alike and have vertical walls. The mid-section has the same cross-section as the ends, and its axis makes an angle of 15° with the horizontal. Draw the front and top views of the offset, and make a half-size pattern that shows how the three pieces may be cut, without waste, from a rectangular sheet of metal.

**6.** A047 and E307, respectively, are the rear left-hand corners of two horizontal rectangles that are 1″ wide by 2″ deep. These rectangles are the end openings of a three-piece offset pipe connection. The two end pieces are to be alike and vertical. The middle section is to have the same cross-section as the ends, and two of its faces are to make an angle of 30° with the horizontal. Draw the front and top views. Make a one-half size pattern that shows how the three pieces may be cut from a rectangular sheet without waste.

**7.** A048 C146 and E207 G305 are the respective diagonals of two 1″ by 2″ horizontal rectangles. These rectangles are the open ends of a three-piece compound offset pipe connection. The end pieces of the connection are to be alike and vertical. The upper and lower faces of the middle piece are to make 45° with the horizontal, and are to be 1″ apart. Draw the front and top views of the connection, and show the development for the middle section and the two similar ends.

## Group 66. Prisms and Pyramids.

**1.** A245 C248 is the diagonal of a horizontal square. This square is the upper end of a right prism of 4″ altitude. E436 H416 is the diagonal of a profile hexagon. This hexagon is the right end of a right prism that is 4″ long. In the front, top, and right-side views, show the two intersecting prisms.

**2.** Show the development of the hexagonal prism of the preceding problem, cut so that it will fit the square prism.

**3.** A246 B606 is the axis of an oblique prism. Each base is a horizontal hexagon having one corner 1½″ in front of its center. C646 D206 is the axis of a second oblique prism. Each base of the second prism is a horizontal hexagon having one corner 1½″ to the right of its center. Show, in the front and top views, the intersection of the prisms.

**4.** Develop that part of the prism C D, of the preceding problem, that is above the prism A B. Make the development with the aid of a right section of the prism.

**5.** A546 B506 is the axis of a right prism. The edges of the hexagonal bases are 1″ long, and two edges of each hexagon are frontal. C3,3,5½ D7,1,5½ is the axis of a second right prism. The edges of its hexagonal bases are 1⅛″ long, and two faces of the prism are frontal. Show the two intersecting prisms in the front and top views.

**6.** Show the development of the prism A B of the preceding problem, with a part cut out to fit around the prism C D.

**7.** A457 B717 and C417 D757 are the axes of two right hexagonal prisms. The sides of the hexagons are 1″ long, and two sides of each hexagon are horizontal. Show the front and top views of the intersecting prisms, and the development of that part of the first prism that is above the second prism.

**8.** A324 O326 is the axis of a right prism. The diagonals of the hexagonal bases are 1½″ long, and one diagonal of each base is horizontal. The axes of two similar prisms are horizontal, make an angle of 120° with A O, and have one end at O. Draw the front and top views of the intersecting prisms, and make one development for the three prisms.

**9.** V636 is the vertex, and C606 is the center of the base of a right hexagonal pyramid. The diagonals of the base are 3″ long, and one diagonal is frontal. A4,2½,6 B8,2½,6 is the axis of a right hexagonal prism. The rectangular faces of the prism are 1½″ wide, and two faces are vertical. The pyramidal base is a support for the prism. In the front, top, and left-side views, show only the pyramidal base. Also show its development.

**10.** V3,2,5½ is the vertex, and C8,2,5½ is the center of the hexagonal base of a right pyramid. One corner of the base is 1½″ above its center. K606 L646 is the axis of a right prism. One corner of the lower square base is 2″ in front of K. In the front and top views, show the two intersecting surfaces.

**11.** From the preceding problem, obtain the development of the parts of the pyramid that are outside the prism.

**12.** V646 is the vertex, and A416 D816 is one diagonal of the hexagonal base of a right pyramid. X4,2½,6 is the vertex, and G846 J816 is the diagonal of the hexagonal base of a second right pyramid. Show the front, top, and left-side views of the intersecting pyramids.

**13.** Make a pattern of the surface of the second pyramid of the preceding problem, showing the hole that must be cut in the pattern to fit it around the first pyramid. Add the hexagonal base to the pattern.

# PROBLEMS—CHAPTER X

## SINGLE-CURVED SURFACES

### Group 67. Developments of Right Circular Cones.

**1.** V225 is the vertex, and C227 is the center of the 3″ circular base of a right cone. Develop the surface of the cone.

**2.** V646 is the vertex, and C616 is the center of the 3″ circular base of a right cone. Develop the surface.

**3.** C216 is the center of the 4″ horizontal circular base of a right cone. The altitude is ¾″. Make a pattern of the conical surface.

**4.** B625 is the vertex, and C626 is the center of the 4″ circular base of a right cone. Develop the conical surface.

**5.** C837 is the center of one-fourth of a 6″ horizontal circle. The circle is the base, and V807 is the vertex of one-fourth of a right cone. Draw its front and top views, and show the development of the conical surface and the vertical plane sides.

**6.** Design a well-proportioned funnel that is to be used in a hole ¾″ in diameter. Make all necessary patterns.

**7.** Design a well-proportioned funnel to be used in an opening that is ⅝″ in diameter. Make the necessary stretchouts.

**8.** A214 and B414 are the vertices, and E204 and F404 are the centers of the 2″ horizontal circular bases of two right cones. One-half of each conical surface is an end of a sloping lid. Draw the front and top views of the lid. Make a one-piece development of the entire lid.

**9.** A 30° sector is cut from a 4″ frontal circle with center at C638. The remaining sector is the development of a right cone with a vertical axis, and with its vertex at V246. Draw the front and top views of the cone.

**10.** V606 is the vertex of a right cone. The center of its base is above the vertex. C004 is the center of one-half of a horizontal 6″ circle. This half-circle is the development of the cone. Draw its front and top views.

**11.** V026 is the vertex and C226 is the center of the base of a right circular cone. P506 is the center of a 5″ horizontal circle. A sector of this circle is the development of the cone. Draw the front, top, and right-side views of the cone, and complete the development.

**12.** Draw the front and top views of the right cone having its vertex at A246, and the center of its 3″ circular base at B206. Develop the conical surface. On the development draw a straight line extending from tip to tip of the development. Locate this line in the front and top views.

**13.** V613 is the vertex, and C618 is the center of the half-circular base of the upper one-half of a right cone. The diameter of the base is 3″. Draw the front and top views of the half-cone, and develop the surface. On the development, draw a straight line from the point where one straight side meets the curved side, to the middle point of the other straight side. Show this line in the front and top views.

## Group 68. Cone and Plane Intersections.

**1.** V326 is the vertex of a right cone. In the front, top, and right-side views, show that part of the cone that lies between the plane Q414 R418 S0,3½,8, and the 2½″ circular base with center at C026. Show also the development of this part of the cone.

**2.** V246 is the vertex, and C206 is the center of the 4″ circular base of a right cone. Draw its front and top views, and develop that part of the conical surface that lies to the left of a profile plane that is ¼″ to the right of the axis.

**3.** C646 is the center of the 3½″ circular base, and V606 is the vertex of a right cone. In the front, top, and left-side views, show the part of the cone that is below the plane Q345 R805 S808. Also show the development of this part of the cone.

**4.** V426 is the vertex and C725 is the center of the 2½″ circular base of a right cone. In the front, top, and auxiliary views, show the intersection of this cone with the profile plane that is ½″ to the left of C. Develop that part of the cone that lies between the vertex and the profile plane.

**5.** V524 is the vertex, and C226 is the center of the 3″ circular base of a right cone. Show the front and top views of the cone, and find its intersection with the frontal plane ½″ behind V.

**6.** A026 and B526 are the centers of 2″ profile circles. The circles are the bases of the two nappes of a right cone. Draw its front, top, and right-side views. In the front and top views, show the intersections of the cone with the following planes: a vertical plane C107 D6,0,4½; a horizontal plane ½″ above the axis; and a vertical plane E106 F608. Also show the true shape of each section. Cross-hatch the true shapes, and letter each curve with its name.

**7.** A625 and X628 are the centers of 3″ frontal circles. These circles are the bases of the two nappes of a right cone. Draw its front, top, and left-side views. In all three views show the intersections of the cone with the following planes: a vertical plane through E538 and F736; a vertical plane through B8,4,4¾ and C348; and a profile plane ½″ to the left of the axis. Find the true shape of each curve. Cross-hatch the true shape, and letter each curve with its name.

**8.** C206 D246 is the center of two 3½″ circles. These circles are the bases of the two nappes of a right cone. Draw its front, top, and right-side views. In the front and right-side views, show the intersections of the cone with the following planes: the profile plane ½″ to the right of the axis; the plane E358 F3,½,4 G158; and the plane J048 K0,0,4¼ L248. Cross hatch the true shape of each section, and letter each curve with its name.

**9.** V636 is the vertex, and C616 is the center of the 3½″ circular base of a right cone. In the front, top, and auxiliary views, show the intersection of the cone with the plane J336 K838 L525.

**10.** Develop that part of the cone of the preceding problem that lies between the given plane and the base.

**11.** B826 is the center of the 3″ circular base, and A526 is the vertex of a right cone. This cone is cut into two parts by the plane Q435 R837 S705. In the front, top, left-side, and auxiliary views, show the larger part of the cone.

**12.** Make a pattern for the part of the cone that is shown in the preceding problem. Include the base.

**13.** V227 is the vertex, and O224 is the center of the ½″ circular base of a right cone. In the front, top, and auxiliary views, show the part of this cone that is between the vertex and the plane R116 S518 T334. Also develop the part of the cone indicated above. To the development add the base.

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## Group 69. Developments of Oblique and Non-circular Cones.

**1.** X536 is the vertex, and O627 is the center of the frontal 4″ circular base of an oblique cone. Show the front and top views and the development of the cone.

**2.** C214 is the center of the 3″ horizontal circular base, and V036 is the vertex of an oblique cone. Draw its front and top views and develop the surface.

**3.** The vertex of an oblique cone is at A634, and the center of its profile 2½″ circular base is at B826. Draw the front, top, and left-side views of the cone, and develop the surface.

**4.** C606 is the center of the horizontal 4″ circular base, and V416 is the vertex of an oblique cone. Show its front and top views and the development.

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**5.** A116 B416 is the major axis of a horizontal ellipse. The minor axis is 1½″ long. The ellipse is the base of a cone. The vertex is 1½″ above the center of the base. Develop the conical surface.

**6.** C526 D726 is the minor axis of a frontal ellipse. The major axis is 4″ long. This ellipse is the base, and V625 is the vertex of a cone. Draw its front and top views and develop the surface.

**7.** A117 B114 is the major axis of a horizontal ellipse. The minor axis is 2″ long. The ellipse is the base, and V2½,3,5½ is the vertex of an oblique cone. Draw its front and top views and show the development of the surface.

**8.** A626 B606 is the axis of one turn of a right-hand helix of 2″ lead that starts 1½″ to the left of B. This helix is the base, and V646 is the vertex of a cone. Draw the front and top views of the cone and develop its surface.

**9.** P246 is the pole, and A046 is a point on one turn of a right-hand horizontal Archimedean spiral of 2″ lead. The spiral is the base, and V226 is the vertex of a cone. Draw the front and top views of the cone, and develop the surface.

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## Group 70. Oblique Cone Transitions.

**1.** A symmetrical sheet-metal pipe connection is 4″ long, 2″ square at one end, and 2″ in diameter at the other end. Draw its front and top views, and show one-half of the complete pattern.

**2.** The front and top views and one-half of the stretchout are required for a symmetrical sheet-metal transition that is 4″ long. One end is rectangular, 2″ by 3″, and the other end is 2″ in diameter.

**3.** The front and top views and one-half of the development are required for a symmetrical sheet-metal hopper 4″ deep, 3″ square at the inlet, and 2″ in diameter at the outlet.

**4.** Draw the front, top, and right-side views of a symmetrical sheet-metal transition that is 3″ tall, the upper end being a 1″ by 4″ horizontal rectangle, and the lower end a $2\frac{1}{4}$″ horizontal circle.

**5.** Show one-half of the pattern for the transition of the preceding problem.

**6.** A536 D836 is the diagonal of a horizontal hexagon. X$6\frac{1}{2}$,1,6 is the center of a 2″ circle. The circle and the hexagon are the ends of a hopper. Draw its front, top, and left-side views and show one-half of the development.

**7.** A016 B416 is the major axis and C215 D217 is the minor axis of an ellipse. E236 is the center of a $1\frac{1}{2}$″ horizontal square having two edges frontal. Show the top and front views of a transition piece connecting the ellipse with the square, and develop the front half of the surface.

**8.** C$1\frac{1}{2}$,4,6 is the center of a $2\frac{1}{2}$″ horizontal circle. This circle is the open end, and the line A$\frac{1}{2}$,0,6 B$2\frac{1}{2}$,0,6 is the closed end of a paper container. Draw the front and top views of the container and make a pattern.

**9.** A135 C337 and E014 G418 are the diagonals of two horizontal squares. Round the corners of the larger square to a $\frac{5}{8}$″ radius. The two figures are the open ends of a transition. Draw its front and top views. Show the development of one face and of one of the rounded corners of the transition.

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**10.** B045 D247 is the diagonal of a horizontal square, P416 is the center of a horizontal 2″ circle. The circle and square are the open ends of a transition piece. Draw its front, top, and right-side views.

**11.** Make a stretchout for the transition piece of the preceding problem.

**12.** A146 E446 is the diagonal of a horizontal square. C516 is the center of a $2\frac{1}{4}$″ horizontal circle. The circle and square are the open ends of a pipe connection. Draw its front and top views.

**13.** Make a symmetrical pattern for the pipe connection of the preceding problem.

**14.** B425 D407 is the diagonal of a profile square. P646 is the center of a $2\frac{1}{4}$″ horizontal circle. The circle and the square are the open ends of a sheet-metal transition. Draw its front, top, and left-side views.

**15.** Make a pattern for the sheet-metal transition of the preceding problem.

**16.** X246 is the center of a 3″ horizontal circle. B335 D317 is the diagonal of a profile square. The square and the circle are the open ends of a transition. Draw its front, top, and right-side views.

**17.** Make a complete development for the transition of the preceding problem.

**18.** D345 F547 is the diagonal of a horizontal square. C216 and O616 are the centers of 2″ horizontal circles. The square is the inlet, and the circles are the outlets of a sheet-metal breeching, or Y. Draw the front and top views of the breeching. Line shade the conical parts.

**19.** Make a pattern for the right half of the breeching of the preceding problem.

**20.** A145 C347 and E545 F747 are the respective diagonals of two horizontal squares. P416 is the center of a 3″ horizontal circle. The squares are the inlets, and the circle is the outlet of a symmetrical breeching that is to be made from sheet metal. Draw its front and top views.

**21.** Make the stretchout for the right half of the breeching of the preceding problem.

## Group 71. Intersecting Right Cones.

**1.** A3,0,5½ is the center of the 5″ circular horizontal base of a right cone of 2″ altitude. B4,0,6½ is the center of the 2″ circular horizontal base of a right cone of 3½″ altitude. Draw the intersecting cones in the front and top views.

**2.** Develop the smaller cone of the preceding problem to fit the larger cone. Show also the hole that must be cut in the development of the surface of the larger cone to fit the smaller cone.

**3.** A4,2,5½ is the vertex, and B4,0,5½ is the center of the 5″ circular base of a right cone. E535 is the vertex, and F505 is the center of the 4″ circular base of a right cone. In the front and top views, show the intersection of the cones.

**4.** Develop the surface of the taller cone of the preceding problem to fit the shorter cone.

**5.** V546 is the vertex, and C506 is the center of the 4″ circular base of a right cone. A4,3,5½ is the vertex, and O4,0,5½ is the center of the 3″ circular base of a right cone. In the front and top views, show the intersecting cones.

**6.** Develop that part of the surface of the smaller cone that is outside the surface of the larger cone of the preceding problem.

**7.** V846 is the vertex, and C806 is the center of the 6″ circular base of a right cone. X646 is the vertex, and O606 is the center of the 2″ circular base of a right cone. Show the front, top, and left-side views of the intersecting cones.

**8.** Develop the smaller cone of the preceding problem to fit the larger cone. Develop also enough of the surface of the larger cone to show the hole that must be cut in it to fit the smaller cone.

**9.** A246 is the vertex, and C206 is the center of the 4″ circular base of a right cone. B248 is the vertex, and D208 is the center of the 8″ circular base of a right cone. Draw the front, top, and right-side views of as much of the intersecting cones as may be shown within limits.

**10.** Develop that part of the smaller cone of the preceding problem that is outside the larger cone.

**11.** A246 is the vertex, and B206 is the center of the 4″ circular base of a right cone. E324 is the vertex, and F328 is the center of the 3″ circular base of a right cone. Draw the front and top views of the intersecting cones, and develop the surface of the smaller cone to fit the larger cone.

**12.** V424 and P346 are the vertices, and C428 and K306 are the respective centers of the 4″ circular bases of two right cones. Find the intersection of the cones.

**13.** A2½,2,4 and V1½,4,6 are the vertices, and O2½,2,8 and C1½,0,6 are the respective centers of the 3″ circular bases of two right cones. Show the front and top views of the intersecting cones. Also develop the surface of the A O cone, and show the hole that is cut into it by the V C cone.

**14.** A646 is the vertex, and B616 is the center of the 3″ circular base of a right cone. X436 is the vertex, and C626 is the center of the 2″ circular base of a right cone. In the front and top views, show the intersecting cones.

**15.** C206 is the center of the 3″ circular base, and V246 is the vertex of a right cone. O216 is the center of the 2″ circular base, and X436 is the vertex of a right cone. Show the intersection of the cones in the front and top views. This problem may be solved within the limits of the paper by using as the base of the first cone a circle that is 1″ above the given base, and using as a base for the second cone a circle that is halfway between O and X.

## Group 72. Intersecting Oblique Cones.

**1.** A336 and E746 are the vertices, and B606 and F406 are the respective centers of the 4″ horizontal bases of two oblique cones. Draw the front and top views of the intersecting cones.

**2.** J336 is the vertex, and K506 is the center of the 4″ horizontal circular base of an oblique cone. A536 is the vertex, and B306 is the center of the 3″ horizontal circular base of an oblique cone. Draw the front and top views of the intersecting cones.

**3.** E336 and B536 are the vertices, and F506 and C306 are the respective centers of the horizontal 3″ circular bases of two oblique cones. Show the intersecting cones in the front and top views.

**4.** The intersecting cones of the preceding problem are to be made from sheet metal. Make the necessary pattern.

**5.** E246 is the vertex, and F506 is the center of the 4″ horizontal circular base of an oblique cone. F is the vertex, and E is the center of the 4″ horizontal circular base of an oblique cone. In the front and top views, show the intersecting cones.

**6.** A446 and B326 are the vertices, and X306 and Z506 are the respective centers of the 4″ horizontal circular bases of two oblique cones. Draw the front and top views of the intersecting cones.

## Group 73. Funnels and Spouts.

**1.** C646 is the center of a 3″ horizontal circle. P216 is the center of a 1″ profile circle. These circles are the ends of a three-piece conical funnel.

The sections are right cones. Draw a well-proportioned design of the funnel in the front view, and then complete the top view.

**2.** Show the development of the smaller end section and the middle section of the funnel of the preceding problem.

**3.** O847 is the center of a 2″ profile circle. P407 is the center of a $\frac{3}{4}''$ horizontal circle. These circles are the ends of a three-piece conical spout. Draw its front and top views. Show also the front view of the right cone from which the spout is made, and indicate how the cone is cut.

**4.** A326 is the center of a 3″ profile circle. B724 is the center, and X625 is a point on the axis of a 1″ circle. These circles are the open ends of a three-piece conical transition. Design a well-proportioned transition in the top view, and then complete the front view.

**5.** C6,$4\frac{1}{2}$,X is the center of a $2\frac{3}{4}''$ frontal circle. The center of a $\frac{1}{2}''$ horizontal circle is $4\frac{1}{2}''$ below and $2\frac{1}{2}''$ behind C. Draw the front and left side views of the circles. These circles are the ends of a four-piece conical spout. The conical sections are parts of right cones. In the side view, design the spout so that it looks well. Then complete the front view.

**6.** Develop the two largest sections of the spout of the preceding problem.

**7.** A806 is the center of a 4″ profile circle. B503 is the center of a $\frac{1}{2}''$ frontal circle. These circles are the ends of a five-piece conical horn. Draw its top view. Show also the development of the smallest section.

**8.** A284 B244 is the major axis of a frontal ellipse. The minor axis is $2\frac{1}{2}''$ long. O216 is the center of a $1\frac{3}{4}''$ horizontal circle. The ellipse and the circle are the ends of a six-piece ship's funnel. Design a well-proportioned funnel. Draw only the front and side views.

**9.** A784 B754 is the major axis of a frontal ellipse. The minor axis is 2″ long. O7,2,$5\frac{1}{2}$ is the center of a $1\frac{1}{2}''$ horizontal circle. The ellipse and the circle are the ends of a four-piece funnel. Design the funnel. Draw its front and left-side views, and show the true shape of the curve of each seam.

**10.** A24X B28X is the major axis of a frontal ellipse. The minor axis is 2″ long. The point O, 4″ below and 2″ behind A, is the center of a $1\frac{1}{2}''$ horizontal circle. The ellipse and the circle are the ends of a ventilating funnel. Design a well-proportioned funnel of six pieces. Draw its front and side views.

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## Group 74. Cylinder and Plane Intersections and Developments.

**1.** C246 D206 is the axis of a $1\frac{1}{2}''$ right circular cylinder. Develop the part of the cylinder lying below the plane E145 F305 G308, and show the true shape of the intersection of the plane and cylinder.

**2.** C0,1,$2\frac{1}{2}$ D6,1,$2\frac{1}{2}$ is the axis of a 1″ right circular cylinder. Show the front and top views, and develop the surface of all of the cylinder that is below the plane E0,$\frac{1}{4}$,4 F824 G828.

**3.** A525 X025 is the axis of a $1\frac{1}{2}''$ hollow cylindrical pipe with walls $\frac{1}{4}''$ thick. The right end of the pipe is sawed off in the vertical plane through E034 F536. Show the front, top, and right-side views of the remainder of the pipe.

**4.** A614 X617 is the axis of a double-walled right cylindrical sheet-metal pipe of 2″ outside diameter and 1¼″ inside diameter. The front end of the pipe is cut off by a vertical plane through S407 T804. Draw the front and top views of the part of the pipe that is left. Make the developments for the outside and inside walls, and for the two ends of the part of the pipe that is shown in the front and top views.

**5.** C227 is the center of a 2½″ frontal circle. This circle is the rear end of a right cylinder that is 2″ long. In the front and top views, show all of the cylinder that is behind the vertical plane through Q034 R337. Also develop the surface of this part of the cylinder, and show the true shape of the plane section.

**6.** A628 B624 is the axis of a 2″ right circular cylinder. Draw the front and top views, and develop that part of the cylinder that is behind the vertical plane through E408 F7½,0,4. Show also the true shape of the intersection.

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**7.** C225 D726 is the axis of a 1¾″ right circular cylinder. In the front and top views, show that part of the cylinder that lies behind the frontal plane through C.

**8.** Develop the part of the cylinder that is shown in the preceding problem.

**9.** E116 F536 is the axis of a 2″ right circular cylinder. In the front and top views, show the part of the cylinder that lies below the horizontal plane that is 1″ above E.

**10.** Develop the surface of the cylinder that is shown in the preceding problem.

**11.** X136 Y626 is the axis of a 1½″ right circular cylinder. Draw the front and top views of that part of the cylinder which lies below the plane Q024 R734 S737.

**12.** Develop the part of the cylinder that is shown in the preceding problem.

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## Group 75. Lines on Cylindrical Surfaces.

**1.** O536 P516 is the axis of a 4″ right circular cylinder. Show the path of the shortest distance on the surface of this cylinder from the point A514 to the point B63X on the rear of the cylinder.

**2.** E217 F247 is the axis of a 4″ right circular cylinder. In the front and top views, show the front half of the cylinder. Develop the surface. On the development draw a diagonal. Show this diagonal in the front view.

**3.** A606 B646 is the axis of a 2″ right circular cylinder. Develop the surface of the cylinder. Make the seam along the rear element. Centrally on the development, draw an equilateral triangle with one of its 3½″ sides parallel to one of the longer sides of the development. In the front view, show this triangle on the surface of the cylinder.

**4.** X246 Y206 is the axis of 2½″ right circular cylinder. Draw its front and top views. Develop the surface. Make the seam along the rear element. In the middle of the development draw a 3½″ circle. Show this circle in the front view of the cylinder. This represents the shape of a sheet-metal pipe that will fit a round hole in the surface of the 2½″ cylinder.

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## Group 76. Elbows and Bends.

**1.** C147 is the center of a 2″ horizontal circle. O317 is the center of a 2″ profile circle. These circles are the open ends of a four-piece, sheet-metal elbow. The least radius of the bend is $\frac{3}{4}''$. Draw the front, top, and right-side views of the elbow, and show the development of one of the middle sections.

**2.** O527 is the center of a 2″ profile circle, and C724 is the center of a 2″ frontal circle. These circles are the open ends of a three-piece elbow. The least radius of the bend is $\frac{1}{2}''$. Draw the front and top views of the elbow, and develop the middle section.

**3.** C446 is the center of a 3″ horizontal circle. D0,$1\frac{1}{2}$,6 is the center of a 3″ profile circle. These circles are the ends of a four-piece pipe elbow. The inner radius of the elbow is $\frac{1}{2}''$. Show two views of the elbow, and one-half of the development of one of the middle sections.

**4.** O224 is the center of a 3″ frontal circle. P6,2,$6\frac{1}{2}$ is the center of a 3″ profile circle. These circles are the open ends of a five-piece elbow. The least radius of the elbow is $\frac{1}{2}''$. Draw the front and top views of the elbow, and show one-half of the development of one of the middle sections.

**5.** C447 is the center of a 2″ profile circle. D707 is the center of a 2″ horizontal circle. These circles are the open ends of a seven-piece pipe elbow. The least radius of the elbow is $1\frac{1}{2}''$. Draw the front and top views of the elbow, and develop one of the middle sections.

**6.** A547 is the center of a 2″ profile circle, and B707 is the center of a 2″ horizontal circle. These circles are the open ends of a six-piece sheet-metal elbow. The least radius of the bend is $1\frac{1}{4}''$. Draw the front and top views of the elbow, and show the development of one of the middle sections.

**7.** A215 is the center and A X416 is the axis of a 2″ circle. B713 is the center of a 2″ frontal circle. These circles are the ends of a four-piece, sheet-metal pipe elbow. The least radius of the bend is $1\frac{1}{4}''$. Draw the front and top views of the elbow.

**8.** Develop the shorter end section and one of the middle sections of the elbow of the preceding problem.

**9.** A147 is the center of a $1\frac{3}{4}''$ horizontal circle. B514 is the center of a $1\frac{3}{4}''$ frontal circle. These circles are the open ends of an offset elbow connection that is made from two similar four-piece elbows. There is, however, no joint in the middle section of the connection. The least radius of each bend is $\frac{3}{4}''$. Draw the front and top views of the connection, and show the development of the middle section.

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**10.** A215 is the center, and A X416 is the axis of a 2″ circle. B713 is the center of a 2″ frontal circle. These circles are the open ends of a sheet-metal, three-piece, pipe elbow. The least radius of the bend is 1″. Draw the front and top views of the elbow.

**11.** Show the developments of the shorter end-section and the middle section of the elbow of the preceding problem.

**12.** C247 and D647 are the centers of $1\frac{1}{2}''$ circles. The axes of the circles intersect at X407. These circles are the open ends of a five-piece pipe elbow.

The least radius of the bend is 1″. Draw the front and top views and develop one of the middle sections.

**13.** A124 and B524 are the centers of two 1½″ frontal circles. These circles are the open ends of a thirteen-piece, U-shaped pipe bend. The center of the bend is at O325. Draw a complete top view, a partially completed front view, and develop one of the middle sections.

**14.** E346 and F746 are the centers of two 2″ horizontal circles. These circles are the open ends of a nine-piece, sheet-metal, U-shaped bend. The center of the bend is at C536. Draw a complete front view, a partially complete top view, and develop one of the middle sections.

**15.** O257 and P507 are the centers of two 2″ horizontal circles. These circles are the open ends of an offset pipe connection that is made from two similar three-piece elbows. Draw its front and top views and develop one of the sections.

**16.** K128 and L424 are the centers of two 2″ frontal circles. These circles are the open ends of an offset pipe connection that is made from two similar two-piece pipe elbows. Draw its front and top views, and develop one of the sections.

**17.** A416 B537 and B C848 are the center lines of 1½″ cylindrical pipes. The pipes form a two-piece elbow. In a normal view of the center lines, show the pipes and the angle at which they are joined.

**18.** The center lines of a two-piece pipe elbow are B426 C615 and C D836. In a normal view of these lines, show the elbow.

**19.** D518 E645 and E F828 are the center lines of two 2″ cylindrical pipes that are joined together. In a normal view of the center lines, show the two-piece joint. The elbow is too sharp to be practical. In the same view, change the joint to a three-piece elbow.

**20.** A212 D412 and B023 C025, respectively, are the upper edges of a frontal and a profile 1″ by 2″ rectangle. These rectangles are the open ends of a pipe bend of uniform cross-section. The least radius of the bend is ¾″. Draw the front and top views of the bend, and make a complete set of patterns.

**21.** A315 B515 is the upper edge of a 1″ by 2″ frontal rectangle. E146 F246 is the front edge of a 1″ by 2″ horizontal rectangle. These rectangles are the openings of a two-bend pipe connection of uniform cross-section. The least radius of the bends is ¾″. Draw the front and top views of the connection.

**22.** Referring to the pipe connection of the preceding problem, develop the surface of which A B is one edge. Develop also the surface of which E F is one edge.

**23.** The open ends of a three-bend pipe connection of uniform cross-section are rectangular. A5½,3,4 B834 is the upper edge of the frontal opening, and E5,1,5½ F5,0,5½ is the front edge of the profile opening. The least radius of the bends is 0″. Draw the front, top, and left-side views of the connection.

**24.** Referring to the pipe connection of the preceding problem, develop the surface of which A B is one edge, and also develop the surface of which E F is one edge.

## Group 77. Developments of Oblique Cylinders.

**1.** C236 and D417 are the centers of two 2″ horizontal circles. The circles are the bases of an oblique cylinder. Draw the front and top views, the normal auxiliary elevation, and the end view of the cylinder.

**2.** Draw the development of the cylindrical surface of the preceding problem.

**3.** Two horizontal 2″ circles centered respectively at A517 and B726 are the bases of an oblique cylinder. Draw its front and top views, its normal auxiliary elevation, and its end view.

**4.** Develop the surface of the oblique cylinder of the preceding problem.

**5.** E627 and F735 are the centers of the $1\frac{1}{2}''$ horizontal circular bases of an oblique cylinder. Draw the front and top views of the cylinder. Draw also an auxiliary elevation taken in a direction perpendicular to the axis, and an end view taken in a direction parallel to the axis. Develop the surface of the cylinder.

**6.** C135 and O217 are the centers of the $1\frac{3}{4}''$ horizontal circular bases of an oblique cylinder. Draw the front and top views, a normal auxiliary elevation, and an end view of the cylinder.

**7.** Develop the cylindrical surface of the preceding problem.

**8.** A126 B116, C326 D316, and E246 F236 are the axes of three $1\frac{1}{2}''$ right circular cylinders. The cylinders are the ends of a Y. Connect the lower cylinders with the upper by means of two equal oblique cylinders. Draw the front and top views of the Y. Obtain a right section of the oblique cylinders and develop the surface of one of them.

**9.** C236 is the center of a 2″ horizontal circle. E116 and F316 are the centers of two horizontal half-circles, 2″ in diameter, that form the rounded ends of a rectangle. The circle and the rectangle with rounded ends are the open ends of a transition. Draw its front and top views, and show a right section of the oblique cylindrical surfaces. Make a development showing one of the oblique cylindrical surfaces and one of the triangular faces.

**10.** A146 is the center of a 2″ horizontal circle. C025 D425 is the front edge of a 2″ by 4″ horizontal rectangle. Round the ends of the rectangle so that they are half-circular. The circle and the rectangle with rounded ends are the open ends of a transition. Draw its front and top views. Line shade the front view. Draw a right section of the inclined cylindrical surface. Develop all of the surface, with the exception of the right cylindrical surface.

## Group 78. Intersecting Cylinders.

**1.** $A2,4,5\frac{3}{4}$ $B2,1,5\frac{3}{4}$ is the axis of a $1\frac{1}{2}''$ right circular cylindrical pipe. C016 D416 is the axis of a 2″ right circular cylindrical pipe. Show the intersection of the pipes. Develop the smaller pipe.

**2.** A246 B206 is the axis of a 2″ right circular cylinder. C026 D426 is the axis of a 1½″ right circular cylinder. Find the intersection of the cylinders. Develop one end of the smaller cylinder.

**3.** A625 B627 is the axis of a 3″ right circular cylinder. C436 D836 is the axis of a 1″ right circular cylinder. Draw the front and top views of the intersecting cylinders. Develop the entire surface of the smaller cylinder. Develop enough of the surface of the larger cylinder to show the shape of the hole that is cut in it.

**4.** A304 B308 is the axis of one-half of a 6″ right circular cylinder. C026 D626 is the axis of a 1¾″ right circular cylinder. Find the intersection of the surfaces.

**5.** A504 B508 is the axis of one-half of a 6″ right circular cylinder. C226 D826 is the axis of a 2½″ right circular cylinder. Find the intersection of the cylindrical surfaces.

**6.** A646 B606 is the axis of a 3½″ right circular cylinder. C3,3,5½ D5,1,5½ is the axis of a 2″ right circular cylinder. In the front and top views, show the intersecting cylinders.

**7.** Develop the smaller cylinder of the preceding problem, and develop enough of the larger cylinder to show the shape of the hole that must be cut into it to fit the smaller cylinder.

**8.** A346 B306 and C1,1,5½ D5,3,5½ are respectively the axes of 2½″ and 1½″ right circular cylinders. Draw the front and top views of the intersecting cylinders, and develop one end of the smaller cylinder.

**9.** Draw the front and top views of a 3″ right circular cylinder with axis A246 X206, intersecting a 2″ right circular cylinder with axis O5,3,5½ P2,1½,5½.

**10.** Show the development of the smaller cylinder of the preceding problem, and enough of the development of the larger cylinder to show the shape of the hole in the development.

**11.** C117 D457 and E417 F157 are the axes of 1¾″ right circular cylinders. Show the intersecting cylinders in the front and top views. Show also a development from which each of the four parts of the cross may be made.

**12.** A246 B206, C226 D546, and E246 F526 are the axes of three 2″ right circular cylinders. Show their intersections in the front and top views.

**13.** Make developments for the C D and E F cylinders of the preceding problem.

**14.** A646 B606 is the axis of a 3″ right circular cylinder. E3,3½,6 F626 is the axis of a 2″ right circular cylinder. K2,1½,6 L536 is the axis of a 1½″ right circular cylinder. Draw the front and top views of the intersecting cylinders.

**15.** Develop the surfaces of the E F and K L cylinders of the preceding problem.

**16.** B447, C227, and D425 are the centers of the 1½″ circular openings in a three-way, sheet-metal, pipe fitting. The axes of the three cylindrical parts intersect at A427. Draw the front and top views of the fitting, and make the necessary developments. Is it possible to design this fitting so that one development will serve for the three parts?

**17.** A357 B337 is the axis of a 2″ right circular cylinder. Two other cylinders of the same size, each having a frontal axis that makes 120° with A B, and with one end at B, form with the A B cylinder a three-way pipe fitting. Draw its front and top views, and show one development for the three similar parts.

**18.** A457 B437, C117 D107, E717 F707, B C, and B E are all axes of 2″ right circular cylinders that form a sheet-metal breeching. Where two cylinders join, divide the angle of the joint equally between them. Show the front and top views of the breeching.

**19.** Make a complete set of patterns for the breeching of the preceding problem.

**20.** The lines A4,5,$6\frac{1}{2}$ B4,3,$6\frac{1}{2}$, C2,1,$6\frac{1}{2}$ D2,0,$6\frac{1}{2}$, E6,1,$6\frac{1}{2}$ F6,0,$6\frac{1}{2}$, B C, and B E are the axes of five $2\frac{1}{2}''$ right circular cylinders. These cylinders form a breeching. Where the cylinders meet, divide the angle of the joint equally between them. Draw the front and top views of the breeching.

**21.** Make a complete set of patterns for the breeching of the preceding problem.

## Group 79. Intersecting Cylinder and Cone.

**1.** V224 is the vertex, and C228 is the center of the 4″ base of a right circular cone. A036 B436 is the axis of a 3″ right circular cylinder. Find the intersection of the cone and cylinder.

**2.** Develop the surface of the cone of the preceding problem, showing the hole that must be cut in it so that it will fit around the cylinder.

**3.** V424 is the vertex, and C428 is the center of the 4″ base of a right circular cone. A346 B306 is the axis of a 3″ right circular cylinder. Find the intersection of the cone and cylinder.

**4.** Show the shape of the hole that must be cut in the development of the cylinder of the preceding problem.

**5.** V424 is the vertex, and C428 is the center of the 4″ circular base of a right cone. Find the intersection of this cone with the 3″ right circular cylinder that has A3,$1\frac{1}{2}$,4 B3,$1\frac{1}{2}$,8 for its axis.

**6.** C536 is the center of the 4″ circular base, and V506 is the vertex of a right cone. In the front and top views, show the cone and its intersection with the 2″ right circular cylinder of which E216 F816 is the axis.

**7.** Develop the part of the cone that is above the cylinder of the preceding problem.

**8.** V246 is the vertex, and C206 is the center of the 3″ circular base of a right cone. A016 B416 is the axis of a $1\frac{3}{4}''$ right circular cylinder. Show the intersection of the cone and cylinder.

**9.** Develop the cone and one end of the cylinder of the preceding problem.

**10.** V246 is the vertex, and C206 is the center of the 4″ base of a right circular, sheet-metal cone. A226 B426 is the axis of a sheet-metal pipe $1\frac{3}{4}''$ in diameter. Draw the front and top views of the intersecting surfaces.

**11.** V624 is the vertex, and C628 is the center of the $3\frac{1}{2}''$ circular base of a right cone. A546 B506 is the axis of a $2\frac{1}{2}''$ right circular cylinder. Show three views of the surfaces and their intersection.

**12.** Make a pattern for the cylinder of the preceding problem so that it will fit around the cone.

**13.** V646 is the vertex, and C606 is the center of the 4″ circular base of a right cone. A546 B506 is the axis of a $1\frac{1}{2}''$ right circular cylinder. Show the front and top views of the intersecting surfaces. Develop the cylinder to fit the cone.

**14.** C405 is the center of the 6″ horizontal circular base, and V425 is the vertex of a cone. E$2\frac{1}{2}$,2,5 F$2\frac{1}{2}$,0,5 is the axis of a 2″ right circular cylinder. Draw the front and top views of the intersecting surfaces.

## Group 80. Developments by Triangulation.

**1.** O225 is the center of a $2\frac{1}{2}''$ frontal circle. A207 B247 is the major axis of a frontal ellipse. The minor axis is $1\frac{1}{2}''$ long. The ellipse and the circle are the open ends of a pipe connection. Draw its front, top, and right-side views, and develop the upper half of the surface.

**2.** A426 B826 is the major axis of a horizontal ellipse. The minor axis is 2″ long. C636 is the center of a 3″ horizontal circle. The ellipse and the circle are the ends of a sheet-metal transition. Draw its front and top views, and develop the rear half of the surface.

**3.** A416 B816 and E638 F634 are the major axes of two horizontal ellipses having minor axes 2″ long. Show the front and top views of a transition piece connecting the two ellipses, and develop the rear half of the surface.

**4.** A126 B326 and E245 F247 are the minor axes of two horizontal ellipses. The major axes are 3″ long. The ellipses are the open ends of a pipe transition. Draw the front and top views and develop the rear half of the surface.

**5.** P524 is the center of a $2\frac{1}{2}''$ frontal circle. O627 is the center, and X425 is a point on the axis of a $2\frac{1}{2}''$ circle. The circles are the open ends of a transition. Draw its front and top views, and develop the upper half of the surface.

**6.** A746 is the center of a 2″ horizontal circle. B626 is the center of a 3″ horizontal circle. These circles are the open ends of a transition. Show its front and top views, and develop the surface.

**7.** A136 and B436 are the centers of two 2″ horizontal circles. C$2\frac{1}{2}$,0,6 is the center of a 3″ horizontal circle. These circles are the open ends of a Y. Draw its front, top, and right-side views.

**8.** Make a pattern for one of the two similar parts of the Y of the preceding problem. Since the vertices of the oblique cones of this Y are out of reach, it is necessary to develop the surface by triangulation.

## Group 81. Ventilators.

**1.** The base diameter of a standard ship's ventilator, made up of four girth sections, is 3′. To a scale of $\frac{1}{2}'' = 1'$, draw the profile of the ventilator, and one-half of the development of the mouth section.

**2.** The base diameter of a standard ship's ventilator, made up of four girth sections, is 4′. To a scale of $\frac{1}{2}'' = 1'$, draw the profile of the ventilator, and one-half of the development of the next to the largest section.

**3.** The base diameter of a standard ship's ventilator is 18″. It is made up of four girth sections. Show the profile view of the ventilator, and one-half of the development of the next to the smallest section. Scale $1\frac{1}{2}''$ = 1′.

**4.** A standard ship's ventilator of 4′ base diameter is made up of six girth sections. Draw the profile of the ventilator to a scale of $\frac{1}{2}''$ = 1′, and make the pattern for one-half of the largest section.

## Group 82. Convolutes.

**1.** The line A646 B616 is the common axis of a 2″ and a $3\frac{1}{2}''$ cylinder. A convolute lies between the cylinders. The directrix is a left-hand helix of 3″ lead. One end of the helix is 1″ behind A. Draw the front and top views, and develop one turn of the convolute. Cut the development radially.

**2.** C228 D225 is the common axis of a $1\frac{1}{2}''$ and a $3\frac{1}{2}''$ cylinder. A right-hand helix of 3″ lead on the smaller cylinder is the directrix of a convolute. One end of the helix is to the right of C. The larger cylinder limits the surface of the convolute. Draw the front and top views, and develop one flight of the convolute. Cut the development radially.

**3.** A414 B418 is the axis of a right-hand helix of 4″ lead. One end of the helix is $\frac{1}{2}''$ below A. The helix is the directrix of a convolute. Draw the front and top views of the part of the convolute that is between the helix and the frontal plane through A. Show sixteen elements and the intersection of the surface with the given frontal plane.

**4.** The line E436 F406 is the axis of a left-hand helix of 3″ lead. One point of the helix is $\frac{1}{2}''$ behind E. The helix is the directrix of a convolute. Draw the front and top views of the part of the convolute lying between the helix and the horizontal plane through F. Show sixteen elements and the intersection of the surface with the given horizontal plane.

# PROBLEMS—CHAPTER XI

## WARPED SURFACES

### Group 83. Hyperbolic Paraboloid.

**1.** A628 B246 and C515 D107 are the directrices of an hyperbolic paraboloid. Divide each directrix into sixteen equal parts. A D and B C are elements of the surface. Draw fifteen other elements.

**2.** The lines C028 D836 and E024 F816 are the directrices, and C F and E D are two elements of an hyperbolic paraboloid. Draw fifteen other elements.

**3.** A707 B434 and C008 D634 are the directrices of an hyperbolic paraboloid. Divide each directrix into twenty-four equal parts. B C and A D are elements of the surface. Draw twenty-three other elements.

**4.** E145 F418 and K404 L737 are the directrices of an hyperbolic paraboloid. E K and F L are elements. Draw eleven other elements equally spaced. Show the intersection of the surface with the horizontal plane $\frac{1}{2}''$ above F.

**5.** The straight lines K037 L437 and A004 B406 are respectively the upper and lower edges of the wing wall of a dam. The surface of the wall is an hyperbolic paraboloid. The plane director is a profile plane. Draw seventeen equally spaced elements of the surface. Show the intersections of the surface with five equally spaced frontal planes.

**6.** C538 D734 and E718 F514 are the elements of one generation of an hyperbolic paraboloid, and C E and F D are elements of the other generation. In the front, top, and left-side views, draw fifteen other elements of each generation. Show also the intersection of the surface with the profile plane that is $\frac{3}{4}''$ to the right of C.

**7.** The line R138 S638 represents the ridge, and the line E315 F716 represents the eaves of a roof. Draw single lines representing six equally spaced rafters. Draw also single lines representing the upper edges of eight equally spaced sheathing boards. Show the intersection of the roof with the vertical plane through E and S.

**8.** The square A106 B308 C506 D304 represents the eaves, and the line E126 F526 represents the ridge of a roof. The roof is to be made of two similar pairs of hyperbolic paraboloids. Show nine equally spaced rafters for each quarter of the roof. Draw the auxiliary elevation taken in the direction D A. In three views, show the intersection of the roof with the frontal plane $\frac{1}{2}''$ in front of A F.

## Group 84. Conoid.

**1.** The 3″ frontal circle centered at O228, and the line A024 B424 are the directrices, and a profile plane is the plane director of a conoid. Draw the front, top, and right-side views of the conoid. Show twenty-four elements equally spaced around the circle. Show also the intersection of the surface with the frontal plane that is halfway between the two directrices.

**2.** C506 is the center of a 4″ horizontal circle. This circle and the line A244 B248 are the directrices, and a frontal plane is the plane director of a conoid. Draw thirty elements spaced $\frac{1}{4}''$ apart along A B. Show the intersection of the surface with a horizontal plane that is 2″ above C.

**3.** The line D434 E808 and the $2\frac{1}{2}''$ horizontal circle centered at C646 are the directrices of a conoid. All of the elements are frontal. In the front, top, and left-side views, show twenty-four elements of the surface equally spaced around the circle. Show also the intersection of the conoid with the horizontal plane 1″ below C.

**4.** C528 is the center of a $3\frac{1}{2}''$ frontal circle. This circle and the line A147 B404 are the directrices, and a horizontal plane is the plane director of a conoid. Draw twenty-eight elements equally spaced along the directrix A B. Show the intersection of the surface with a frontal plane through A.

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**5.** The square A145 B445 C415 D115 and the 3″ frontal circle centered at $O2\frac{1}{2},2\frac{1}{2},8$ are the open ends of a cast-iron pipe. The upper and lower surfaces of the hole through the pipe are conoidal, and the sides are triangular. Show the front, top, and right-side views of the interior surface of the pipe, by means of elements spaced $\frac{1}{4}''$ apart.

**6.** The square A745 B445 C415 D715 and the 3″ frontal circle centered at $O5\frac{1}{2},2\frac{1}{2},8$ are the open ends of a cast-iron pipe. The entire inner surface of the pipe is made up of parts of conoids. Design the interior of the pipe, choosing the proper directrices and plane director, and then draw the front, top, and left-side views by choosing elements spaced $\frac{1}{8}''$ apart.

**7.** C214 is the center of the upper half of a 4″ frontal circle. This curve and the straight line E027 F427 are the directrices of the conoidal ceiling of a hall. All elements of the surface are perpendicular to E F. Draw the front, top, and right-side views of the conoidal surface, spacing the elements $\frac{1}{4}''$ apart.

**8.** O226 is the center of a 3″ horizontal circle. This curve and a part of the line A006 B406 are the directrices of a right conoid. In the front, top, and right-side views, draw sixteen elements equally spaced around the circle.

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## Group 85. Helicoid.

**1.** A446 B406 is the axis, and O326 and P306 are the ends of one turn of a left-hand helix. The helix and its axes are the directrices of a helicoid. A O is one element. In the front and top views, show sixteen equally spaced elements of the surface extending from the axis to the horizontal plane through B. Show the intersection of the surface with this plane. Show also the cone director with vertex at V735, and the center of its base at C715.

**2.** A446 X406 is the axis of one turn of a right-hand helix of 3″ lead. The lower end of the helix is $\frac{3}{4}$″ to the left of X. V735 is the vertex, and C705 is the center of the 2″ circular base of a right cone. The helix and its axis are the directrices and the cone is the cone director of a helicoid. Draw the front and top views of twenty-four equally spaced elements of the part of the helicoid that lies between the axis and the horizontal plane through X, and show the intersection of the surface with this plane.

**3.** A425 X428 is the axis of a right-hand helix of 3″ diameter and 1″ lead. One end of the helix is directly to the left of X. The helix and its axis are the directrices of a helicoid, all elements of which make an angle of 45° with the axis. In the front and top views, show sixteen equally spaced elements extending from the axis to the frontal plane through X. Show also the intersection of the surface with this plane.

**4.** K346 L306 is the axis of a 1″ right circular cylinder. On the cylinder is one turn of a left-hand helix of 3″ lead. The upper end of the helix is directly behind K. C736 is the center of the 2″ circular base, and V716 is the vertex, of a right cone. The cone is the cone director, and the helix and its axis are the directrices of a helicoid. In the front and top views, show twenty-four equally spaced elements of the surface extending from the axis to the horizontal plane through K. Show also the intersection of the helicoid with this plane.

## Group 86. Helicoidal Chutes and Conveyors.

**1.** A436 B416 is the common axis of a 1″ and a 4″ right circular cylinder. On the larger cylinder is a right-hand helix of 2″ lead. The upper end of the helix is to the right of A. The helix and its axis are the directrices of a right helicoidal chute. Regarding the cylinders as transparent, show, in the front and top views, thirty-two equally spaced elements of the part of the helicoidal surface that lies between the two cylinders.

**2.** A246 B206 is the common axis of a $1\frac{1}{2}$″ and of a 4″ right circular cylinder. On each cylinder is a right-hand helix of 4″ lead. The upper ends are to the right of A. The helices are the limits of a right helicoidal, or "spiral", chute. Draw its front and top views, showing thirty-two equally spaced elements, and develop one-fourth of the flight by triangulation.

**3.** A246 X206 is the common axis of a $1\frac{1}{2}$″ and a 4″ right circular cylinder. Between these cylinders is one turn of a right helicoidal chute. The top end of the chute is to the right of A, and slopes forward and downward. Design the chute so that the slope of the surface halfway between the cylinders is $22\frac{1}{2}$° to the horizontal. Show sixteen equally spaced elements and the limiting helices of one turn of the chute, and the development of one-fourth of the surface by triangulation.

**4.** A646 X606 is the common axis of a $1\frac{1}{2}$″ and a 4″ right circular cylinder. Between these cylinders is one turn of a left-hand right helicoidal, or "spiral", chute. The top end of the chute is to the left of A. Design the chute so that the slope of the surface halfway between the cylinders is 15° to the horizontal. Show sixteen equally spaced elements and the limiting helices of one-half turn of the chute in the front and top views. Show also the development by triangulation of one-fourth of one turn of the surface.

**5.** A right-hand screw conveyor having a right helicoidal surface, has an outside diameter of 16″, and a length of 4′. The center shaft is 3″ in diameter, and the lead is 12″. Draw the front and end views of the conveyor to a scale one-eighth full size. Erase all hidden lines.

**6.** The line A028 B528 is the common axis of a $\frac{1}{2}$″ and a $2\frac{1}{2}$″ right circular cylinder. A right helicoidal screw conveyor is wrapped left handed around the smaller cylinder, and is limited by the larger cylinder. The lead is such that the outer edge has a slope of $22\frac{1}{2}$° measured with a plane that is perpendicular to the axis. Draw the front and end views of the conveyor, and develop one-fourth of a turn by triangulation.

## Group 87. Screws.

**1.** A right-hand, single, square-threaded screw, 4″ in diameter and 4″ long, has $\frac{2}{3}$ thread per inch. Draw the front and end views of the screw.

**2.** C446 D406 is the axis of a 3″ right circular cylinder on which is cut a screw thread having the following specifications: left-hand, single square thread, one thread per inch. Show the front and top views of the screw.

**3.** Draw the front and end views of a left-hand, quadruple, square-thread screw of 4″ lead, $2\frac{3}{4}$″ diameter, and 5″ length.

**4.** A458 B408 is the axis of a 6″ right circular cylinder. Draw its front view and the front half of the top view. Into this cylinder is cut a left-hand, double, square thread of 4″ lead. Complete the views of the screw.

**5.** Draw the front and top views of a right-hand, triple, square-threaded screw of 3″ lead, 2″ diameter, and 5″ length.

**6.** A457 X407 is the axis of a 2″ right circular cylinder. In the middle of this cylinder turn a rectangular groove 1″ wide by $\frac{3}{8}$″ deep. On one end of the cylinder cut a right-hand, single square thread, $1\frac{1}{2}$ threads per inch. On the other end cut a similar left-hand thread. Draw the front and top views.

**7.** E026 F526 is the axis of a 2″ right circular cylinder. Into this cylinder are cut a right-hand and a left-hand helical groove, $\frac{1}{2}$″ square, and of 2″ lead. Show the front, top, and end views of the grooved cylinder.

**8.** A screw is cut with both right-hand and left-hand single square threads of 1″ lead on a cylinder 3″ in diameter and 5″ long. Show the front, side, and end views of the screw.

**9.** A44X B04X is the axis of a 3″ right circular cylinder. On this cylinder is cut a 60°, sharp V, single, right-hand thread, one thread per inch. Draw the front and end views of the screw.

**10.** A458 B408 is the axis of a 5″ right circular cylinder. Show the front view and the front half of the top view. On this cylinder is cut a 60°, sharp V, single, right-hand thread, of 2″ lead. Complete the front and top views of the screw. This screw has very little core. It is typical of the appearance of the small screws that are used in watches.

## Group 88. Springs.

**1.** Draw the front and top views of a left-hand, helical spring of 4″ outside diameter, 3″ length, and $\frac{1}{2}$″ lead. The spring is to be made from bronze of $\frac{1}{8}$″ by $\frac{3}{4}$″ rectangular cross-section, wound on edge.

**2.** Show the front and end views of five turns of a helical spring, 2″ outside diameter, 1″ lead, wound left handed from stock $\frac{3}{8}$″ square.

**3.** Show the front and end views of five turns of a helical spring, 3″ outside diameter, $\frac{3}{4}$″ lead, wound right handed on edge from rectangular stock $\frac{1}{4}$″ by $\frac{3}{4}$″.

**4.** A346 is the vertex, and B316 is the center of the 4″ circular base of a right cone. A conical spring is wound right handed around this cone. The lead is $\frac{7}{8}$″ measured parallel to the axis. The material of the spring is $\frac{1}{8}$″ by $\frac{3}{8}$″ rectangular steel, and is wound flatwise so that a cross-section of the spring shows the $\frac{3}{8}$″ edges of the steel parallel to the axis of the cone. Draw the front and top views of the spring with the cone removed.

## Group 89. Hyperboloid of Revolution.

**1.** A4,3,5$\frac{1}{2}$ B4,0,5$\frac{1}{2}$ is the axis and C635 D205 is the generatrix of an hyperboloid of revolution. In the front and top views, draw twenty-four elements of each generation.

**2.** C426 is the center of a 1$\frac{1}{2}$″ horizontal circle, and A446 and B406 are the centers of 4″ horizontal circles. The circles are the directrices of an hyperboloid of revolution. In the front and top views, draw twenty-four elements of one generation.

**3.** A446 and C426 are the centers of 3″ horizontal circles, and B436 is the center of a 1″ horizontal circle. The three circles are the directrices of an hyperboloid of revolution. In the front and top views, draw sixteen elements of each generation.

**4.** A446 is the center of a 2″ horizontal circle. B4X6 is the center of a 1″ horizontal circle. C406 is the center of a 4″ horizontal circle. These circles are the directrices of an hyperboloid of revolution. In the front and top views, show thirty-two elements of one generation.

## Group 90. Skew Gears.

**1.** A4,1$\frac{1}{2}$,6 B7,1$\frac{1}{2}$,6 and C535 D637 are the center lines of two $\frac{3}{4}$″ round shafts. On the shafts are to be mounted two equal skew gears, each 3″ long. In the front and top views, show the outlines of the rolling surfaces of the gears. Show also the holes through the gears.

**2.** A5,2,6$\frac{1}{2}$ B7,2,6$\frac{1}{2}$ and C6,4,7$\frac{1}{2}$ D6,4,5$\frac{1}{2}$ are the center lines of two shafts. In the front, top, and left-side views, show two equal hyperboloids of revolution, each 2″ long, mounted on the shafts, and so proportioned that the surfaces will roll together and rotate at the same speed. In the top view, show the visible parts of twelve elements equally spaced around the upper gear. These elements are to be taken so that when the upper gear rotates clockwise as seen in the front view, the lower gear will rotate counterclockwise as seen in the left-side view. Should the teeth of these two gears be cut alike, or should one be cut right hand and one left hand?

**3.** Design the skew gears of the preceding problem so that when the upper gear rotates in a clockwise direction as seen in the front view, the lower gear will also rotate in a clockwise direction as seen in the left-side view. Draw the elements in the proper direction. Should the teeth of the two gears be cut alike, or should they slant in opposite directions?

**4.** C036 D436 and E128 F324 are the center lines of two shafts that are to be geared together by two equal skew gears that are $2\frac{1}{2}''$ long. Show the front, top, and end views, of the rolling surfaces of the two gears by drawing twelve equally spaced elements on each gear.

# PROBLEMS—CHAPTER XII

## DOUBLE CURVED SURFACES

### Group 91. Intersecting Sphere and Plane.

**1.** C426 is the center of a $3\frac{3}{4}''$ sphere. Find its intersection with the plane D716 E447 F335.

**2.** O425 is the center of a $2''$ sphere. Show the larger part of the sphere that is left when a part is cut off by the plane R346 S516 T617.

**3.** O226 is the center of a $3\frac{1}{2}''$ sphere. By means of an auxiliary elevation, determine whether or not the plane Q418 R715 S346 intersects the sphere. If so, draw the front and top views of the intersection.

**4.** Determine whether or not the plane R138 S437 T2,$3\frac{1}{2}$,6 intersects the $2\frac{1}{2}''$ sphere centered at C625. If so, show the front and top views of the intersection.

**5.** The side view is an edge view of the plane Q035 R028. C226 is the center of a $2''$ sphere. In the front, top, and right-side views, show all of the sphere that lies below the given plane.

**6.** A626 is the center of a $3''$ sphere. In the front, top, and left-side views, show all of the sphere that is above the plane R824 S416 T708.

**7.** C326 is the center of a $3''$ sphere. In the front and top views, show the section that is cut from the front half of this sphere by the vertical planes through C A224 and C B524.

**8.** A408 is the center of the front half of the upper half of a $6''$ sphere. In the front and top views, show the notch that is cut into this part of the sphere by the vertical planes D437 E135 and D F535.

### Group 92. Intersecting Sphere and Prism.

**1.** A $2\frac{1}{2}''$ cube has the top end chamfered to a spherical surface of $2''$ radius. Show the plan, elevation, and elevation at 45° to the vertical faces.

**2.** A right hexagonal prism, $4''$ across the corners and $2''$ tall, has the top end chamfered to a spherical surface of $3''$ radius. Show the front, top, and right-side views of the chamfered prism.

**3.** The vertical axis of a hexagonal hole through a $4''$ sphere passes through the center C626 of the sphere. The faces of the hexagonal hole are $1\frac{1}{2}''$ wide, and two of the faces are frontal. In the front, top, and left-side views, show what is left of the sphere after the hole is cut through it.

**4.** O226 is the center of a $3''$ sphere. A106 C306 is the diagonal of a horizontal square. The square is the base of a right prism of $4''$ altitude. Draw the front, top, and right-side views of the intersecting sphere and prism.

## Group 93. Intersecting Spheres.

**1.** C307 is the center of a 6″ sphere. O4,2,$5\frac{1}{2}$ is the center of a 3″ sphere. Find the intersection of the spheres in the front and top views.

**2.** C226 is the center of a 3″ sphere. O$2\frac{1}{2}$,$2\frac{1}{2}$,$5\frac{1}{2}$ is the center of a $1\frac{1}{2}$″ sphere. Show the front, top, and right-side views of the spheres and their intersection.

**3.** O408 is the center of an 8″ sphere and C516 is the center of a 3″ sphere. Show the front and top views of the intersection.

**4.** C318 is the center of a 6″ sphere. D436 is the center of a 4″ sphere. Find the intersection of the spheres in the front and top views.

**5.** C336 and D417 are the centers of 3″ spheres. In the front and top views, show the intersection of the spheres.

**6.** In the front and top views, show the 4″ sphere centered at C426, and the hole that must be cut into it to fit the $2\frac{1}{2}$″ sphere centered at D6,$1\frac{1}{2}$,5.

**7.** C226, D4,2,$6\frac{1}{2}$, and E6,2,$5\frac{1}{2}$ are the centers of 3″ spheres. In the front and top views, show the part of each sphere that is external to the other spheres.

**8.** Draw the front and top views of the three 2″ intersecting spheres centered respectively at A$3\frac{1}{2}$,$2\frac{1}{2}$,$5\frac{1}{2}$, B427, and C536.

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**9.** Show the front and top views of two 4″ spheres that pass through the points A314 and B427, and rest on a horizontal plane 1″ below A.

**10.** A $2\frac{3}{4}$″ sphere rolls on a horizontal plane at the zero level. Show the front and top views of the two possible positions of the sphere when it rolls against the points E216 and F425.

**11.** A 3″ sphere rests on the points D326, E425, and F527. Draw the front and top views of the sphere.

**12.** Show the front and top views of the $3\frac{1}{2}$″ sphere that rests on the points A516, B414, and C215.

**13.** Show the front, top, and right-side views of the $2\frac{3}{4}$″ sphere that rests on the points C116, D327, and E235.

**14.** Draw the front and top views of the 3″ sphere that rests on the points J325, K437, and L516.

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## Group 94. Intersecting Sphere and Cylinder.

**1.** A025 B425 is the axis of a $1\frac{1}{2}$″ right circular cylinder. C226 is the center of a $3\frac{1}{2}$″ sphere. Show the intersection of the cylinder and the sphere in the front, top, and right-side views.

**2.** A334 B338 is the axis of a 2″ right circular cylinder. C326 is the center of a 4″ sphere. Find the intersection of the sphere and the cylinder, and develop the front half of the cylinder.

**3.** C2,$1\frac{1}{2}$,6 is the center of a 3″ sphere. A224 B228 is the axis of a 2″ right circular cylinder. Draw the front, top, and right-side views of the intersecting surfaces.

**4.** A224 X228 is the axis of a $2\frac{1}{2}$″ right circular cylinder. C$1\frac{1}{2}$,$1\frac{1}{2}$,6 is the center of a 3″ sphere. Find the intersection of the two surfaces. Develop that part of the surface of the cylinder that is outside the sphere.

**5.** C5½,2,6 is the center of a 4″ sphere. E746 F706 is the axis of a 2″ right circular cylinder. Show the surfaces and their intersection. Develop enough of the surface of the cylinder to show the shape of the hole in the cylinder.

**6.** A247 B505 is the axis of a cylinder, the bases of which are horizontal 2″ circles. C426 is the center of a 3½″ sphere. Show the intersecting surfaces in the front and top views.

**7.** C526 is the center of a 4″ sphere. E305 F745 is the axis of a cylinder having 2″ circular horizontal bases. In the front and top views show the intersecting surfaces.

**8.** C216 is the center of the upper half of a 4″ sphere. The center of a 3″ spherical milling cutter moves along the line A036 B436, and cuts a groove in the hemisphere. In the front, top, and right-side views, show what is left of the hemisphere after the groove is cut in it.

## Group 95. Intersecting Sphere and Cone.

**1.** V246 is the vertex, and C206 is the center of the 4″ circular base, of a right cone. O2½,2,5½ is the center of a 3″ sphere. Show the front, top, and right-side views of the intersecting surfaces.

**2.** Develop the cone of the preceding problem. Show the hole that must be cut to permit the cone to fit the sphere.

**3.** C228 is the center of the 3½″ circular base of a right cone, and the vertex is at V224. E326 is the center of a 3″ sphere. Draw the front, top, and right-side views of the intersecting surfaces.

**4.** Develop that part of the surface of the cone of the preceding problem that lies outside the sphere.

**5.** O226 is the center of a 3″ sphere. C306 is the center of the 2″ circular base, and A346 is the vertex of a right cone. Show the top, front, and right-side views of the surfaces and their intersection.

## Group 96. Approximate Development of Sphere.

**1.** O226 is the center of a 3½″ sphere. Design for this sphere a close-fitting casing made from eight equal cylindrical sections. Draw the front and top views of the casing and show the development of one of the sections.

**2.** O326 is the center of a 3½″ sphere. Design for this sphere a close-fitting jacket made from twelve equal cylindrical sections. Show the front and top views of the jacket, and develop one of the sections.

**3.** C226 is the center of a 3″ sphere. Design for the sphere a close-fitting casing made of six equal cylindrical sections. Show the top, front, and right-side views of the casing, and the development of one of the sections.

**4.** Develop the surface of a 2½″ sphere by the zone method. Divide the surface into three pairs of cones. Show the front and top views of the approximate sphere with center at C226. Show also the development of one of each pair of cones.

**5.** The center of a 3″ sphere is at O626. Design for this sphere a casing consisting of a cylindrical equatorial section, two pairs of conical sections, and a pair of circular discs at the poles. Draw the front and top views of the casing, and show the developments for the conical sections.

**6.** C236 is the center of the lower half of a 4″ sphere. This hemisphere is to be made from four hammered zone sections. Show the front and top views of the hemisphere before it is hammered. Show also the patterns for the bottom section and the section next to it.

## Group 97. Ellipsoid.

**1.** A246 B206 is the major axis of an ellipse. The minor axis is 2″ long. In the front, top, and right-side views, show the ellipsoid that is generated by revolving this ellipse about its major axis. Show also the intersections of the ellipsoid with three frontal planes, respectively $\frac{1}{4}$″, $\frac{1}{2}$″, and $\frac{3}{4}$″ in front of the axis.

**2.** The point C626 is the center of a frontal ellipse. The major axis is $3\frac{1}{2}$″ long and horizontal, and the minor axis is $1\frac{1}{2}$″ long. In the front, top, and left-side views, show the ellipsoid that is generated by revolving this ellipse about its minor axis. Also show the intersections of the ellipsoid with three frontal planes, respectively $\frac{1}{2}$″, 1″, and $1\frac{1}{2}$″ in front of the center.

**3.** A406 B446 is the major axis of a prolate ellipsoid. The minor axis is $2\frac{1}{2}$″ long. In the front and top views, show all of the ellipsoid that lies below the plane J217 K504 L646.

**4.** C126 D326 is the minor axis of an oblate ellipsoid. The major axis is 4″ long. In the front, top, and right-side views, show the intersection of this ellipsoid with the plane Q104 R406 S047.

## Group 98. Annular Torus.

**1.** C615 is the center of a 2″ frontal circle. This circle generates an annular torus when revolved about the line A425 B405. Find the intersection of the torus with the plane D426 E328 F204.

**2.** C$2\frac{1}{2}$,2,6 is the center of a 1″ horizontal circle that revolves about the line A427 B425 and generates an annular torus. In the front and top views, show the intersection of the torus with the plane D538 E817 F626.

**3.** O326 is the center of a $1\frac{1}{2}$″ frontal circle. This circle revolved about the line A436 B416 generates an annular torus. C404 D408 is the axis of a $5\frac{1}{2}$″ right circular cylinder. Find the intersection of the torus and the cylinder.

**4.** C226 is the center, and X445 is a point on the axis of a 3″ circle. This circle is the path of the center of a 1″ circle that revolves about C X and generates an annular torus. Show the front, top, and right-side views of the torus.

**5.** A connecting rod of $1\frac{1}{4}$″ diameter has a stub end of $1\frac{1}{2}$″ by $2\frac{1}{2}$″ rectangular cross-section. The rod and end are joined by a turned fillet of 3″ radius. Show the curves where the fillet joins the flats of the stub end. Show the front, top, and end views of the stub end.

**6.** A connecting rod, $1\frac{1}{2}$″ in diameter, has a stub end of 2″ by 3″ rectangular cross-section. The rod and the stub end are joined by a turned fillet of 2″ radius. In the front, top, and end views, show the finished drawing of the stub end.

## Group 99. Surfaces of Revolution.

**1.** $C4,2\frac{1}{2},6$ is the center of a 3″ sphere. A406 is the center of the $3\frac{1}{2}''$ horizontal base of a right circular cylinder that is $\frac{1}{4}''$ tall. The cylinder and the sphere are parts of an ornamental turned knob. Make an attractive design for the knob in the front view. Two frontal planes, respectively $\frac{1}{2}''$ and 1″ in front of the axis of the knob, represent glued joints. Show where these joints will be seen on the surface in the front view.

**2.** C407 is the center of a $2\frac{1}{2}''$ horizontal circle. This circle is the base of a turned vase that is 5″ tall. Design the front view of the vase, and show the intersection of the surface with four frontal planes spaced each $\frac{1}{4}''$ in front of C.

**3.** C426 is the center of a 4″ ball. Let A547 B507 be the center line of the heart wood of the block from which the ball is turned. In the top view, show the annular rings of the wood by means of cylinders spaced $\frac{1}{2}''$ apart. In the front view show the grain of the wood on the surface of the ball.

**4.** The tip of a pointed round dome is at V646. The base is represented by a 2″ horizontal circle with center at C606. The largest horizontal section is represented by a 4″ circle $1\frac{1}{2}''$ above C. Design the outline of the dome in the front, top, and left-side views, and show the lines on the dome where it intersects six equally spaced planes that pass through the axis.

**5.** C028 is the center of an arc that is the upper right one-fourth of a 4″ frontal circle. This arc, revolving around the axis A448 X418, generates a collar that is to be approximated by twenty-four equal cylindrical sections. Draw the front view and one-half of the top view of the collar, and show the development of one of the equal sections.

**6.** $C1\frac{1}{2},4,8$ is the center of the lower half of a 3″ frontal circle. This arc, revolving around the axis A448 X428, generates the approximate surface of an annular cup that is to fit around a 2″ cylinder. The cup is to be made from sixteen equal cylindrical sections. Draw the front view and one-half of the top view of the cup, and make the pattern for the sections.

**7.** C215 is the center of a $1\frac{3}{4}''$ frontal circle that represents the section of a rubber tire. A405 B425 is the center line of the axle. Centered in the front view is a square having a $\frac{3}{4}''$ vertical diagonal. This square represents a molded lug that projects $\frac{1}{8}''$ and is continued around the tread. Complete the front and top views of the tire.

**8.** Flat-formed knives are mounted on the sides of the square shaft of a sticker. These knives cut molding from strips of wood that are forced between the knives and the horizontal table of the machine. The shaft is 2″ square, and its center line is 3″ above the table. Design the cross-section of a simple molding $2\frac{1}{2}''$ wide and $1\frac{1}{2}''$ thick, and make a drawing to show the shape of the knife necessary to cut the molding. This problem may be considered as the intersection of a surface of revolution and a plane. The surface of revolution is generated by revolving the curved outline of the molding about the axis of the shaft. The plane is the cutting face of the knife.

**9.** A cast-iron hand wheel, 10″ outside diameter, has a rim of $1\frac{1}{4}''$ circular cross-section. The hub fits a $\frac{3}{4}''$ shaft and is $1\frac{1}{2}''$ in diameter by $1\frac{1}{2}''$ long, and its nearest end is $3\frac{1}{2}''$ from the middle plane of the rim. Six curved tapering arms of rectangular cross-section connect the rim with the hub. Design the arms and show the complete end and side views of the hand wheel. Scale $6'' = 1'$.

**10.** A cast-iron hand wheel has a rim of $1\frac{1}{8}''$ circular cross-section and $8''$ outside diameter. The cylindrical hub is $1\frac{3}{4}''$ diameter by $1\frac{1}{2}''$ long. The nearest end of the hub is $1\frac{1}{2}''$ from the middle plane of the rim. Three tapering curved arms of rectangular cross-section, $\frac{1}{2}''$ by $1''$ at one end and $\frac{3}{4}''$ by $1\frac{1}{4}''$ at the other end, connect the rim to the hub. Show the front view of the hand wheel in section, and one-half of the top view. Scale full size.

**11.** A symmetrical casting consists of a $5''$ by $10''$ cylinder located centrally on a $16''$ by $2''$ cylindrical disc. A $2\frac{1}{2}''$ diameter axial hole goes through both cylinders. Eight braces, or webs, are cast in the angle between the cylinders. The webs are $1''$ thick and the outside edges are concave circular arcs of $12''$ radius. Show the front and top views of the casting. Scale $3'' = 1'$.

## Group 100. Spring and Serpentine.

**1.** The outside diameter of a left-hand coiled spring is $4''$, and the lead is $\frac{3}{4}''$. The spring is made from wire $\frac{1}{2}''$ in diameter. Show the front half of the top view, and in the front view show five and one-half turns of the spring.

**2.** A heavy coiled spring is wound right handed from stock $\frac{3}{4}''$ in diameter. The lead is $1''$, and the outside diameter is $3\frac{1}{4}''$. In the front and end views, show three and one-half turns of the spring.

**3.** A316 and B5,$4\frac{1}{2}$,6 are the ends of three and one-half turns of a left-hand helix. The axis of the helix is vertical. The helix is the center line of the coils of a spring that is made from round wire $\frac{3}{8}''$ in diameter. Draw the front and top views of the spring.

**4.** C467 D407 is the axis of a $1''$ right circular cylinder. On the cylinder are two right-hand helices of $12''$ lead. The upper end of one helix is $\frac{1}{2}''$ to the right of C, and the upper end of the other is $\frac{1}{2}''$ to the left of C. The center of a $1''$ sphere follows these helices, and the sphere generates a serpentine. Draw the front and top views of the serpentine.

**5.** A446 B406 is the axis of a right circular cylinder. On the cylinder are two left-hand helices of $8''$ lead. One helix passes through a point $1''$ to the right of A, and the other passes through a point $1''$ to the left of A. The center of a $2''$ sphere follows the helices, and the sphere generates a serpentine. Draw the front and top views of the serpentine.

**6.** The center of a $1''$ sphere follows a left-hand helix of $3''$ diameter and $2''$ lead, and the sphere generates a serpentine. Show the front view, and one-half of an end view of two and one-half turns of the serpentine.

# PROBLEMS—CHAPTER XIII

## MINING PROBLEMS

### Group 101. Mining Problems.

In the following problems, the scale of the drawings is to be taken as 1 inch = 100 feet. Letter the words "strike," "dip," "shaft," "tunnel," etc., in their proper places as an aid in interpreting the drawing. Lengths and depths should be stated in feet and tenths of a foot. Slopes are stated as a ratio in which the first figure is measured horizontally and the second vertically. The views may overlap when necessary. In practice, the scale of the drawing should be large enough so that the results may be determined with a sufficient degree of accuracy.

**1.** A445, B837, and C608 are points of outcrop in the center of a vein. Show the strike through B, and find the angle of dip.

**2.** Points of outcrop in the foot wall of an ore vein are located at C105, D328, and E536. Show the strike through D, and the angle of dip.

**3.** The points B138, C414, and D525 are located as points of outcrop in the hanging wall of a bed of coal. Show the strike of the bed through the point D. Show also the angle of dip.

**4.** A thin vein of ore outcrops at C224, D418, and E736. Show the strike and dip. At S645 a vertical shaft is to be sunk to the vein. What will be the depth of this shaft in feet?

**5.** Points in the foot wall of a vein are located at D317, E628, and F746. T634 is to be the mouth of the shortest possible horizontal tunnel to reach the ore. Show the tunnel in all views. State its length in feet. Also show the strike and dip.

**6.** E025, F134, and G208 are points of outcrop in the hanging wall of a bed of coal. S347 is the mouth of a proposed vertical shaft. Show the strike through E, the angle of dip, and the shaft. State the depth of the shaft in feet.

**7.** Points of outcrop of a vein are located at D335, E518, and F027. Show the strike and dip. T1,$2\frac{1}{2}$,4 is to be the mouth of an incline. P326 is a point on the center line of the incline. Locate the point S where the incline pierces the ore body, and state its length.

**8.** A405, B626, and C138 are points of outcrop in one wall, and D526 is a point of outcrop in the other wall of a vein. Show the strike, dip, and thickness of the vein. Show the shortest incline shaft from T025 to the ore body. State the per cent grade of the shaft.

**9.** A434, B725, and C806 are points of outcrop in the foot wall of a thick vein. At D616 is an outcrop in the hanging wall. Show the strike, dip, and cross-section of the vein. State the thickness of the vein in feet. E427 is to be the mouth of the shortest tunnel on a 5 per cent up-grade to reach the vein. Show it in all views and state its length.

**10.** C425, D647, and E016 are points of outcrop in the foot wall of a vein. At P337 is an outcrop in the hanging wall. T034 is the mouth of a tunnel. The tunnel is to be on a 5 per cent up-grade, and is to be as short as possible. Locate S where the tunnel strikes the ore body. State the length of the tunnel, and show it in all views. Show also the thickness of the vein, the strike, and the dip.

**11.** One wall of a vein of ore outcrops at A718, B605, and C326. P517 is the location of an outcrop in the other wall. S826 is to be the mouth of the shortest incline shaft that leads to the ore body. Show the views of the shaft, and state its length and per cent grade. Show also the strike, dip, and thickness of the vein.

**12.** A426 is a point of outcrop in the center of a vein. The vein strikes N 67½° E, and the dip is 60° to the north. The vein is 24′ thick. Show the strike, dip, and thickness of the vein. In the top view show the points C30X, D61X, and E73X in the center of the vein.

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**13.** Make an approximate copy of the contour map of Fig. 141 on the upper half of the problem space. At J646, K235, and L416 are traces of a thin vein of ore. Locate the probable line of outcrop on the map. Show also the direction of the strike, and find the angle of the dip.

**14.** In the top half of the problem space draw a contour map approximately similar to that shown in Fig. 141. The points A235, B547, and C314 are points of outcrop of the hanging wall of a vein. On the 100′ contour, 50′ from C and to the north and east, is an outcropping of the foot wall of the same vein. Show the lines of outcropping of both walls of the vein. Show also the thickness and dip of the vein.

**15.** On the upper half of the problem sheet copy approximately the contours of Fig. 141. T448 is the mouth of a tunnel. A horizontal trestle 50′ long extends south from T. The angle of repose of the material dumped from the end of the trestle is 1½ in 1. Show the outline of the dump in the front and top views.

**16.** In the upper half of the problem space draw a contour map approximately like the one in Fig. 141. S657 is the mouth of a shaft from which is dumped the mine refuse that has a normal slope of 1¼ in 1. On the map draw the outline of the dump. Show also the increased size of the dump if the material is dumped from the edge of a flat of 50′ radius centered at the shaft mouth.

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**17.** On the upper half of the problem sheet draw a contour map like that shown in Fig. 142. P626 is a point of outcrop of a thin vein. The strike is N 75° W. The dip is 22½° to the SW. Locate the probable line of outcrop.

**18.** On the upper half of the problem sheet draw a contour map similar to the one shown in Fig. 142. At A727 a vertical bore hole strikes the hanging wall of a vein of ore at 60′, at B645 at 20′, and at C126 at 110′. The drill passes through the vein in a distance of 28′. Determine the strike, angle of dip, and thickness of the vein. Show also the probable line of outcrop of the hanging wall.

**19.** On the upper half of the problem space draw a map similar to the one shown in Fig. 142. One thin vein of ore outcrops at the points J626, K238, and L314; and a second thin vein outcrops at C104, D746, and E328. Show the strike of each vein and the probable lines of outcrop. Also find the front and top views of the line of intersection of the two veins. Enrichment of the ore may be expected along this line.

**20.** On the top half of the problem sheet draw a contour map similar to the one shown in Fig. 142. The property line passes through P626 and bears N 52½° W. For convenience in construction it should be noted that this angle is halfway between 45° and 60°. The property NE. of this line belongs to another company. On the 300′ level, west of the property line, locate a point D where the mine refuse may be dumped so that the limits of the dump may come just to the property line. The material hangs up at a slope of 2 in 1¾. Show the outlines of the dump.

# PROBLEMS—CHAPTER XIV

## PERSPECTIVE DRAWINGS

### Group 102. Parallel Perspective of Straight-line Objects.

**1.** A225 D525 is the diagonal of a frontal hexagon. This hexagon is the front end of the hole through a hexagonal pipe that is 3″ long and has walls ⅛″ thick. Show the picture of this pipe on a frontal plane 1″ in front of A when viewed from E3,2½,0.

**2.** A335 B535 C505 D305 is the front end of a pipe that is 3″ long. Show the picture of this pipe as seen on a frontal plane 1″ in front of A when viewed from E620.

**3.** A335 B535 C505 D305 is the front end of the outside of a pipe that is 3″ long. The walls are ¼″ thick. Show the perspective view of this pipe as seen on a frontal picture plane 1″ in front of A when viewed from E220.

**4.** A225 D525 is the diagonal of a frontal hexagon. This hexagon is the front end of the outside of a pipe that is 3″ long. The walls are ¼″ thick. Make a perspective drawing of this pipe on a frontal plane through A when the point of view is at E3,2½,0.

**5.** A306 D506 is the front edge of a horizontal square. Four other similar squares are located respectively 1″, 2″, 3″, and 4″ directly above the first square. Show the views of these four squares on a frontal picture plane through A when seen from S412.

**6.** A326 C528 is the diagonal of a horizontal square. The square is the base of a right prism of ¾″ altitude. One-quarter inch inside the sides of the top face of the prism is the square base of a right pyramid of 1½″ altitude. The pyramid stands on the prism. Make a perspective drawing of this object on a frontal picture plane through A. The eye is at E240.

**7.** A238, B438, C437, D337, E336, and F236 are the upper corners of an L-shaped block that is 2″ high. Show the perspective view of this block, as seen from V543, on a frontal picture plane that passes through E.

**8.** A228 B528 is one edge of a horizontal square that is the lower base of a right prism 1″ tall. Divide the prism into three equal parts by using profile planes, and discard the front two-thirds of the middle third of the prism. Make a perspective drawing of the U-shaped block that remains. The frontal picture plane is taken 2″ in front of A. The observer is at P3¾,3¾,1.

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### Group 103. Parallel Perspective of Circular Objects.

**1.** A425 X428 is the axis of a tube of 2″ outside diameter and 1½″ inside diameter. Show the view of the tube on a frontal picture plane 1″ in front of A, when viewed from V630.

**2.** C427 D428 is the axis of a ring of 4″ outside diameter and $2\frac{1}{2}$″ inside diameter. It is viewed from X310, and the frontal picture plane is $\frac{1}{2}$″ is front of C. Make the perspective drawing of the ring.

**3.** C426 D428 is the axis of a pipe of 3″ outside diameter and 2″ inside diameter. Show the perspective drawing of the pipe when the station point is at E201 and the frontal picture plane is 1″ in front of C.

**4.** A217 B517 is the lower edge of a frontal square. This square is the front face of a rectangular block that is 1″ deep. A $1\frac{1}{2}$″ circular hole, centered on the front face, is cut through the block. Show the view of the block, as it is seen from S651, on a frontal picture plane 1″ in front of A.

**5.** C338 D638 is one edge of a $1\frac{1}{2}$″ by 3″ horizontal rectangle. The rectangle is the upper face of a right prism that is 1″ tall. Divide the prism into three equal parts by means of profile planes. Discard the front half of the middle third of the prism. Show a $\frac{3}{4}$″ hole drilled from front to back through the longer legs of the U-shaped block. Make a perspective drawing of this piece. The frontal picture plane passes through C, and the station point is at S$3\frac{1}{2}$,$3\frac{1}{2}$,1.

**6.** B448 D347 is the diagonal of the horizontal, square, upper base of a right prism that is 3″ tall. Centered respectively on the upper and lower thirds of the front face of the prism are two cylindrical bosses $\frac{3}{4}$″ in diameter by 2″ long. Make a perspective drawing of this magnet core on a frontal picture plane through D when viewed from E$2\frac{1}{2}$,2,0.

**7.** A238 C227 is the diagonal of the profile, square, left end of a right prism that is 3″ wide. Centered respectively on the right and left thirds of the front face of the prism are two cylindrical bosses $\frac{3}{4}$″ in diameter by 2″ long. Make a perspective drawing of the specified object on a frontal plane through A. Take the station point at S340.

**8.** C428 is the center of a 3″ frontal circle. The circle is the rear face of a right cylindrical disk that is $\frac{3}{4}$″ long. A cylindrical hub, 1″ in diameter by $1\frac{1}{4}$″ long, is mounted centrally on the front face of the disk. Six equally spaced $\frac{5}{8}$″ holes are drilled through the disk parallel to the axis, and centered 1″ from the axis. Show the view of this piece on a frontal picture plane through C when viewed from V301.

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## Group 104. Angular Perspective of Straight-line Objects.

**1.** A306 C607 is the diagonal of a horizontal square. Other similar squares are located at the 1″, 2″, 3″, and 4″ levels directly above the first square. Show the views of these squares when seen from E$4\frac{1}{2}$,2,3 on a frontal picture plane through the front corners of the squares.

**2.** A416 C717 is the diagonal of a horizontal square. Three other similar squares are located 1″, 2″, and 3″ directly above the first square. Make perspective drawings of the four squares on the frontal picture plane through A. The eye is at E623.

**3.** A336 C736 is the diagonal of a horizontal square. Two inches directly above and two inches directly below this square are similar squares. Show the appearance of the three squares, on a frontal picture plane 1″ behind A, when viewed from E620.

**4.** A316 C617 is the diagonal of a horizontal square. Two other parallel squares of the same size are located, respectively, 2″ and 4″ directly above the first square. Show the appearance of the three squares on a frontal picture plane 1″ behind C when the observer is at E$4\frac{1}{2}$,2,0.

**5.** A326 C627 is the diagonal of a horizontal square. Show the appearance of the square on a frontal picture plane through D, when viewed from the point E503, and from points 1″, 2″, 3″, and 4″ above E.

**6.** To show the effect of varying the distance of the station point from the object, make three perspective drawings of a cube as follows: A$4\frac{1}{4}$,0,7 C$4\frac{1}{4}$,0,8 is the diagonal of a horizontal square. The square is the base of a cube. The frontal picture plane is taken through A. Show the picture of the cube when seen from E413. Elevate the eye and the cube 1″, move the eye 2″ to the rear, and show the picture from this position. Elevate the eye and the cube 2″ above the first positions, move the eye $3\frac{1}{2}$″ to the rear, and show the picture from this position.

**7.** To show the effect of moving the frontal picture plane, make three perspective drawings of a cube as follows: A407 C408 is the diagonal of a horizontal square. The square is the base of a cube. E$3\frac{3}{4}$,1,4 is the point of sight. Take the picture plane through the front edge of the cube. Elevate the cube and the eye 1″ and take the picture plane halfway between the front and the rear edges of the cube. Elevate the cube and the eye 2″ more, and take the picture plane through the rear edge of the cube.

**8.** To show the effect of changing the elevation of the point of view, make four perspective drawings of a cube as follows: A$4\frac{1}{4}$,0,7 C$4\frac{1}{4}$,0,8 is the diagonal of a horizontal square. This square is the base of a cube. The frontal picture plane is taken through A. Show the picture when seen from E403. Elevate the cube 1″, elevate the eye 2″, and show the corresponding picture. Elevate the cube 1″ more, elevate the eye to the 4″ level, and show the corresponding picture. Elevate the cube 1″ more, elevate the eye to the 6″ level, and show the picture that is seen from this level.

**9.** A616 C316 is the diagonal of a horizontal square that is the base of a cube. Four walls of the cube are $\frac{1}{4}$″ thick, and two opposite vertical faces are open. Show the view of the hollow cube on a frontal picture plane 2″ in front of A when seen from E541.

**10.** A616 C316 is the diagonal of a horizontal square. This square is the base of a cube. A centrally located recess $1\frac{1}{2}$″ square by $\frac{1}{4}$″ deep is cut into the vertical faces of the cube. Show the picture of the recessed cube as it would be seen on a frontal plane through P744 when the eye is at E521.

**11.** A335 B534 C504 D305 is the front end of the outside of a rectangular pipe that is 3″ long. The walls are $\frac{3}{16}$″ thick. Show the picture of this pipe as it would appear on a frontal plane through B when seen from E520.

**12.** A335 B534 C504 D305 is the front end of a rectangular pipe that is 3″ long. Show the picture of this pipe as it would appear on a frontal plane through B when seen from E420. .

**13.** The rectangle A117 B318 C$3\frac{1}{2}$,1,7 D$1\frac{1}{2}$,1,6 is the base of a rectangular block that is $\frac{3}{4}$″ high. Temporarily turn the sheet until the line A B in the top view becomes frontal. Now discard the front halves of the left and right thirds of the block, leaving a T-shaped block. A T-shaped hole is cut through the block, leaving the vertical walls ⅛″ thick. Show the appearance of the hollow block on a frontal picture plane through A when viewed from S234.

**14.** A518 B717 is one edge of a horizontal square. Divide this square into sixteen equal squares and make a checkerboard pattern by cross-hatching alternate squares. Show the perspective view of this pattern when viewed from O5½,2,3¼. The frontal picture plane is 1″ in front of B.

**15.** A508 B707 is one edge of the horizontal base of a cube. Cut out of this cube four steps of equal rise and run. Make the treads parallel to A B. Show the view of the steps on a frontal picture plane 3″ in front of A when viewed from O612.

**16.** A107 B308 is one edge of the horizontal base of a cube. Cut out of this cube four steps of equal rise and run. The treads are to be parallel to A B, and are to go up from the front. Show the perspective drawing of the steps on a frontal plane 2½″ in front of A when viewed from E2,3,1½.

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**17.** Discard the front two-thirds of the left half of the rectangle of which A527 C724 is the diagonal and two sides are frontal. On the L-shaped figure that remains design a simple one-story house. Show the views of this house on a vertical picture plane through E408 F800 as seen by an observer O3½,2½,4.

**18.** A1,2½,4 C3,2½,7 is the diagonal of a horizontal rectangle of which two sides are frontal. Discard the front two-thirds of the right one-half of the rectangle. On the remaining L-shaped area design a simple one-story cottage. Draw the front elevation. Show the perspective view of the cottage on a vertical picture plane through K214 L418, when the observer is at P433.

**19.** A238 C436 is the diagonal of a horizontal square. Discard the front half of the right half of the square. On the remaining L-shaped area design the front elevation of a simple one-story house. Show one door and one window. Make a perspective drawing of this house on a vertical picture plane through C F133 when the point of sight is at S544.

**20.** A4,2½,5 C7,2½,7 is the diagonal of a horizontal rectangle of which two sides are frontal. On this rectangle design a plain one-story house having a hip roof. Make a perspective drawing of the house on a vertical picture plane through A K603. The station point is at P333.

**21.** A rectangular block 1″ tall has for its base the horizontal square of which A527 C725 is the diagonal. Discard all of the block excepting the right one-fourth and the rear one-fourth. Find the perspective view of the remaining L-shaped block on the vertical picture plane through K408 L802, when viewed from V4,3½,4½.

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## Group 105. Angular Perspective of Circular Objects.

**1.** C426 D627 is the axis of a 2″ right circular cylinder. The observer is at A5,3½,0, and the frontal picture plane passes through C. Make the perspective drawing of the cylinder.

**2.** A426 X327 is the axis of a 2″ right circular cylinder. Make a perspective drawing of the cylinder on a frontal picture plane $1\frac{1}{2}$″ in front of A. The station point is at S340.

**3.** E426 F406 is the axis of a 3″ right circular cylinder. The point of view is at V330, and the frontal picture plane is taken 2″ in front of the axis. Make a perspective drawing of the cylinder.

**4.** A416 B446 is the axis of a $3\frac{1}{2}$″ right circular cylinder. Show the appearance of this cylinder on a frontal picture plane through the axis when viewed from V420. Show also circles on the cylinder at the 2″ and 3″ levels.

**5.** A436 B446 is the axis of a $3\frac{1}{2}$″ right circular hollow cylinder. The inside diameter is 2″. Make a perspective drawing of the hollow cylinder on a frontal picture plane through A B. The observer is stationed at V220.

**6.** C237 D$2\frac{1}{2}$,3,6 is the axis of a 4″ hollow right circular cylinder. The inside diameter is 2″. Discard the lower half of the hollow cylinder. Form an arch by mounting the upper half on posts that are 1″ square and 3″ tall. Make a perspective drawing of the arch on a frontal picture plane 2″ in front of D. The station point is at E220.

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## KEY FOR CORRECTING PROBLEMS

**X** Incorrect. Find the mistake. Draw correctly.

**C** Contrast lacking. Make lines of proper weights.

**D** Drawing incomplete. Show all lines.

**L** Letter points as required.

**R** Redraw. Inaccurate, or not neat.

**S** Sharp pencil of proper grade should be used.

**T** Too late. Hand in work on time.

a . . . correct + return . . . . . . . . . . .

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